Chapter 4

Marine Magnetic Anomalies

4.1 Introduction

This chapter develops the equations needed to compute the scalar magnetic field that would be recorded by a magnetometer towed behind a ship, given a magnetic timescale, a spreading rate, and a skewness. A number of assumptions are made to simplify the mathematics. The intent is to first review the origin of natural remnant magnetism (NRM), to illustrate that the magnetized layer is thin compared with its horizontal dimension. Then the relevant differential equations are developed and solved under the ideal case of seafloor spreading at the north magnetic pole. This development highlights the Fourier approach to the solution to linear partial differential equations. The same approach will be used to develop the Green's functions for heat flow, flexure, gravity, and elastic dislocation. For a more general development of the geomagnetic solution, see the reference by Parker [53].

4.2 Crustal Magnetization at a Spreading Ridge

As magma is extruded at the ridge axis, its temperature falls below the Curie point, and the uppermost part of the crust becomes magnetized in the direction of the present-day magnetic field. Figure 4.1, from Kent et al. [37], illustrates the current model of crustal generation. Partial melt that forms by pressure-release in the uppermost mantle (~40 km depth) percolates to a depth of about 2000 m beneath the ridge, where it accumulates to form a thin magma lens. Beneath the lens a mush-zone develops into a 3500 m thick gabbro layer, by some complicated ductile flow. Above the lens, sheeted dikes (~1400 m thick) are injected into the widening crack at the ridge axis. Part of

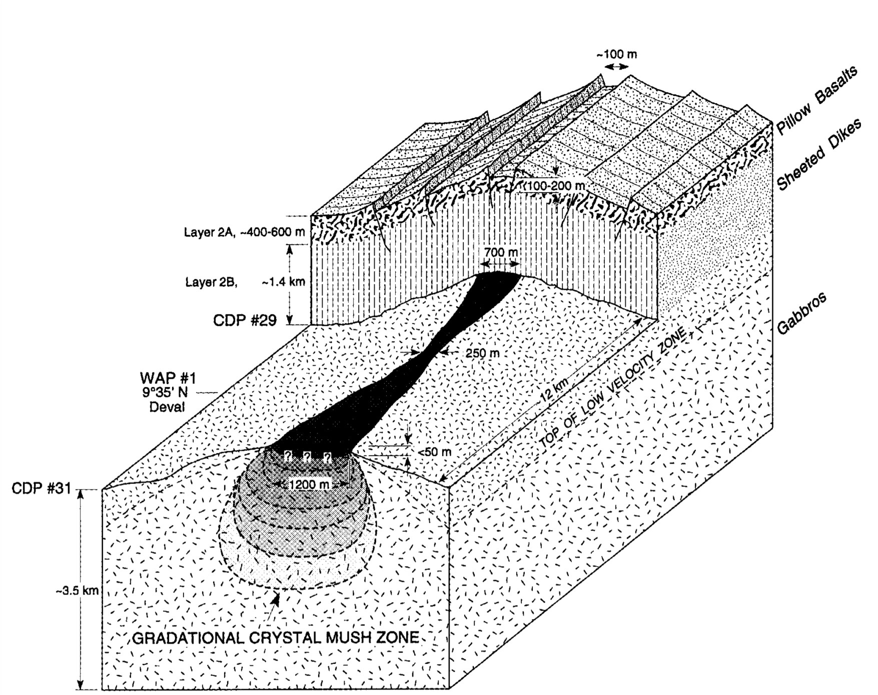


Figure 4.1: Model of crustal structure derived from reflection and refraction seismology [37].

this volcanism is extruded into the seafloor as pillow basalts. The pillow basalts and sheeted dikes cool rapidly below the Curie temperature as cool seawater percolates to a depth of at least 2000 m. This process forms the basic crustal layers seen by reflection and refraction seismology methods.

The highest magnetization occurs in the extrusive layer 2A (Figure 4.2, Table 4.1), although the dikes and gabbro layers provide some contribution to the magnetic anomaly

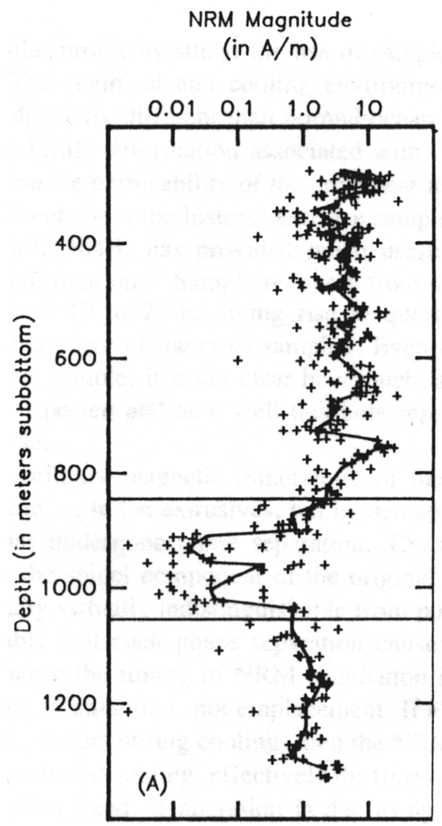


Figure 4.2: NRM values (in A/m) from Hole 504B. Depths are measured from the seabottom and include 274.5 m of sediment. The horizontal lines separate the upper units, the transition zone, and the dike complex. (From [75].)

measured on the ocean surface. Note that the reversals recorded in the gabbro layer do not have sharp vertical boundaries (Figure 4.3). The tilting reflects the time delay when the temperature of the gabbro falls below the Curie point. The sea-surface magnetic-anomaly model shown in Figure 4.3 [28] includes the thickness and precise geometry of the magnetization of all three layers. For the calculation below, we assume all of the magnetic field comes from the thin extrusive layer.

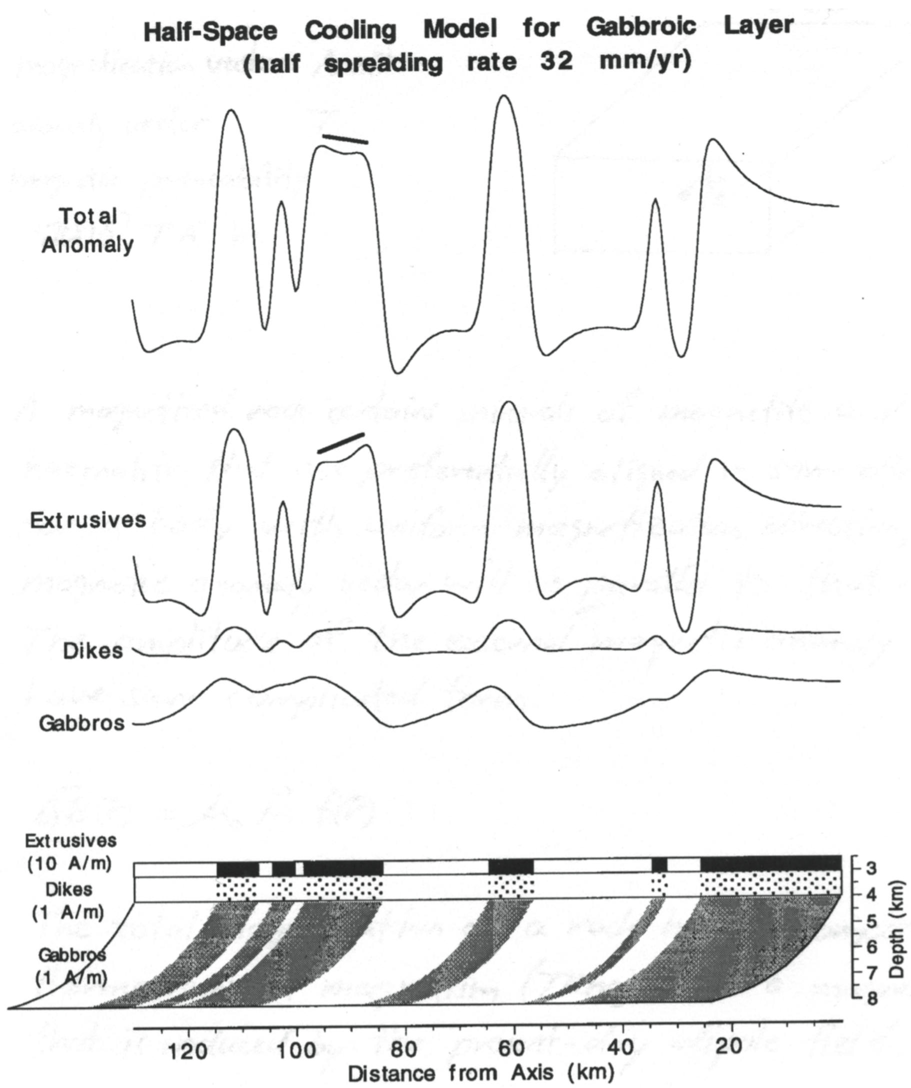


Figure 4.3: Magnetic anomalies generated by a realistic model of crustal magnetization. The primary magnetization signature comes from the thin layer of extrusives. Dipping magnetization in the Gabbros reflects the position of the Curie isotherm at depth away from the ridge axis. (Jeff Gee, personal communication.)

Layers	**Thickness/Velocity**	**Description**	**Thermoremnant Magnetism (TRM)**
Layer 1	variable <2.5 km/s	sediment	N/A
Layer 2A	400–600 m 2.2–5.5 km/s	extrusive, pillow basalts	5–10 A/m
		Magma Lens	
Layer 2B	1400 m 5.5–6.5 km/s	intrusive, sheeted dikes	~1 A/m
Layer 3	3500 m 6.8–7.6 km/s	intrusive, gabbro	~1 A/m

Table 4.1

The other assumptions are:

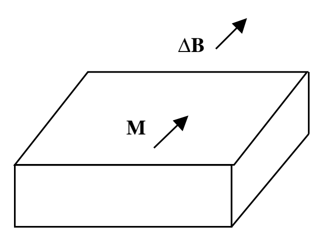
1. The ridge axis is 2-D, so there are no along-strike variations in magnetization.
2. The spreading rate is uniform with time. Before going into the calculation, we briefly review the magnetic field generated by a uniformly magnetized block.

4.3 Uniformly Magnetized Block

M magnetization vector (Am^{-1})

ΔB magnetic anomaly vector (T)

μ_o magnetic permeability ($4\pi \times 10^{-7}$ $TA^{-1}m$)



A magnetized rock contains minerals of magnetite and haematite that can be preferentially aligned in some direction. For a body with a uniform magnetization direction, the magnetic anomaly vector will be parallel to that direction. The amplitude of the external magnetic field will have some complicated form:

$$\Delta\mathbf{B}(\mathbf{r}) = \mu_o \mathbf{M} f(\mathbf{r}) \tag{4.1}$$

where $f(\mathbf{r})$ is a function of position that depends on geometry. The total magnetization of a rock has two components: thermoremnant magnetism (TRM) $\mathbf{M}_{TRM}$, and magnetization that is induced by the present-day dipole field $\mathbf{M}_I$:

$$\mathbf{M} = \mathbf{M}_{TRM} + \mathbf{M}_I \qquad \mathbf{M}_I = \chi\mathbf{H} \tag{4.2}$$

where χ is the magnetic susceptibility and H is the applied dipole field of the Earth. The Koenigberger ratio Q is the ratio of the remnant field to the induced field. This value should be much greater than 1 to be able to detect the crustal anomaly. Like the magnetization, the value of Q is between 5 and 10 in Layer 2A, but falls to about 1 deeper in the crust.

4.4 Anomalies in the Earth's Magnetic Field

When a magnetometer is towed behind a ship, one measures the total magnetic field $\mathbf{B}$, and must subtract out the reference Earth magnetic B_e field to establish the magnetic anomaly ΔB:

$$\mathbf{B} = \mathbf{B}_e + \Delta\mathbf{B} \tag{4.3}$$

Most marine magnetometers measure the scalar magnetic field. This is an easier measurement, because the orientation of the magnetometer does not need to be known. The total scalar magnetic field is

$$|\mathbf{B}| = \left(|\mathbf{B}_e|^2 + 2\mathbf{B}_e \bullet \Delta\mathbf{B} + |\Delta\mathbf{B}|^2\right)^{1/2} \tag{4.4}$$

The dipolar field of the Earth is typically 50,000 nT, while the crustal anomalies are only about 300 nT. Thus, $|\Delta\mathbf{B}|^2$ is small relative to the other terms, and we can develop an approximate formula for the total scalar field:

$$|\mathbf{B}| \cong |\mathbf{B_e}|\left(1 + \frac{2\Delta\mathbf{B} \bullet \mathbf{B}_e}{|\mathbf{B}_e|^2}\right)^{1/2} \cong |\mathbf{B}_e|\left(1 + \frac{\Delta\mathbf{B} \bullet \mathbf{B}_e}{|\mathbf{B}_e|^2}\right) \tag{4.5}$$

Equation (4.5) can be rearranged to relate the measured scalar anomaly $\mathbf{A}$ to the vector anomaly $\Delta\mathbf{B}$, given an independent measurement of the dipolar field of the Earth $\mathbf{B}_e$:

$$\mathbf{A} = |\mathbf{B}| - |\mathbf{B}_e| = \frac{\Delta\mathbf{B} \bullet \mathbf{B}_e}{|\mathbf{B}_e|} \tag{4.6}$$

4.5 Magnetic Anomalies Due to Seafloor Spreading

To calculate the anomalous scalar field on the sea surface due to thin magnetic stripes on the seafloor, we go back Poisson's equation relating magnetic field to magnetization. The model is shown in Figure 4.4. We have an xyz coordinate system with z pointed

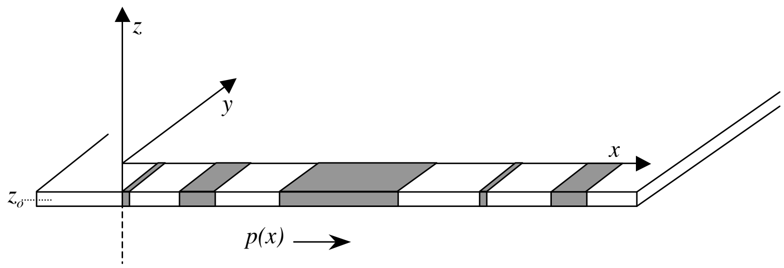


Figure 4.4

upward. The $z = 0$ level corresponds to sea level and there is a thin magnetized layer at a depth of z_o.

We define a scalar potential U and a magnetization vector M. The magnetic anomaly ΔB is the negative gradient of the potential. The potential satisfies Laplace's equation above the source layer and it satisfies Poisson's equation within the source layer.

$$\Delta \mathbf{B} = -\nabla U \tag{4.7}$$

$$\nabla^2 U = 0 \quad z \neq z_o \tag{4.8}$$

$$\nabla^2 U = \mu_o \nabla \bullet \mathbf{M} \quad z = z_o \tag{4.9}$$

$U(x, y, z)$	magnetic potential	Tm
μ_o	magnetic permeability	$4\pi \times 10^{-7}$ TA^{-1}m
$\mathbf{M}$	magnetization vector	Am^{-1}

In addition to assuming the layer is infinitesimally thin, we assume that the magnetization direction is constant, but that the magnetization varies in strength and polarity as specified by the reversal function $p(x)$. The approach to the solution is:

1. Solve the differential equation and calculate the magnetic potential U at $z = 0$.
2. Calculate the magnetic anomaly vector $\Delta\mathbf{B}$.
3. Calculate the scalar magnetic field A = $(\Delta\mathbf{B} \bullet \mathbf{B}_e)/|\mathbf{B}_e|$.

Let the magnetization be of the following general form:

$$\mathbf{M}(x, y, z) = \left(M_x\hat{\imath} + M_y\hat{\jmath} + M_z\hat{k}\right) p\,(x)\,\delta(z - z_o) \tag{4.10}$$

The differential equation 4.9 becomes:

$$\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2} = \\ = \mu_o \left\lfloor \frac{\partial}{\partial x} M_x p(x)\,\delta(z - z_o) + \cancel{\frac{\partial}{\partial y} M_y p(x)\,\delta(z - z_o)} + \frac{\partial}{\partial z} M_z p(x)\,\delta(z - z_o) \right\rfloor \quad (4.11)$$

The y-source term vanishes, because the source does not vary in the y-direction (i.e., the y derivative is zero). Thus the component of magnetization that is parallel to the ridge axis does not produce any external magnetic potential or external magnetic field. Consider a N-S oriented spreading ridge at the magnetic equator. In this case, the TRM of the crust has a component parallel to the dipole field, which happens to be parallel to the ridge axis, so there will be *no external magnetic field anomaly.* See Figure 4.5.

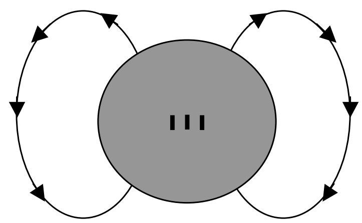


Figure 4.5

This explains why the global map of magnetic anomaly picks [7] has no data in either the equatorial Atlantic or the equatorial Pacific, where ridges are oriented N-S. Now with the ridge-parallel component of magnetization gone, the differential equation reduces to

$$\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial z^2} = \mu_o \left\lfloor \frac{\partial}{\partial x} M_x p(x)\,\delta(z - z_o) + \frac{\partial}{\partial z} M_z p(x)\,\delta(z - z_o) \right\rfloor \quad (4.12)$$

This is a second-order differential equation in two dimensions, so four boundary conditions are needed for a unique solution:

$$\lim_{|x|\to\infty} U(\mathbf{x}) = 0 \quad \text{and} \quad \lim_{|z|\to\infty} U(\mathbf{x}) = 0 \quad (4.13)$$

Take the 2-dimensional Fourier transform of the differential equation where the forward and inverse transforms are defined as

$$F(\mathbf{k}) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\mathbf{x}) e^{-i2\pi(\mathbf{k}\cdot\mathbf{x})} \mathrm{d}^2\mathbf{x} \quad F(\mathbf{k}) = \Im_{\mathbf{2}}\left[f(\mathbf{x})\right]$$

$$f(\mathbf{x}) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F(\mathbf{k}) e^{i2\pi(\mathbf{k}\cdot\mathbf{x})} \mathrm{d}^2\mathbf{k} \quad f(\mathbf{x}) = \Im_{\mathbf{2}}^{-\mathbf{1}}\left[F(\mathbf{k})\right] \quad (4.14)$$

where $\mathbf{x} = (x, z)$ is the position vector, $\mathbf{k} = (k_x, k_z)$ is the wave number vector, and $(\mathbf{k} \bullet \mathbf{x}) = k_x x + k_z z$. The derivative property is $\Im_2[\mathrm{d}U/\,\mathrm{d}x] = i2\pi k_x\, \Im_2[U]$. The Fourier transform of the differential equation is

$$-\left[(2\pi k_x)^2 + (2\pi k_z)^2\right] U(k_x, k_z) = \mu_o p(k_x) e^{-i2\pi k_z z_o}\,(i2\pi \mathbf{k} \bullet \mathbf{M}) \tag{4.15}$$

The Fourier transform in the z-direction was done using the following identity:

$$\int_{-\infty}^{\infty} \delta\,(z - z_o) e^{-i2\pi k_z z}\, \mathrm{d}z \equiv e^{-i2\pi k_z z_o} \tag{4.16}$$

Now we can solve for $U(\mathbf{k})$:

$$U(\mathbf{k}) = \frac{-i\mu_o}{2\pi} p(k_x)(\mathbf{k} \bullet \mathbf{M}) \frac{e^{-i2\pi k_z z_o}}{\left(k_x^2 + k_z^2\right)} \tag{4.17}$$

Next, take the inverse Fourier transform with respect to k_z, using the Cauchy Residue Theorem:

$$U(k_x, z) = \frac{\mu_o}{2\pi i} p\,(k_x) \int_{-\infty}^{\infty} \frac{(\mathbf{k} \bullet \mathbf{M}) e^{i2\pi k_z (z - z_o)}}{\left(k_x^2 + k_z^2\right)}\, \mathrm{d}k_z \tag{4.18}$$

The poles of the integrand are found by factoring the denominator:

$$k_x^2 + k_z^2 = (k_z + ik_x)(k_z - ik_x) \tag{4.19}$$

We see that $U(\mathbf{k})$ is an analytic function with poles at $\pm ik_x$. The integral of this function about any closed path in the complex k_z plane is zero, unless the contour includes a pole, in which case the integral is $i2\pi$ times the residue at the pole:

$$\oint \frac{f(z)}{z - z_o} dz = i2\pi f(z_o) \tag{4.20}$$

One possible path integral is shown in Figure 4.6.

There are two ways to close the path at infinity. The selection of the proper path—and thus the residue—depends on the boundary condition, equation (4.13). First, consider the case where $k_x > 0$. If we close the path of integration in the upper imaginary plane, then the pole will be ik_x. The residue will have an exponential term that vanished as z goes to plus infinity. This is what we need, since the observation plane is above the source.

$$\oint ()\, \mathrm{d}k_z = \frac{e^{-2\pi k_x (z - z_0)}}{2ik_x} (k_x M_x + ik_x M_z) \tag{4.21}$$

Next, consider the case where $k_x < 0$. To satisfy the boundary condition as z goes to plus infinity, the $-ik_x$ pole should be used, and the integration path will be clockwise instead of counterclockwise, as in equation (4.20).

$$\oint ()\, \mathrm{d}k_z = \frac{e^{+2\pi k_x (z - z_0)}}{2ik_x} (k_x M_x - ik_x M_z) \tag{4.22}$$

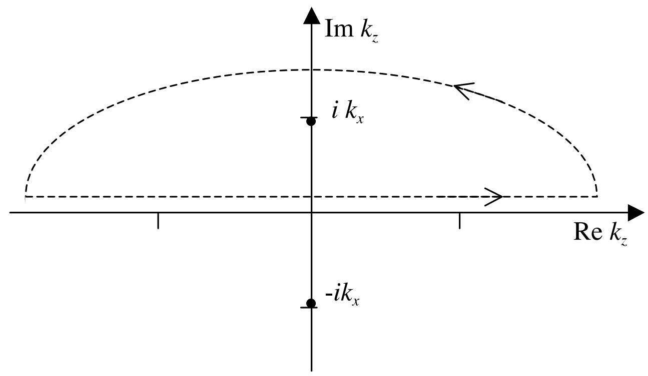


Figure 4.6

One can combine the two cases by using the absolute value of k_x:

$$U(k,z) = \tfrac{\mu_o}{2}\, p\,(k)\, e^{-2\pi\,|k|\,(z-z_0)} \left(M_z - i\tfrac{k}{|k|} M_x\right) \tag{4.23}$$

where we have dropped the subscript on the x-wavenumber.

This is the general case of an infinitely long ridge. To further simplify the problem, let's assume that this spreading ridge is located at the magnetic pole of the Earth, so the dipolar field lines will be parallel to the z-axis and there will be no x-component of magnetization. The result is

$$U(k,z) = \frac{\mu_o M_z}{2}\, p\,(k) e^{-2\pi|k|(z-z_o)} \tag{4.24}$$

Next calculate the magnetic anomaly $\Delta B = -\nabla U$:

$$\Delta\mathbf{B} = (-i2\pi k, 2\pi\,|k|)\; U(k,z) \tag{4.25}$$

The scalar magnetic field is given by equation (4.6). Since only the z-component of the Earth's field is non-zero, the anomaly simplifies to

$$\underbrace{A(k,z)}_{\text{observed anomaly}} = \underbrace{\frac{\mu_o M_z}{2}\, p\,(k)}_{\text{reversal pattern}} \times \underbrace{2\pi\,|k|\, e^{-2\pi|k|(z-z_o)}}_{\text{Earth filter}} \tag{4.26}$$

The reversal pattern is a sequence of positive and negative polarities. To generate the model anomaly, one would take the Fourier transform of the reversal pattern, multiply by the Earth filter, and take the inverse transform of the result. An examination of

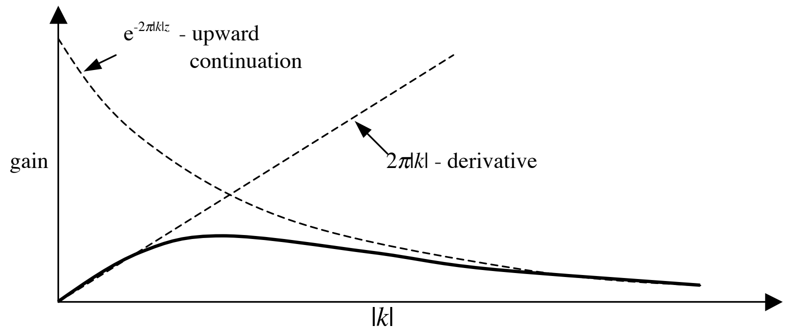


Figure 4.7

the Earth filter in Figure 4.7 illustrates why a square-wave reversal pattern becomes distorted.

This Earth filter attenuates both long and short wavelengths, so it acts like a band-pass filter. In the space domain it modifies the shape of the square-wave reversal pattern, as shown in Figure 4.8.

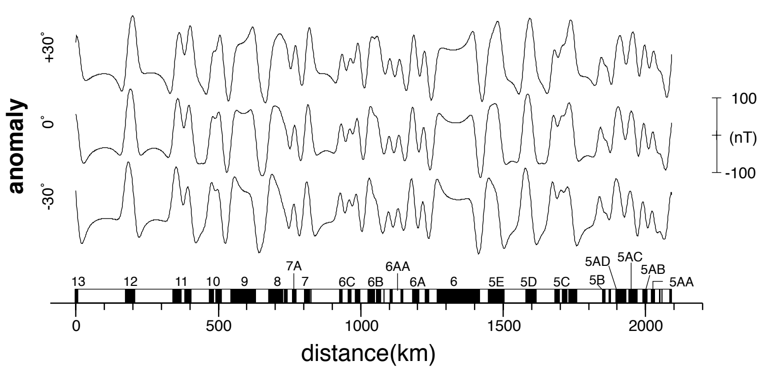


Figure 4.8: Synthetic magnetic anomalies generated from the reversal pattern (lower) using a 0.5 km thick magnetized layer at a depth of 4.25 km and a spreading rate of 100 mm/yr. The three curves have skewness of +30, 0, and -30 degrees. From Horner et al., 2003[33].

When the seafloor spreading ridge is not at the magnetic pole, both the magnetization and the Earth's magnetic field will have an x-component. This introduces a phase shift, or skewness Θ, in the output magnetic anomaly. At the ocean surface, the skewed magnetic anomaly is

$$A(k) = \frac{\mu_o M_z}{2} p(k) e^{i\Theta \frac{k}{|k|}} 2\pi |k| e^{+2\pi|k|z_o} \tag{4.27}$$

The skewness depends on both the geomagnetic latitude and the orientation of the spreading ridge when the crust was magnetized. Moreover, this parameter will vary over time. If one knows the skewness, then the model profile can be skewed to match the observed profile. Alternatively, the observed magnetic anomaly can be de-skewed. This is called *reduction to the pole*, because it synthesizes the anomaly that would have formed on the magnetic pole.

$$A_{pole}(k) = A_{observed}(k) e^{-i\Theta \frac{k}{|k|}} \tag{4.28}$$

4.6 Discussion

The ability to observe magnetic reversals from a magnetometer towed behind a ship relies on some rather incredible coincidences related to reversal rate, spreading rate, ocean depth, and Earth temperatures (mantle, seafloor, and Curie). In the case of marine magnetic anomalies, four scales must match.

First, the temperature of the mantle (1200°C), the seafloor (0°C), and the Curie temperature of basalt (~500°C) must be just right for recording the direction of the Earth's magnetic field at the seafloor spreading ridge axis. Most of the thermoremnant magnetism (TRM) is recorded in the upper 1000 meters of the oceanic crust. If the thickness of the TRM layer was too great, then as the plate cooled while it moved off the spreading ridge axis, the positive and negative reversals would be juxtaposed in dipping vertical layers (Figure 4.3). This superposition would smear the pattern observed by a ship. If the seafloor temperature was above the Curie temperature, as it is on Venus, then no recording would be possible.

The second scale is related to ocean depth and thus the Earth filter. The external magnetic field is the derivative of the magnetization, which, as shown above, acts as a high-pass filter applied to the reversal pattern recorded in the crust. The magnetic field measured at the ocean surface will be naturally smooth (upward continuation), due to the distance from the seafloor to the sea surface; this is a low-pass filter. This smoothing depends exponentially on ocean depth, so for a wavelength of 2π times the mean ocean depth, the field amplitude will be attenuated by $1/e$, or 0.37, with respect to the value measured at the seafloor. The combined result of the derivative and the upward continuation is a band-pass filter with a peak response at a wavelength of 2π times the mean ocean depth, or about 25 km. Wavelengths that are shorter (<10 km) or much longer (>500 km) than this value will be undetectable at the ocean surface.

The third and fourth scales that must match are the reversal rate and the seafloor-spreading rate. Half-spreading rates on the Earth vary from 10 to 80 km per million years. Thus, for the magnetic anomalies to be most visible on the ocean surface, the reversal rate should be between 2.5 and 0.3 million years. It is astonishing that this is the typical reversal rate observed in sequences of lava flows on land!! While most ocean basins display clear reversal patterns, there was a period between 85 and 120 million years ago when the magnetic field polarity of the Earth remained positive, so the ocean surface anomaly is too far from the reversal boundaries to provide timing information. This area of seafloor is called the Cretaceous Quiet Zone; it is a problem area for accurate plate reconstructions.

The lucky convergence of length and time scales makes it very unlikely that magnetic anomalies due to crustal spreading will ever be observed on another planet.

4.7 Exercises

Exercise 4.1. Explain why magnetic lineations cannot be observed from a spacecraft orbiting the Earth at an altitude of 400 km.

Exercise 4.2. Explain why scalar magnetic anomalies are not observed at a N-S oriented spreading ridge located at the magnetic equator.

Exercise 4.3. Write a MATLAB program to generate marine magnetic anomaly versus distance from a spreading ridge axis. Use equation 4.27 relating the Fourier transform of the magnetic anomaly to the Fourier transform of the magnetic timescale. You will need a magnetic timescale and the start of a MATLAB program (`ftp://topex.ucsd.edu/pub/class/geodynamics/hw3`). Assume symmetric spreading about the ridge axis, constant spreading rate, and constant ocean depth.

Use the program and magnetic anomaly profiles across the Pacific-Antarctic Rise (NBP9707.xydm) and the Mid-Atlantic Ridge (a9321.xydm) to estimate the half-spreading rate at each of these ridges. You may need to vary the mean ocean depth and skewness to obtain good fits.

Describe some of the problems that you had fitting the data. Provide some estimates on the range of total spreading rate for each ridge.