Physics of Surfing Waves

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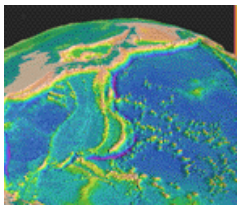
(http://topex.ucsd.edu/ps)



- Physics of waves
- Characteristics of waves
- Generation of waves by storms
- Wave speed - shallow vs. deep ocean
- Sets - dispersion

Exercises: due Oct 11

(each problem is covered in class)

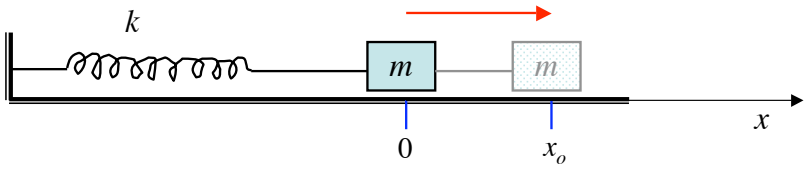
1. Derive the expression for the period of a harmonic oscillator with mass *m* and spring constant *k*.

2. Derive the expression for the speed of a deep water wave in terms of the wave period *T*.

3. What are *sinh, cosh,* and *tanh* in terms of the exponential function? What is *tanh*(10^{-6})? What is *tanh*(10)?

4. Consider two waves of equal height but different period (T_1=12.5 s and T_2=13 s. What is the time between sets? **(we did not cover problem 4 so if you can't do it hand in 1-3.)** Here is a hint: add two cosine functions h(x) = cos(ω_1*t) + cos(ω_2*t) where $\omega_1 = 2*\pi/T_1$,use the trigonometric formula for the sum of two cosines, then interpret or plot the results.

harmonic oscillator



$$m\frac{d^2x}{dt^2} = -kx$$

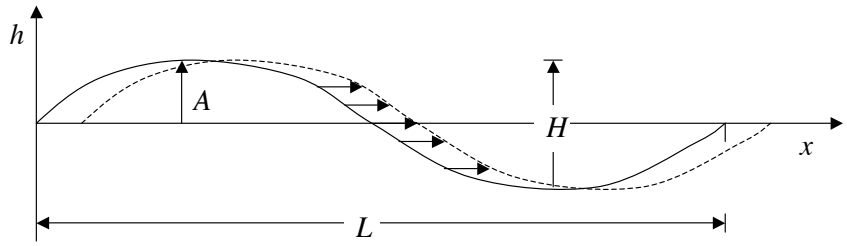
m - mass
k - spring constant

force from acceleration	=	restoring force of spring

guess solution $x(t) = x_o \cos\omega t$

How do we solve for ω?

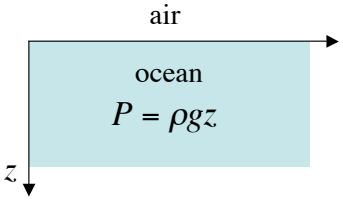
wave characteristics



$$h(x,t) = A\sin\left(\frac{2\pi x}{L} - \frac{2\pi t}{T}\right)$$

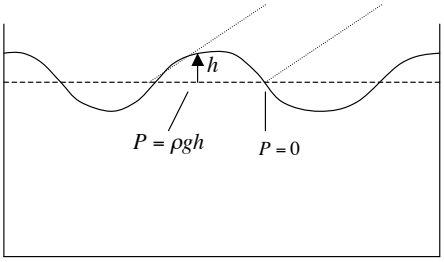
L - wavelength
A - amplitude
H - height
T - period (5 - 18 s)

deep ocean waves



ρ - density (kg m^{-3})
g - acceleration of gravity (9.8 m s^{-2})
z - depth (m)

What are the units of pressure?



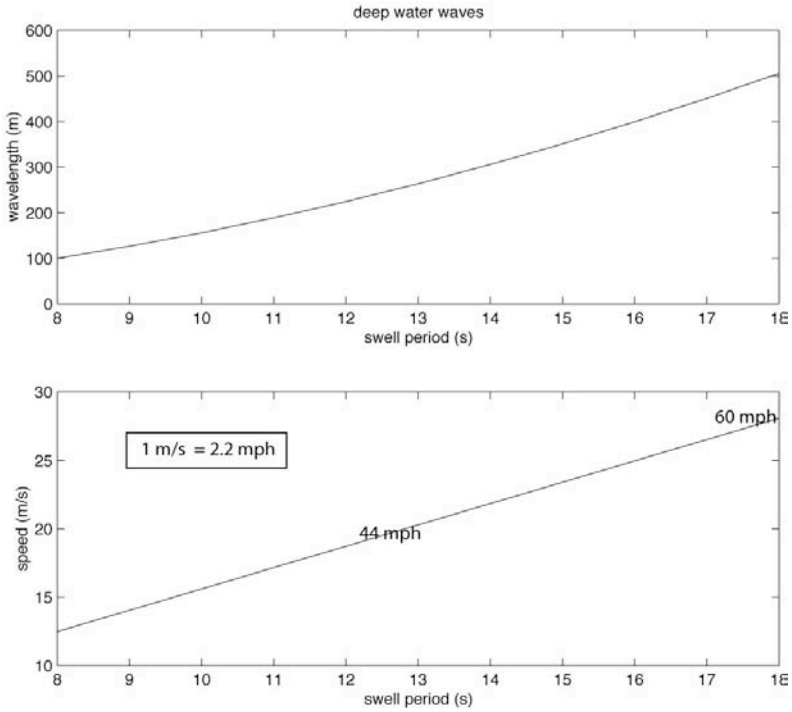
restoring force = $-\rho g h$

acceleration force $= \rho \frac{L}{2\pi}\frac{d^2h}{dt^2}$

$$\rho\frac{L}{2\pi}\frac{d^2h}{dt^2} = -\rho g h$$

guess: $h(t) = A\cos\omega t$

What is ω?



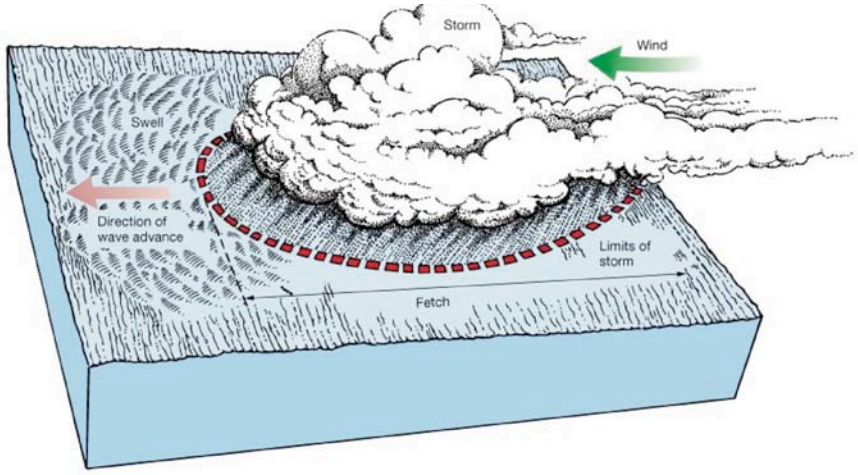
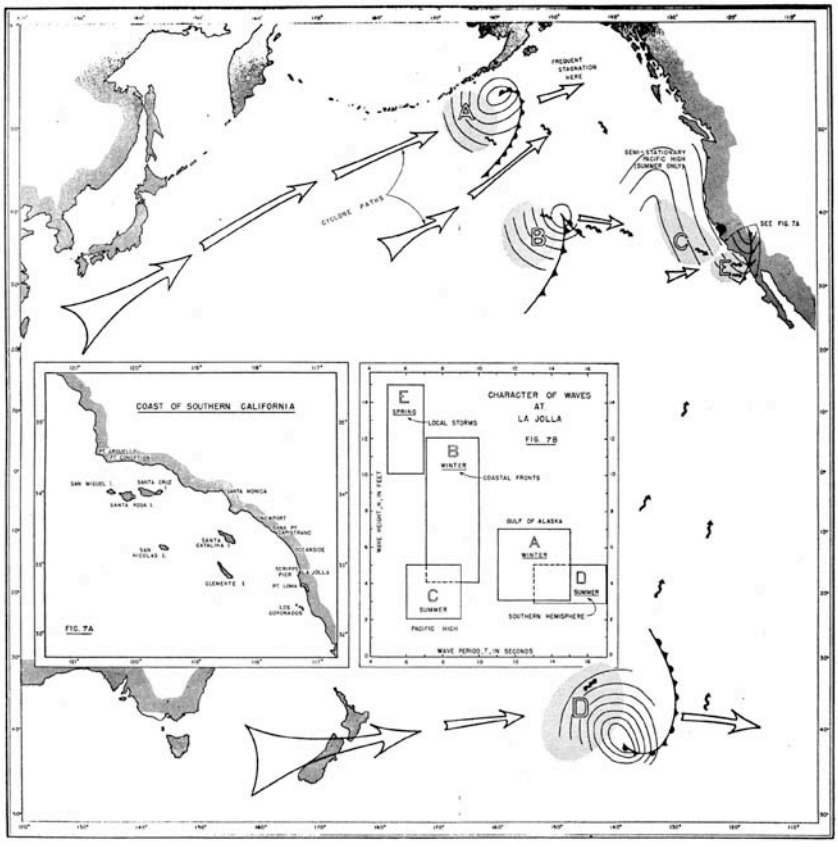
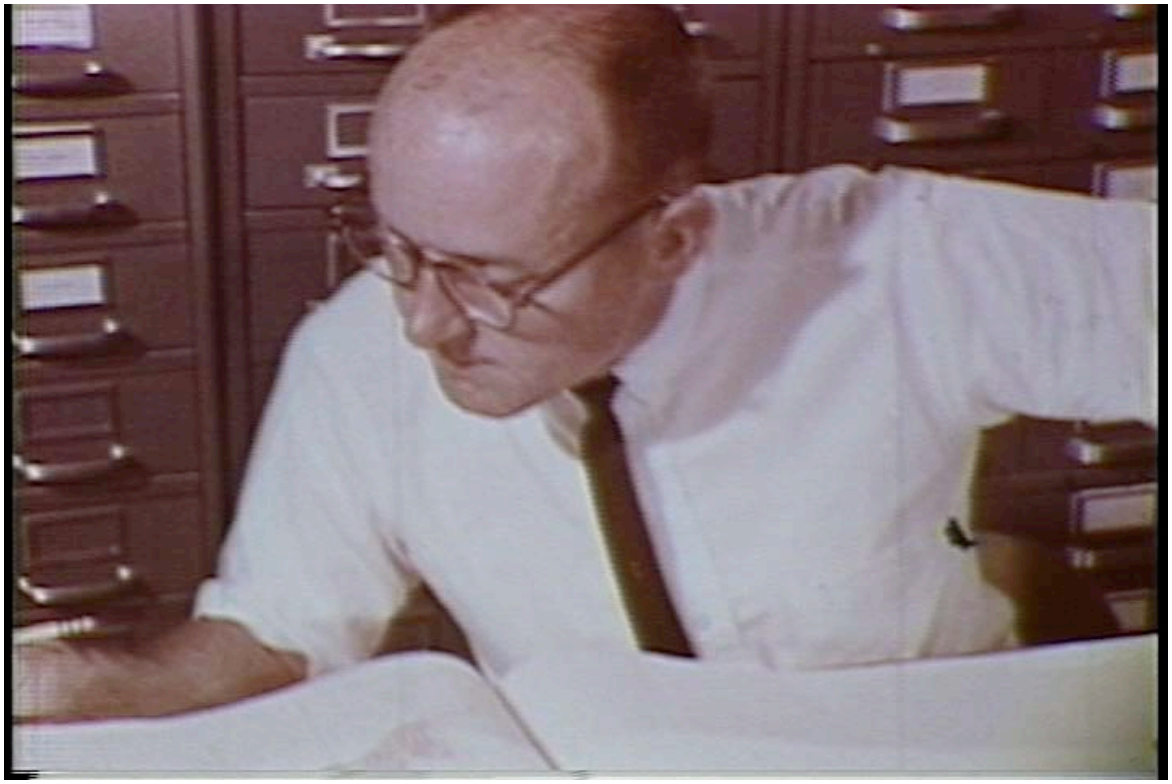


Table 8–1 Description of a fully developed sea for a given wind speed.

Wind speed in km/h (mi/h)	*Average height in m (ft)*	*Average length in m (ft)*	*Average period in sec*	*Highest 10% of waves in m (ft)*
20 (12)	0.33 (1.0)	10.6 (34.8)	3.2	0.75 (2.5)
30 (19)	0.88 (2.9)	22.2 (72.8)	4.6	2.1 (6.9)
40 (25)	1.8 (5.9)	39.7 (130.2)	6.2	3.9 (12.8)
50 (31)	3.2 (10.5)	61.8 (202.7)	7.7	6.8 (22.3)
60 (37)	5.1 (16.7)	89.2 (292.6)	9.1	10.5 (34.4)
70 (43)	7.4 (24.3)	121.4 (398.2)	10.8	15.3 (50.2)
80 (50)	10.3 (33.8)	158.6 (520.2)	12.4	21.4 (70.2)
90 (56)	13.9 (45.6)	201.6 (661.2)	13.9	28.4 (93.2)

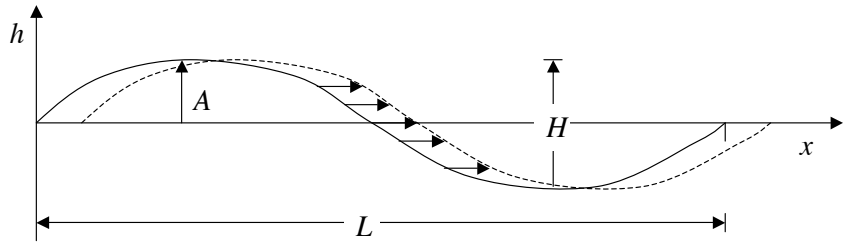
Munk, W. H. and M. A. Traylor,
Refraction of Ocean Waves,
J. Geology, v. LV, No. 1, 1947





wave generation

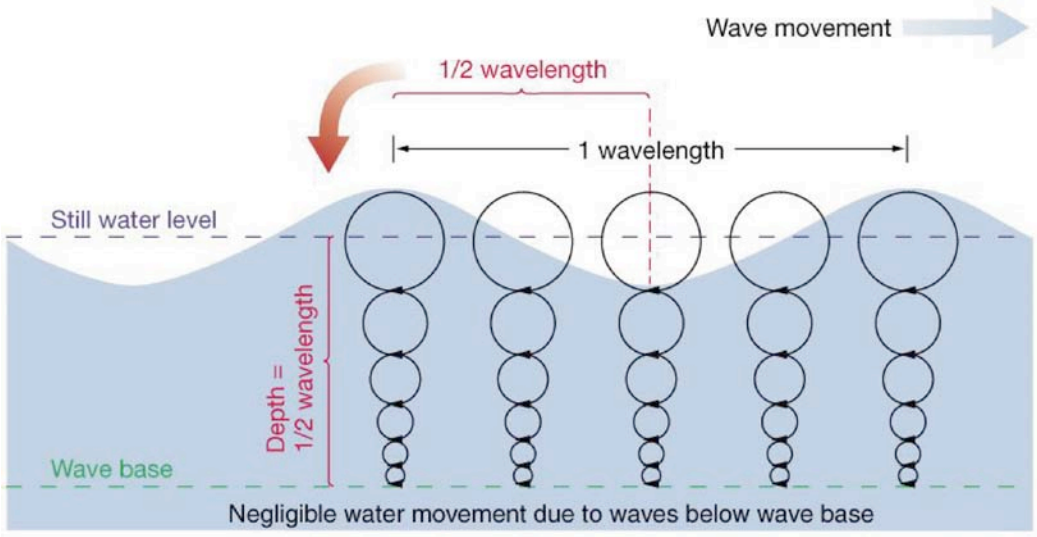
- generated by storms at sea
- far from the storm they are sinusoidal



$$h(x,t) = A\sin\left(\frac{2\pi x}{L} - \frac{2\pi t}{T}\right)$$

L - wavelength
A - amplitude
H - height
T - period (5 - 18 s)

$$c = \frac{L}{T}$$ phase velocity



Airy solution

$$c(d) = \left[\frac{gL}{2\pi}\tanh\left(\frac{2\pi d}{L}\right)\right]^{1/2}$$

L - wavelength
g - acc. gravity
d - ocean depth

deep water waves	shallow water waves
$d >> L/2$	$d << L/2$
$c_d = \sqrt{\frac{gL}{2\pi}}$	$c_s = \sqrt{gd}$

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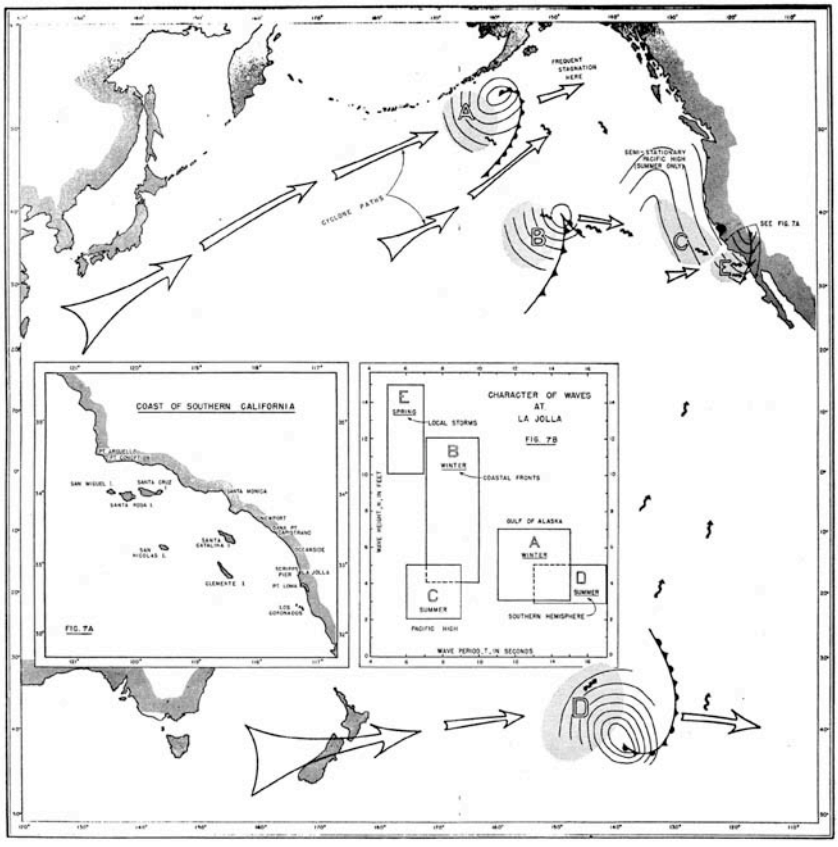
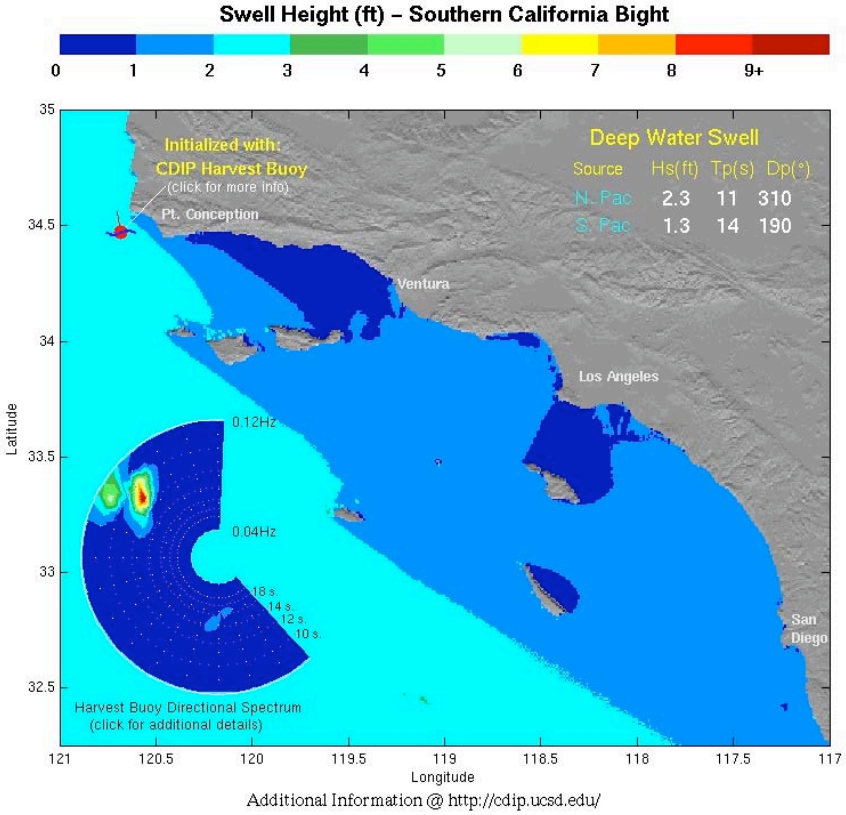
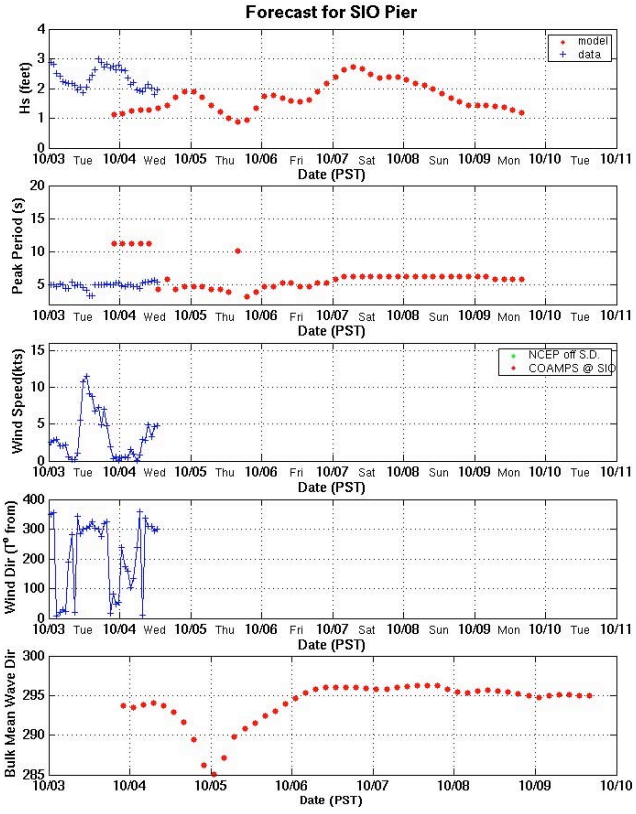


Analysis Time – 4 OCT 2006 : 1323 PST



Additional Information @ http://cdip.ucsd.edu/



Conclusions

- Ocean waves: force of acceleration is balanced by the force of gravity.
- Wind speed >= wave speed. 17-s period waves require wind speed of 27 m/s = 60 mph.
- Wave speed: $d >> L/2$ waves are dispersive; $d << L/2$ speed depends on depth.
- Refraction is important when $d < 10L$.