Slope Correction for Ocean Radar Altimetry

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Abstract

We develop a slope correction model to improve the accuracy of mean sea surface topography models as well as marine gravity models. The correction is greatest above ocean trenches and large seamounts where the slope of the geoid exceeds 100 μrad. In extreme cases, the correction to the mean sea surface height is 40 mm and the correction to the along-track altimeter slope is 1 μrad which maps into a 1 mGal gravity error. Both corrections are easily applied using existing grids of sea surface slope from satellite altimetry.

Introduction

Satellite radar altimeters emit pulses having a rather broad antenna beam width when projected onto the surface of the earth. High range precision is achieved by forming a short radar pulse and recording the travel time of the leading edge of the return echo [*Brown,* 1977]. Over land and ice where the topography has significant height variations within the beam width of the radar, the first arrival will not coincide with the nadir point but will reflect from the point closest to the radar. An approximate way to correct for this effect is to apply a local slope correction [*Brenner et al.,* 1983]. The magnitude of this correction is $|\vec{s}|^2 H/2$ where $\vec{s}$ is the slope vector and H is the altitude of the satellite (e.g., 780 km for Seasat). A more accurate correction also accounts for the curvature of the Earth, R, which reduces the effective height of the satellite by a factor $1/(1+H/R)$ [*Wingham et al.,* 2004]. Over an ice sheet having a nominal slope of 0.014 rad this correction is rather large (68 m) and the correction needs to be applied to produce an accurate topographic map. However over the ocean surface where sea surface slopes have an rms value of 35 μrad [*Smith,* 1998], the correction is usually rather small (0.3 mm). Nevertheless, there are areas near large seamounts and deep ocean trenches where the slope is much greater (100-300 μrad). In the most extreme case this will result in a height error of 40 mm.

In this paper we use our latest model of ocean surface slope derived from altimetry [*Sandwell et al.*, 2013] to calculate a global map of height correction for a nominal effective satellite altitude of 1000 km. This model can be scaled to the actual height of each satellite altimeter. In addition, we assess the impact of this correction on the recovery of the gravity field and show that in extreme cases the correction will change the recovered gravity field by 1 mGal. Since new marine gravity models have accuracies better than 2 mGal [*Andersen et al.,* 2013; *Sandwell et al.,* 2013], this slope correction is significant. In particular, we show that the slope-correction narrows the height and gravity signature over very large seamounts. This is important for more accurate predictions of seafloor depth from gravity. Moreover at mid latitudes a slope error of 1 μrad will map into a geostrophic current error of 0.1 m/s [*Stewart,* 1985]. This magnitude of error is significant given the new geoid models becoming available from GOCE and GRACE [*Knudsen et al.,* 2011].

Slope Correction for a Spherical Earth

We consider a pulse-limited radar altimeter at an altitude H above the Earth of local radius R. Locally the surface of the ocean (principally geoid) is tilted at a slope $\vec{s}$ with respect to the ellipsoid (Figure 1). As developed in *Wingham et al.,* [2004] the footprint offset $\vec{x}_o$ and height correction Δh for the spherical model are given by

$$\vec{x}_o = \vec{s}H_e, \qquad \Delta h = \frac{|\vec{s}|^2 H_e}{2} \tag{1}$$

where the effective altitude is

$$H_e = \frac{H}{(1 + H / R)}. \tag{2}$$

The effective altitude for the satellites of interest is provided in Table 1. There are two methods for applying this correction [*Remy et al.,* 1989]. The "relocation method" shifts the latitude and longitude of the footprint location by an amount $\vec{x}_o$ to be closer to the actual reflection point. The "direct method" improves the height at the nadir location by subtracting the height correction. When mapping ice topography the footprint offset can be larger than the size of a topography grid cell so the relocation method is used. Over the ocean the footprint offset is smaller than the diameter of the pulse-limited footprint so we apply the direct method, as it is easier to implement.

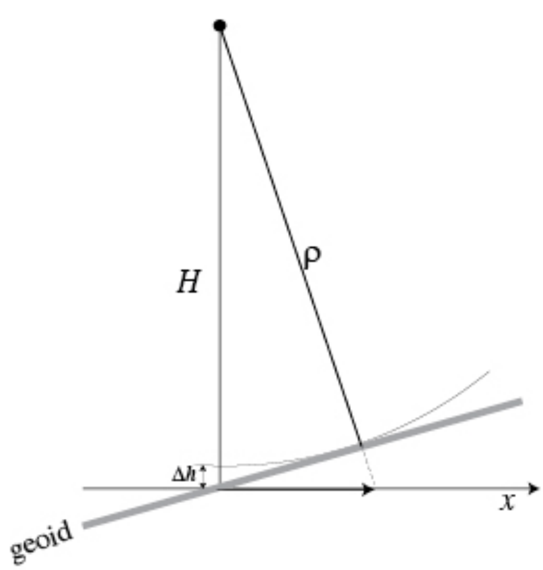


Figure 1 Schematic diagram of footprint offset and range ρ reduction caused by a slope of the geoid relative to the ellipsoid after *Brenner et al*., [1983]. This schematic diagram is for a flat earth but all calculations are done using a spherical earth model.

Table 1. Effective altitude of radar altimeters

altimeter	H (km)	H_e (km)
Seasat, Geosat	784	698
ERS-1/2, Envisat	766	618
Topex, Jason-1/2	1336	1104
CryoSat-2	725	651
HY-2	971	843
Saral	799	710

Note these altimeters have nearly circular orbits so we use the mean effective altitude.

Gaussian Model

For our gravity analysis, we would also like to know the slope correction to be applied to along-track slopes measured by radar altimeters. To illustrate the magnitude and shape of a typical slope correction we use a simple Gaussian model for sea surface height h. The general form of the Gaussian is

$$h(x) = h_o \exp\left(\frac{-x^2}{2\sigma^2}\right) \tag{3}$$

where h_o is the amplitude of the sea surface bump (or trough), σ is the half width of the bump, and x is the distance along the sea surface height profile obtained by the altimeter. The ocean surface slope is the derivative of this height function.

$$s(x) = \frac{-xh_o}{\sigma^2} \exp\left(\frac{-x^2}{2\sigma^2}\right). \tag{4}$$

Given this formula for the slope we can calculate the *x*-offset of the reflection point, *Δx* and the height correction, *Δh*. This height correction needs to be subtracted from the height reported by the altimeter in order to correct for the effect of the sloping surface. The along-track slope correction is then computed from the along-track derivative of the height correction.

$$\Delta x(x) = \frac{-x h_o H_e}{\sigma^2} \exp\left(\frac{-x^2}{2\sigma^2}\right) \tag{5}$$

$$\Delta h(x) = \frac{-x^2 h_o^2 H_e}{2\sigma^4} \exp\left(\frac{-x^4}{4\sigma^4}\right). \tag{6}$$

We combine trench ($h_o = -10m \ \sigma = 20km$) and seamount ($h_o = 1.0m \ \sigma = 4km$) examples along a single profile. Two examples are needed to illustrate that the height and slope corrections are not linearly proportional to slope. The results for an effective altitude of 1000 km are shown in Figures 2a-d. The sea surface slopes are rather large compared with typical rms sea surface slope of about 35 μrad so these can be considered as extreme cases. The trench has an amplitude of about 300 μrad (Figure 2b) and this results in a footprint offset of about 300 m (right scale bar in Figure 2b). The slope amplitude and footprint offset of the seamount are smaller (200 μrad and 200 m). Note that this amount of footprint offset is much smaller than the typical diameter of a pulse-limited radar altimeter of 2000-5000 m. As described above, the reflection point of the radar above a sloped surface is always closer to the radar resulting in an everywhere-positive height correction, which is subtracted from the sea surface height. In the case of a seamount, the corrected bump is narrower than the measured bump although the amplitude stays the same. In the case of a trench, the corrected trough is wider than the measured trough and the amplitude is the same. Also note that the characteristic length scale of the height correction is ½ the characteristic length scale of the sea surface height feature because the height correction depends on the slope squared. For the trench and seamount cases (Figure 2c) the height correction is 40 mm and 10 mm, respectively.

The last plot (Figure 2d) shows the slope correction for these two cases. The slope correction, which is the along-profile derivative of the height correction, has both positive and negative values. Note that the slope corrections for the trench and seamount have comparable magnitudes of ~4 μrad while their height corrections differ by a factor of 4. Indeed as the horizontal scale of a feature decreases its slope correction increases proportionally. We note that this magnitude of sea surface slope is possible but rarely occurs. Next we calculate the actual corrections needed for the oceans.

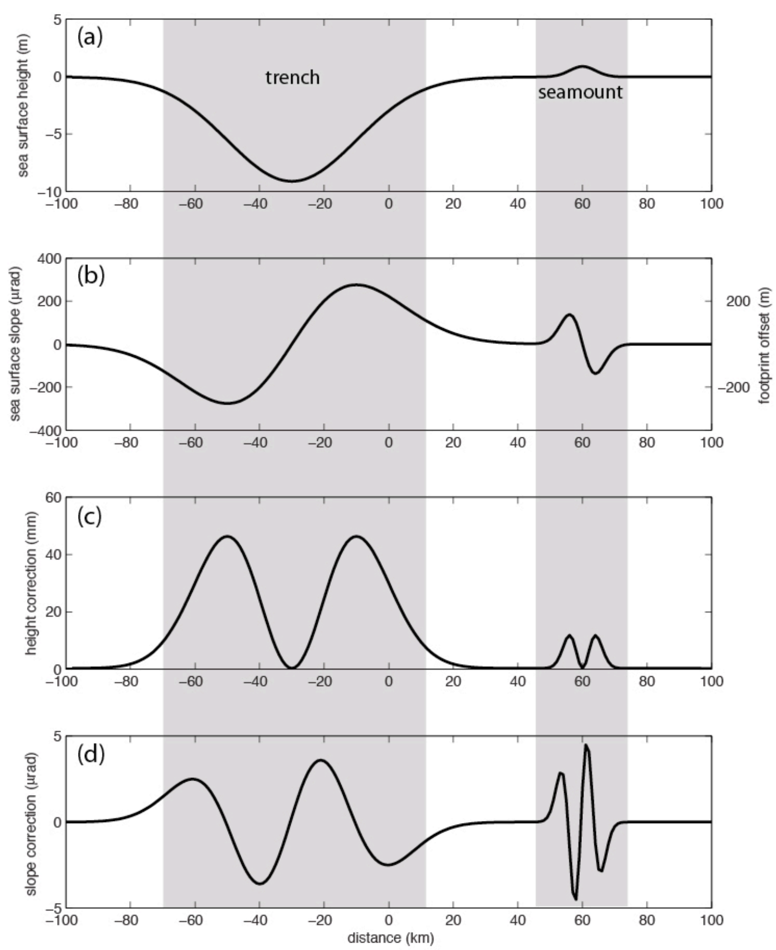


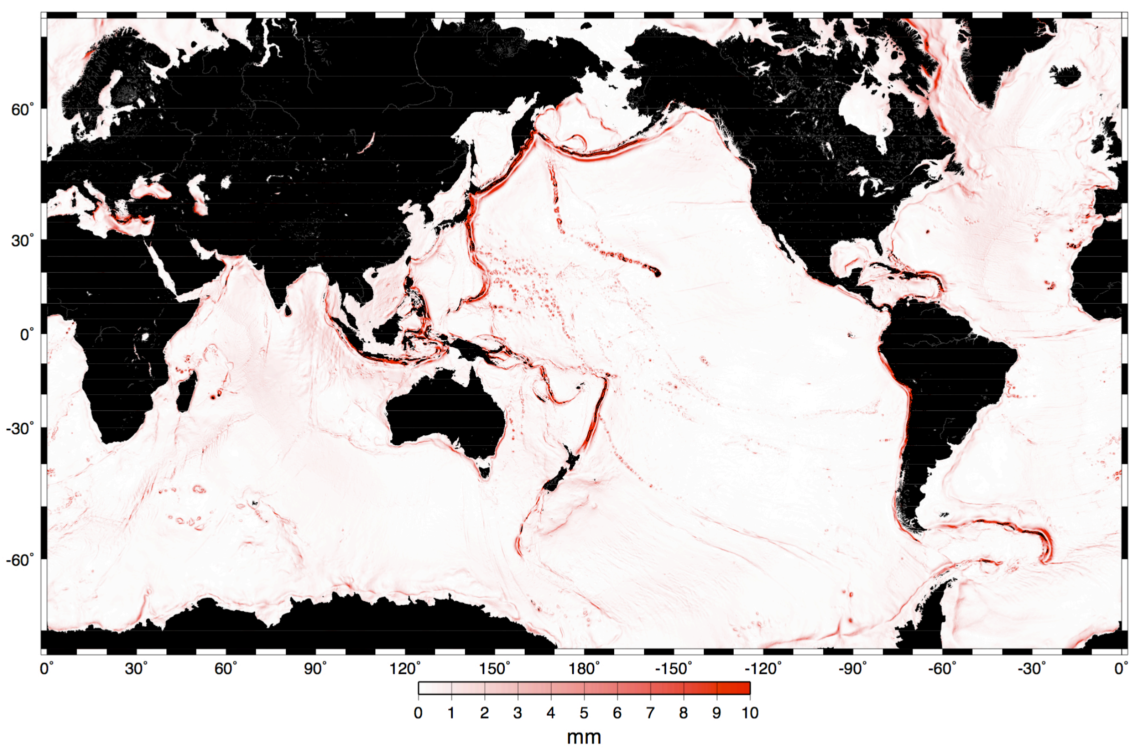
Figure 2. (a) Sea surface height profile across a trench and seamount for an effective satellite altitude of 1000 km. (b) Slope of the sea surface height as well as the footprint offset (labels on right). (c) Height correction for the trench is ~50 mm while the correction for the seamount is ~10 mm. (d) Slope correction for trench and seamount examples have typical amplitude of 4 μrad.

Results

The examples in the previous section show that the corrections to the sea surface height and along-track altimeter slope can be significant in areas of very high geoid slope. (Note that ocean dynamic topography also contributes to the slope of the ocean surface but is typically 50 times smaller than the geoid slope [*Stewart,* 1985].) Moreover we have shown that the correction to the along-track slope is at least 20 times smaller than the slope of the geoid. Therefore the correction is a small perturbation and a one-step correction is sufficient, in contrast to the situation with ice topography, where larger slopes necessitate an iterative correction scheme [*Remy et al.,* 1989]. In this section, we use existing models of north and east sea surface slope (also called deflections of the vertical) derived from Geosat, ERS-1, Envisat, Jason-1, and Cryosat-2 [*Sandwell et al.,* 2013] to develop a grid of height correction and along-track slope correction. The height correction map for an effective altitude of 1000 km is provided in Figure 3 and also available by ftp at (ftp://topex.ucsd.edu/pub/global_VD_1min/DH.grd). The mean value of the height correction is 0.69 mm and the standard deviation is 2.10 mm. As can be seen on the map, there are only a few areas where the height correction exceeds 20 mm and in a very few areas, the correction exceeds 40 mm.

This correction is time-invariant, and scales linearly with effective altitude. Most oceanographic altimeters are in frozen orbits in which the altitude variations are nearly the same on every repeat pass. Also, the height variations around an altimeter's orbit are quite small (±15 km) compared to the effective altitude, and so the correction can be made accurately enough by simply using a mean effective altitude for each satellite. However, when combining data from multiple satellites, each should be given its appropriate altitude (Table 1).

The most important oceanographic usage will be when estimating the mean dynamic ocean currents by subtracting the geoid from the mean sea surface height [e.g., *Knudsen et al.,* 2011]. Ignoring this correction will result in mean dynamic topography errors of more than 20 mm around the ocean trenches. This will result in false currents running parallel to the trench. To estimate the magnitude of this effect, we have computed and then gridded the along-track slope correction for the descending tracks of the Jason-1 altimeter (Figure 4). Most areas show slope corrections of less than 1 μrad. However the correction can be as large as 2 μrad in areas of steep geoid slope. For geostrophic currents at mid latitudes 2 μrad of slope error corresponds to 0.2 m/s of error in estimating current velocity. Given the velocity accuracies of 0.1 m/s being provided by satellite geodesy, this correction should be applied to eliminate a known error source.



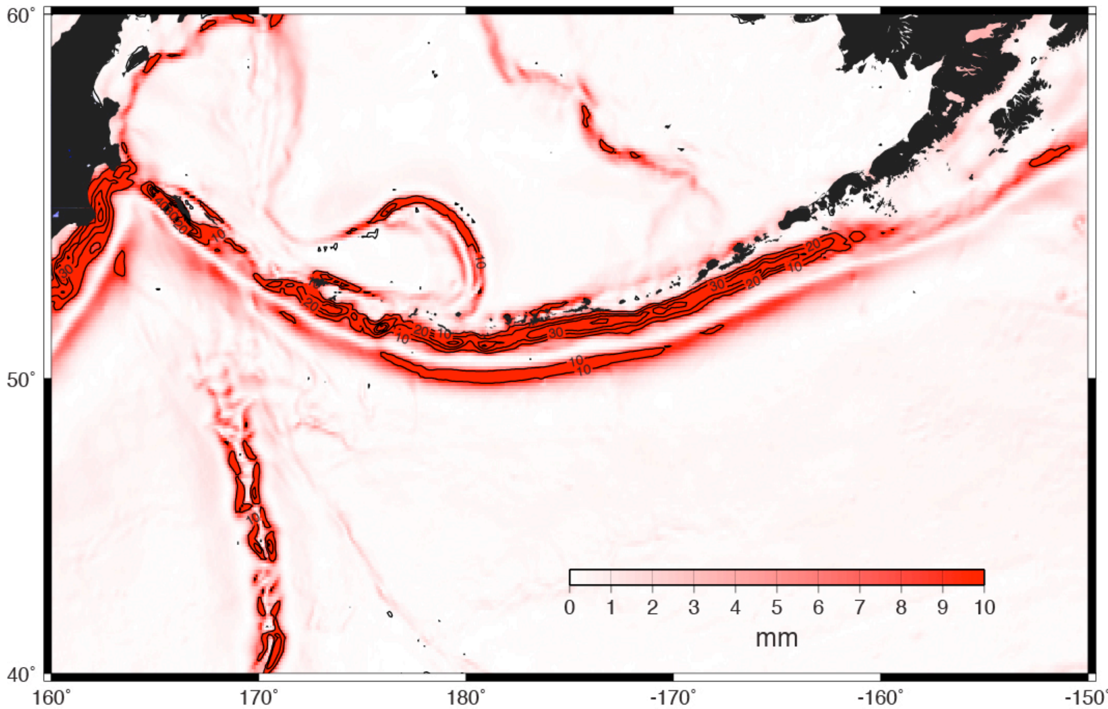


Figure 3. (top) Global map of height correction (10 mm contour interval) for the altitude of 1000 km. (lower) Height correction in the region of the Aleutian Trench shows corrections of up to 40 mm along the trench and corrections up to 20 mm adjacent to the northern Emperor seamounts.

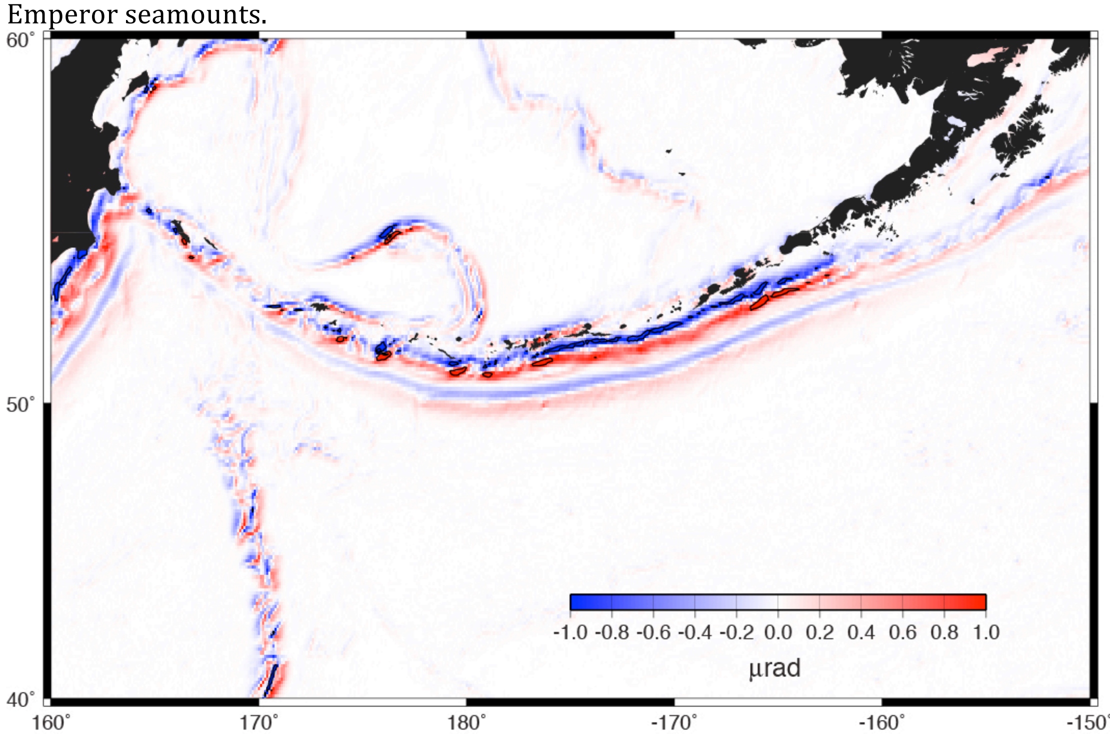


Figure 4. Slope correction (1 µrad contour interval) for descending Jason-1 tracks.

In addition to applying the correction to the mean sea surface topography for oceanographic studies, it is also important to apply the correction to the along-track slopes from each altimeter prior to constructing a gravity anomaly map. In this case the correction will also depend on the direction of the satellite track (e.g., Figure 4). Note that because this along-track correction is from a directional derivative of the height correction, features that are perpendicular to the satellite tracks produce a larger correction. Since the various altimeters have different trackline directions, as well as ascending and descending tracks, the along-track slope correction must be performed on a track-by-track basis. We compute combined impact of this slope correction on the recovery of the marine gravity anomaly by calculating global marine gravity fields using all available altimeter data with and without applying the slope error. The difference between the two gravity models is shown in Figure 5. The gravity difference has similar amplitudes to the slope difference although the phase of the anomalies is shifted by 90˚ during the slope-to-gravity conversion. For example, the gravity difference is slightly positive and usually less than 1 mGal over the crests of the Emperor seamounts and is negative with amplitudes of up to -2 mGal on the steepest flanks of the Emperor seamounts. The gravity differences are larger along the trenches and tend to slightly increase the gravity value along the deepest part of the trench. We attempted to find ship gravity profiles where we could confirm that this correction improves the accuracy of the altimeter-derived gravity field. However, because the amplitudes of the anomalies are hundreds of mGal in the areas of significant correction and both the ship and altimeter gravity have errors of ~2 mGal we could not demonstrate that this correction significantly improved the fit to the ship data. It has been shown [*Sandwell et al.,* 2013] that most shipboard surveys of gravity are not as accurate as the anomalies obtained from altimetry by the latest processing.

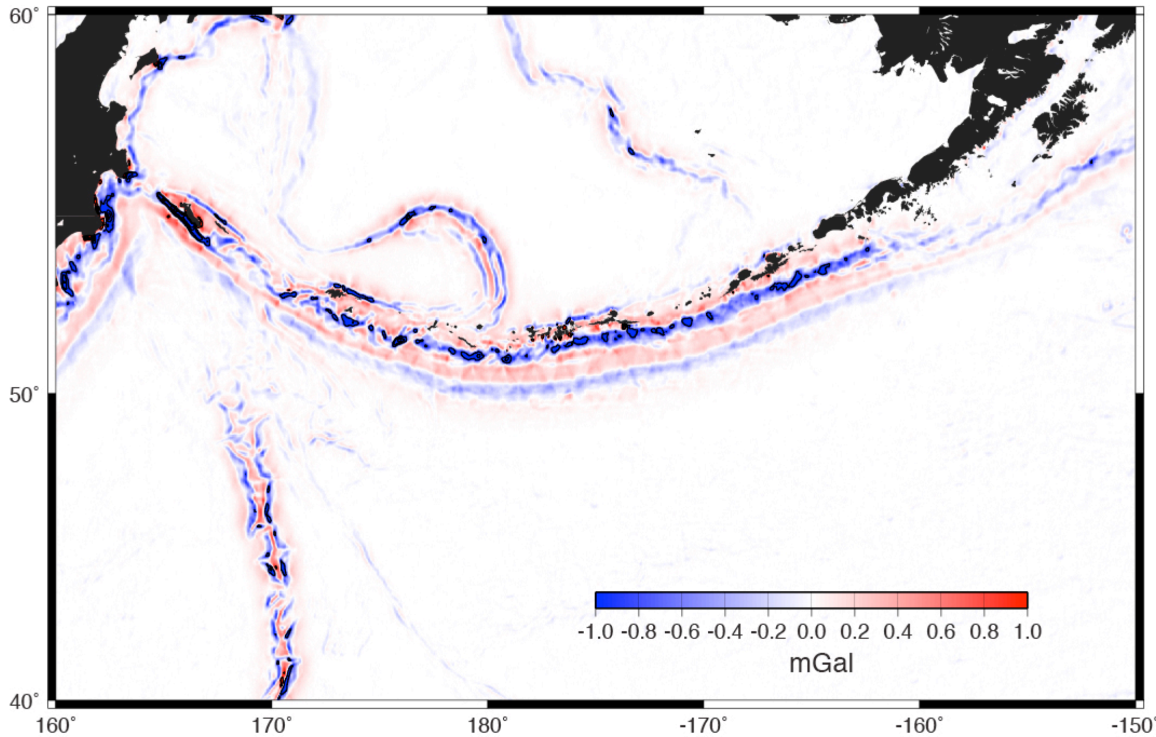


Figure 5. Gravity anomaly differences from models computed with and without the slope correction (1 mGal) contour interval.

Conclusions

This analysis shows that neglecting the offset of the radar reflection point in areas of high sea surface slope (100-300 μrad) will result in errors in estimating sea surface height and slope from pulse-limited radar altimetry. In extreme cases, the correction to the mean sea surface height is 40 mm and the correction to the along-track altimeter slope is 1 μrad which maps into 1 mGal gravity error. This effect has been neglected in all previous ocean studies but now that we have gravity models with typical accuracies of 2 mGal (2 μrad), this effect should be included in the processing of the data; we provide a global correction grid that can be scaled to the effective altitude of any radar altimeter.

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