THERMAL RADIATION SUMMARY

(Rees Chapter 2)

Planck's Law describes the amplitude of radiation emitted (i.e., spectral radiance) from a black body. It is generally provided in one of two forms; $L_\lambda(\lambda)$ is the radiance per unit wavelength as a function of wavelength λ and $L_\nu(\nu)$ is the radiance per unit frequency as a function of frequency ν. The first form is

$$L_\lambda(\lambda) = \frac{2hc^2}{\lambda^5}\left[exp\,\frac{hc}{\lambda kT} - 1\right]^{-1} \quad \text{where}$$

T	-	temperature	
c	-	speed of light	2.99 x 10^{-8} m s^{-1}
h	-	Planck's constant	6.63 x 10^{-34} J s
k	-	Boltzmann's constant	1.38x10^{-23} J °K^{-1}
L_λ	-	spectral radiance	W m^{-3} sr $^{-1}$
L_ν	-	spectral radiance	W m^{-2} Hz^{-1} sr $^{-1}$

To relate the two forms and establish L_ν one takes the derivative of L with respect to ν using the chain rule $\frac{\partial L}{\partial \nu} = -\frac{\partial L}{\partial \lambda}\frac{\partial \lambda}{\partial \nu}$. Note that $\lambda = c/\nu$ so that $\frac{\partial \lambda}{\partial \nu} = -\frac{c}{\nu^2}$ and finally

$$L_\nu(\nu) = \frac{2h\nu^3}{c^2}\left[exp\,\frac{h\nu}{kT} - 1\right]^{-1}$$

The **Stefan-Boltzmann Law** gives the total black body radiance as a function of the temperature T. One can derive this law by integrating the spectral radiance over the entire spectrum. This is left to the reader as an exercise.

$$L = \int_o^\infty L_\lambda \, d\lambda \quad = \frac{2\pi^4 k^4}{15\, c^2 h^3}\, T^4$$

or $L = \sigma T^4$ where σ is the Stefan-Boltzmann constant (5.67 x 10^{-8} W m^{-2} °K^{-4} sr^{-1}).

Wein's Law provides the wavelength (or frequency) there the spectral radiance has maximum value. This can be found by taking the derivative of L_λ with respect to wavelength and determining

47–3 Planck's Radiation Formula

A theoretical explanation for the cavity radiation was the outstanding unsolved problem in physics during the years before the turn of the present century. A number of capable physicists advanced theories based on classical physics, which, however, had only limited success. Figure 47–5, for example, shows the theory of Wien; the fit to the experimental points is reasonably good, within the experimental error of the data, but definitely not exact. Wien's formula is

$$\mathcal{R}_\lambda = \frac{c_1}{\lambda^5} \frac{1}{e^{c_2/\lambda T}},$$

where c_1 and c_2 are constants that must be determined empirically by fitting the theoretical formula to the experimental data.

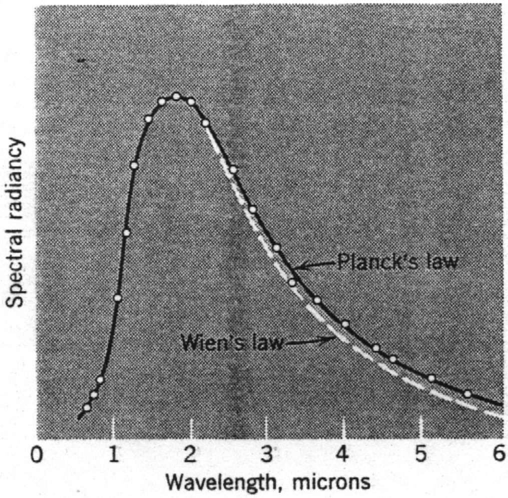


Fig. 47–5 The circles show the experimental spectral radiancy data of Coblentz for cavity radiation. The theoretical formulas of Wien and Planck are also shown, Planck's providing an excellent fit to the data.

In 1900 Max Planck pointed out that if Wien's formula were modified in a simple way it would prove to fit the data precisely. Planck's formula, announced to the Berlin Physical Society on October 19, 1900, was

$$\mathcal{R}_\lambda = \frac{c_1}{\lambda^5} \frac{1}{e^{c_2/\lambda T} - 1}. \qquad (47\text{–}6)$$

This formula, though interesting and important, was still empirical at that stage and did not constitute a theory.

Planck sought such a theory in terms of a detailed model of the atomic processes taking place at the cavity walls. He assumed that the atoms that where this function is zero. This is another excellent exercise; after some algebra you should arrive at the following transcendental equation

$$1 - e^{-\gamma} = \frac{\gamma}{5} \;\Rightarrow\; \gamma = 4.965$$

where

$$\gamma = \frac{hc}{kT\lambda_{max}}.$$

The more common form is $\lambda_{max} = C_w/T$ where $C_w = 2.898 \; x \; 10^{-3}$ ˚Km. Note that one could perform an experiment to measure the total radiance from a black body and establish the Stefan-Boltzmann constant σ. Similarly one could determine the wavelength for maximum black body output to estimate Wein's constant C_W. Then with a knowledge of these two constants one could estimate Planck's constant h and Boltzmann's constant k without every doing any quantum measurements!

The **Rayleigh-Jeans Approximation** provides a convenient and accurate description for spectral radiance when for wavelengths much greater than the wavelength of the peak in the black body radiation formula. To derive the Rayleigh-Jeans approximation, expand the exponential in the denominator of Planck's Law in a Taylor series about zero argument; this is a good appropriation when $\lambda >> \lambda_{\mathrm{max}}$. This is a third exercise left to the reader. The approximate formula is

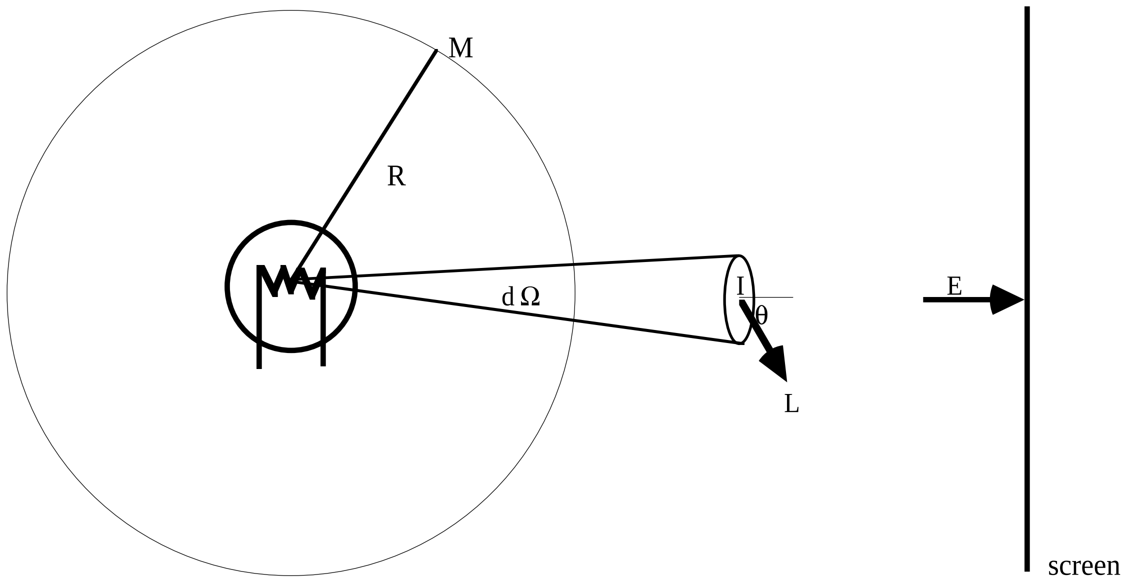
$$L_\lambda = \frac{2kcT}{\lambda^4} \;\; or \;\; L_\nu = \frac{2kT\nu^2}{c^2}.$$

This approximation is better than 1% when $\lambda T > 0.77$ m K. For example, for a body at 300˚K, the approximation is valid when $\lambda > 2.57$ mm; in other words this approximation is good when viewing thermal emissions from the Earth over the microwave band. Microwave radiometers can measure the power received L_λ at an antenna. This is sometimes called the brightness temperature and it is linearly related to the physical temperature of the surface T_p. The Rayleigh-Jeans approximation provides a simple linear relationship between measured spectral radiance and surface temperature as long as the emissivity ε of the surface is known or, in the case of sea ice, one knows the temperature of the surface so the emissivity of the ice can be estimated.

$$L_{measured} = \varepsilon \; \frac{2kcT_p}{\lambda^4}$$

Terminology

Consider a 60 W light bulb. An electric current passes through the tungsten filament and heats it to about 3000˚K. Our bulb is perfect in the sense that it radiates all of this energy, perhaps as a gray body.



radiant flux (total)	ϕ		60	W
radiant intensity	I	$d\phi/d\Omega$	$60/4\pi$	W sr^{-1}
radiant exitance	M	$d\phi/dA$	$60/4\pi R^2$	W m^{-2}
radiance (brightness temp.)	L	$\cos\theta\ d^2\phi/(d\Omega dA)$		W sr^{-1} m^{-2}
irradiance	E	$d\phi/dA$		W m^{-2}