The Development and Evaluation of the Earth Gravitational Model 2008 (EGM2008)

Nikolaos K. Pavlis[1], Simon A. Holmes[2], Steve C. Kenyon[3], and John K. Factor[3]

[1] National Geospatial-Intelligence Agency, 7500 GEOINT Drive, S73-IBG, Springfield, Virginia 22150, USA Email: Nikolaos.K.Pavlis@nga.mil

[2] SGT, Inc., 7701 Greenbelt Rd., Suite 400, Greenbelt, Maryland, 20770, USA

[3] National Geospatial-Intelligence Agency, 3838 Vogel Rd., Arnold, Missouri 63010, USA

Corresponding author: Nikolaos K. Pavlis

Abstract

EGM2008 is a spherical harmonic model of the Earth's gravitational potential, developed by a least squares combination of the ITG-GRACE03S gravitational model and its associated error covariance matrix, with the gravitational information obtained from a global set of area-mean free-air gravity anomalies defined on a 5 arc-minute equiangular grid. This grid was formed by merging terrestrial, altimetry-derived, and airborne gravity data. Over areas where only lower resolution gravity data were available, their spectral content was supplemented with gravitational information implied by the topography. EGM2008 is complete to degree and order 2159, and contains additional coefficients up to degree 2190 and order 2159. Over areas covered with high quality gravity data, the discrepancies between EGM2008 geoid undulations and independent GPS/Leveling values are on the order of ±5 to ±10 cm. EGM2008 vertical deflections over USA and Australia are within ±1.1 to ±1.3 arc-seconds of independent astrogeodetic values. These results indicate that EGM2008 performs comparably with contemporary detailed regional geoid models. EGM2008 performs equally well with other GRACE-based gravitational models in orbit computations. Over EGM96, EGM2008 represents improvement by a factor of six in resolution, and by factors of three to six in accuracy, depending on gravitational quantity and geographic area. EGM2008 represents a milestone and a new paradigm in global gravity field modeling, by demonstrating for the first time ever, that given accurate and detailed gravimetric data, a *single* global model may satisfy the requirements of a very wide range of applications.

1. INTRODUCTION

Accurate knowledge of the gravitational potential of the Earth, on a global scale and at very high resolution, is a fundamental prerequisite for various geodetic, geophysical and oceanographic investigations and applications. Over the past 50 or so years, continuing improvements and refinements to the basic gravitational modeling theory have been paralleled by the availability of more accurate and complete data and by dramatic improvements in the computational resources available for numerical modeling studies. These advances have brought the state-of-the-art from the early spherical harmonic models of degree 8 [*Zhongolovich*, 1952], to the present solution that extends to degree 2190. *Rapp* [1998] provided a brief review of the major developments in global gravitational field modeling over the 20th century.

There are numerous uses for these high degree potential coefficient models [*Tscherning*, 1983]. In recent years, two types of applications have played a major role in emphasizing the need for high resolution, accurate global gravitational models. First, over land areas, GPS positioning and gravimetrically determined geoid heights offer the possibility of determining orthometric heights and height differences without the need for leveling [*Schwarz et al.*, 1987]. A global high degree model may be used here, either as a reference to support the development of more detailed regional geoids, or to provide the geoid heights on its own. Second, over ocean areas, the need to determine the absolute Dynamic Ocean Topography (DOT) and its slopes, from altimeter-derived Sea Surface Heights (SSH) and a global gravitational model, puts very stringent accuracy and resolution requirements on global high degree models [*Ganachaud et al.*, 1997]. Furthermore, a unique, accurate, global high degree gravitational model may be used to provide the reference surface for the realization of a global vertical datum [*Rapp and Balasubramania*, 1992].

The first decade of the new millennium has been called “The Decade of Geopotentials” and has seen the launch of three dedicated gravity field mapping missions: CHAMP [*Reigber et al.*, 1996] launched in July 2000, GRACE [*GRACE*, 1998] launched in March 2002, and GOCE [*ESA*, 1999] launched in March 2009. Considering these advances, and in particular the expected availability of very accurate long wavelength gravitational models from GRACE, the National Geospatial-Intelligence Agency (NGA) decided to embark on the development of a new Earth Gravitational Model (EGM) to serve as: (a) a replacement of EGM96 [*Lemoine et al.*, 1998], and, (b) a candidate (pre-launch) reference model for the analysis of data to be acquired from GOCE. It was decided early on that the new EGM would be developed by combining the best available GRACE-derived satellite-only model, with the most comprehensive compilation of a global 5 arc-minute equiangular grid of area-mean free-air gravity anomalies that NGA could furnish. In this fashion, the highly accurate long wavelength information provided by the GRACE data would be complemented with the short wavelength information contained within the 5 arc-minute gravity anomaly data. The accuracy goal for the new EGM was set to ±15 cm global Root Mean Square (RMS) geoid undulation commission error. The analytical and numerical work required to ensure technical readiness for the development of the new EGM began in earnest around 2000. The status and progress of the project was demonstrated with the development of Preliminary Gravitational Models (PGM) that were presented in 2004 [*Pavlis et al.*, 2005], 2006 [*Pavlis et al.*, 2006a], and 2007 [*Pavlis et al.*, 2007a]. Following the example of EGM96, PGM2007A [*Pavlis et al.*, 2007a] was also provided for evaluation to an independent Special Working Group, functioning under the auspices of the International Association of

Geodesy (IAG) and the International Gravity Field Service (IGFS). Based in part on the feedback received from this Working Group, the development of the final model, designated EGM2008, was completed in late March 2008, and EGM2008 was presented and released to the scientific community on April 17, 2008 [*Pavlis et al.*, 2008].

In the following sections, we present the modeling and estimation aspects of the EGM2008 solution, the preparation and pre-processing of the data used to develop the model, the evaluation of the solution using a variety of independent data, and the error assessment of the model. We also present and discuss briefly some products of the model that have been made available to the community through the World Wide Web. We conclude with a summary of the strengths and weaknesses of the model (at least those that we have identified so far), and with suggestions regarding some aspects of the solution that future work should aim to improve.

2. MODELING AND ESTIMATION ASPECTS

2.1 Solution Design – Rationale and Execution

EGM96, the predecessor of the EGM2008 model, was a composite solution in which different estimation techniques were used to compute different spectral bands of the model (see [*Lemoine* et al., 1998] for details). The lower degree portion of EGM96 (up to degree 70), was estimated from the combination of the satellite-only model EGM96S (complete to degree and order 70), with surface gravity data (excluding altimetry-derived values) and satellite altimetry in the form of "direct" tracking. In this mode, satellite altimeter data are treated as ranges from the spacecraft to the ocean surface whose upper endpoint senses through the orbit dynamics attenuated gravitational signals, both static and time-varying, while their lower endpoint senses the combined effects of geoid undulation, DOT as well as tides and other time-varying effects, without any attenuation. In this manner, altimeter data contribute to the estimation of the satellite's orbit, as well as the estimation of the DOT and of the potential coefficients. The development of EGM96S involved the analysis of various types of satellite tracking data, acquired over many years, from 40 satellites. Fully-occupied normal matrices were formed and combined with appropriate relative weights, in order to estimate this “comprehensive” low degree portion of the EGM96 model. This analysis involved the simultaneous estimation of several parameter sets besides the gravitational potential coefficients and the spherical harmonic coefficients representing the DOT, which were its main products. Beyond degree 70, and up to degree 359, the fully-occupied normal matrix associated with EGM96S was combined with a Block-Diagonal (BD) approximation of the normal equations resulting from the analysis of a complete global 30 arc-minute equiangular grid of gravity anomalies, which used altimetry-derived values over most oceanic areas. Over areas without adequate gravity anomaly data, the 30 arc-minute grid used in EGM96 was filled with composite “fill-in” values, computed from the low degree part of EGM96S, augmented with coefficients of the topographic-isostatic potential (see [*Lemoine et al.*, 1998, sections 7.2 and 8.3] for details). In the specific approximation (BD3) used in EGM96, each block corresponds to all the unknown coefficients of the same order, and the rest of the matrix is all zeroes. Finally, the EGM96 coefficients of degree 360 were estimated from this 30 arc-minute grid using the Numerical Quadrature (NQ) technique. *N. Pavlis* in [*Lemoine et al.*, 1998, section 8] discusses the rational behind the particular choice of the estimation strategy used to develop EGM96. Three specific factors were critical to that choice:

(1) The use of altimeter data in the form of "direct" tracking. This was done so that altimetry would strengthen the determination of the low degree potential coefficients, through the orbit dynamics information that is represented within altimeter range measurements.

(2) The conditioning of the error covariance matrix associated with EGM96S. The EGM96S coefficients, especially those of higher degree, were highly correlated. As a result, this matrix had to be employed in its fully-occupied form, and could not be approximated with any block-diagonal counterpart, without compromising severely the quality of the least squares adjustment results.

(3) The use of marine (non-altimetric) gravity anomalies. The limited resolution and accuracy of the EGM96S model implied that the available marine gravity anomalies offered a useful data resource that could aid the separation between geoid and DOT, within altimeter range measurements.

A significant disadvantage of the composite nature of models developed like EGM96 is the discontinuity present in their error spectra, at the degrees where the estimation technique changes (see, e.g., Figure 10.3.1-2 in [*Lemoine et al.*, 1998]).

The availability of highly accurate "TOPEX-class" orbits [*Fu et al.*, 1994] on the one hand, and the success of the Gravity Recovery And Climate Experiment (GRACE) satellite mission [*GRACE*, 1998; *Tapley et al.*, 2004] on the other, had significant implications for the design of gravitational solutions obtained from the combination of satellite tracking data with surface gravity and satellite altimetry data. In particular, the use of altimetry in the form of "direct" tracking is nowadays unnecessary, and could actually have an adverse effect on the quality of the resulting gravitational model. Errors arising from gravitational model inaccuracies do not dominate the orbit error budget of altimeter satellites anymore. Instead, errors due to, e.g., mis-modeling of non-gravitational forces acting on the spacecraft are likely to be more significant nowadays. In this regard, to allow the orbits of altimeter satellites to contribute (through their dynamics) to the determination of gravitational parameters within a combination solution is not desirable, because the effects of orbit errors of non-gravitational origin could corrupt the solved-for gravitational parameters.

GRACE delivered, for the first time ever, observations that support estimation of gravitational models complete to degree and order 180, purely from space techniques [*Tapley et al.*, 2005; *Mayer-Gürr*, 2007]. Moreover, due to the global coverage (near-polar orbit) and the high degree of homogeneity in the quality of the GRACE data, the resulting GRACE-only gravitational models are accompanied by error covariance matrices that are significantly better conditioned than their pre-GRACE counterparts. The geographic distribution of the propagated errors in the geoid, computed from the error covariance matrices of GRACE-only gravitational models, exhibits a predominantly latitude-dependent (zonal) pattern, symmetric with respect to the equator [*Tapley et al.*, 2005, Figure 4]. Such a pattern implies that the error covariance matrices are predominantly block-diagonal in nature, closely adhering to the BD1 structure discussed by *N. Pavlis* in [*Lemoine et al.*, 1998, section 8.2.2] and in section 2.2 of this paper. This permits their approximation with the corresponding block-diagonal forms, without any appreciable loss of accuracy.

An additional consideration, pertinent to the design of the EGM2008 solution, involved the poor overall quality of the available marine (non-altimetric) gravity anomalies. *Pavlis* [1998] compared the 1° area-mean gravity anomalies used in the development of EGM96, to the satellite-only solution EGM96S, and demonstrated that over the ocean areas, these non-altimetric values were contaminated by significant long wavelength systematic errors. The main reason for using such marine data in EGM96 was to aid the satellite-only solution EGM96S in achieving the separation between the geoid and DOT signals contained within the altimeter range measurements. Nowadays, though, given the very high accuracy of the long wavelength part of the available GRACE-only models, it is questionable whether these marine gravity anomalies could have any such positive impact, in an ocean-wide sense. However, accurate marine gravity data are still quite useful, especially over areas where the altimetry-derived gravity anomalies are either unavailable or inaccurate. Therefore, in EGM2008, their use was restricted to certain coastal areas, and to areas where significant ocean surface variability makes altimetry-derived gravity anomalies less reliable, while marine data of verifiable quality exist, such as the Kuroshio and Gulf Stream areas.

With the above considerations in mind, it became clear that a very high degree (2159) combination solution could now be developed, not as a composite solution anymore, but rather using a single least squares adjustment estimation technique, according to the following iterative procedure:

Step 1: A Mean Sea Surface (MSS) and a GRACE-only gravitational model are used to derive a low degree and order spherical harmonic expansion of the DOT.

Step 2: Satellite altimeter data, along with the estimated DOT model, are used to estimate an ocean-wide set of free-air gravity anomalies.

Step 3: The altimetry-derived free-air gravity anomalies are merged with corresponding values over land, and are supplemented with some “fill-in” values over areas void of any gravity observations, to form a complete global 5 arc-minute equiangular grid of surface free-air gravity anomalies.

Step 4: The 5 arc-minute surface free-air gravity anomalies are continued analytically to the surface of an ellipsoid of revolution.

Step 5: The 5 arc-minute free-air gravity anomalies on the ellipsoid and their associated error estimates are input to a Block-Diagonal (BD) least squares estimator, which produces a “terrestrial” estimate of the gravitational potential coefficients, accompanied by a set of BD normal equations. The fact that a spherical harmonic expansion of the gravitational potential complete to degree and order 2159 involves approximately 4.7 million coefficients, is what necessitates here the BD approximation of the normal equations.

Step 6: The GRACE-only normal equations’ matrix is approximated so that it adheres to the same BD pattern that was used in the “terrestrial” gravity normal equations, by simply equating to zero the elements of the matrix that reside outside the diagonal blocks of interest. The two sets of BD normal equations are then combined and inverted to yield the potential coefficients of the combination solution and their associated error estimates.

Step 7: The MSS from Step 1 and the combination solution from Step 6 are used to estimate a new model of the DOT. Using this new DOT model, one returns to Step 2 and re-iterates the process.

Albeit iterative, the procedure outlined above is quite straightforward and very economic with respect to its computational resource requirements. The estimation of the combined solution according to the above procedure relies on input data that are of the same type as those required to develop the OSU89A/B models [*Rapp and Pavlis*, 1990]. A shortcoming of the above procedure is that it does not permit the simultaneous estimation of the DOT model along with the gravitational potential coefficients. This is a shortcoming that EGM2008 shares with the OSU89A/B models. This shortcoming is offset by the fact that the procedure yields a combined gravitational model as the output of a single least squares adjustment, with an estimated error spectrum free of any discontinuities, and without the need to resort to composite gravitational solutions.

The development of EGM2008 involved essentially two iterations of the procedure outlined above. The timing of the preparation and availability of certain data sets required appropriate modifications to be made to the above procedure as we discuss in following sections, so that the EGM2008 development project could progress without significant delays. During the course of the project, three sets of Preliminary Gravitational Models were developed: PGM2004 [*Pavlis et al.*, 2005], which served as a demonstration of the capability to perform such combination solutions to degree 2160 and indicated the quality of the results to be expected, PGM2006 [*Pavlis et al.*, 2006a], and PGM2007 [*Pavlis et al.*, 2007a]. Unlike PGM2004 and PGM2006, which remain internal to the project, one of the PGM2007 solutions was also released for evaluation to an independent IAG/IGFS Special Working Group.

2.2 Gravity Anomaly and Potential Coefficient Relations

For the benefit of the non-specialist, we briefly discuss here the main concepts associated with Molodensky's theory, which constitutes the theoretical framework upon which the work presented in this paper was based. Unlike the conventional approach, Molodensky's approach aims to determine the external gravity field of the Earth *without* any assumptions concerning the density of the masses above the geoid [*Heiskanen and Moritz*, 1967, section 8-3]. The physical topographic surface of the Earth and the telluroid are central to Molodensky's formulation. The former is the surface where gravity field measurements are made or to which they are reduced if they were made above that surface; the latter is a surface whose normal potential U at every point Q is equal to the actual gravity potential W at the corresponding surface point P, with the points P and Q being situated on the same line that is normal to the ellipsoidal [*ibid.*, 1967, p. 292]. The distance between points P and Q, measured along the ellipsoidal normal is called height anomaly, and corresponds to the geoid undulation that is used in the conventional approach [*ibid.*, 1967, p. 292]. Over the ocean, height anomalies are virtually identical to geoid undulations; over land their difference is a function of the Bouguer anomaly and the elevation [*Heiskanen and Moritz*, 1967, section 8-13]. Unlike the geoid, the telluroid is an "observable" surface (the positions of its points can be calculated, in principle, using, e.g., gravity measurements, spirit leveling, and astronomical observations of latitude). Its adoption implies, formally at least, a more precise free-air correction when gravity anomalies are defined relative to this surface (Molodensky free-air gravity anomalies) rather than to the geoid (classical free-air anomalies) which, in rugged terrain, can be much farther away from the terrain [*Heiskanen and Moritz*, 1967, section 8-3].

The Earth's external gravitational potential, V, at a point P defined by its geocentric distance (r), geocentric co-latitude (θ) (defined as 90°-latitude), and longitude (λ), is given by:

$$V(r,\theta,\lambda) = \frac{GM}{r}\left[1 + \sum_{n=2}^{\infty}\left(\frac{a}{r}\right)^n \sum_{m=-n}^{n} \bar{C}_{nm}^{s}\bar{Y}_{nm}(\theta,\lambda)\right], \tag{1}$$

where GM is the geocentric gravitational constant and a is a scaling factor associated with the fully-normalized, unitless, spherical harmonic coefficients $\bar{C}_{nm}^{s}$. The superscript "s" identifies the coefficients as being *spherical*. a is usually numerically equal to the equatorial radius of an adopted reference ellipsoid. Equation (1) refers to the permanent part of the gravity field, either ignoring or having corrected first for the variable part due to tides, changes in Earth rotation, etc.. The fully-normalized surface spherical harmonic functions are defined as [*Heiskanen and Moritz*, 1967, section 1-14]:

$$\bar{Y}_{nm}(\theta,\lambda) = \bar{P}_{n|m|}(\cos\theta)\cdot\left\{\begin{array}{c}\cos m\lambda \\ \sin|m|\lambda\end{array}\right\} \quad \begin{array}{l}\text{if } m \geq 0 \\ \text{if } m < 0\end{array} \quad . \tag{2}$$

$\bar{P}_{n|m|}(\cos\theta)$ is the fully-normalized associated Legendre function of the first kind, of degree n and order $|m|$. Geocentricity of the coordinate system used, forces the absence of first-degree terms in equation (1). We define the disturbing potential T as the difference between the actual gravity potential of the Earth and the "normal" gravity potential associated with a rotating equipotential ellipsoid of revolution. Detailed formulation of the normal gravity field of such a level ellipsoid (Somigliana-Pizzetti normal gravity field) can be found in [*Heiskanen and Moritz*, 1967, section 2-7]. In our Appendix A, we specify the actual parameters defining the reference ellipsoid and its normal gravity field that was used in the processing of gravity anomalies, the scaling parameters GM and a associated with the potential coefficients in equation (1), and the reference ellipsoid parameters associated with certain model products, as the geoid undulations that are expressed in the WGS 84 Geodetic Reference System. Provided that the rotational speed of the reference ellipsoid is the same as the actual rotational speed of the Earth, so that actual and normal centrifugal potentials would cancel out, the spherical harmonic expansion of T is given by:

$$T(r,\theta,\lambda) = \frac{GM}{r} \sum_{n=2}^{\infty} \left(\frac{a}{r}\right)^n \sum_{m=-n}^{n} \bar{C}^s_{nm} \bar{Y}_{nm}(\theta,\lambda) \ . \tag{3}$$

The zero degree term in equation (3) has been set to zero, forcing the equality of the actual mass of the Earth and the mass of the chosen reference ellipsoid. Furthermore, the even-degree zonal harmonic coefficients in equation (3) represent now the difference between the harmonic coefficients of the actual minus the normal gravitational potentials. We define next the quantity Δg^c (see also [*Rapp and Pavlis*, 1990]) as:

$$\Delta g^c = -\frac{\partial T}{\partial r} - \frac{2}{r} T \ , \tag{4}$$

so that, from equation (3), we have:

$$\Delta g^c(r,\theta,\lambda) = \frac{GM}{r^2} \sum_{n=2}^{\infty} (n-1) \left(\frac{a}{r}\right)^n \sum_{m=-n}^{n} \bar{C}^s_{nm} \bar{Y}_{nm}(\theta,\lambda) \ . \tag{5}$$

The quantity Δg^c is not directly observable. However, it can be estimated, based on the Molodensky surface free-air gravity anomaly Δg [*Heiskanen and Moritz,* 1967, p. 293]. Δg is defined to be the difference of the magnitude of the actual gravity acceleration, which is directly observable using scalar gravimetric techniques, at the surface point P, minus the magnitude of the normal gravity acceleration that can be computed at the corresponding telluroid point Q (see [*Heiskanen and Moritz*, 1967, section 8-3] for details), i.e.,

$$\Delta g = \left|\vec{g}_P\right| - \left|\vec{\gamma}_Q\right| \ . \tag{6}$$

Pavlis [1988, section 2.1.2] provides the specific definition of the telluroid that is employed here. Point values of Δg, at arbitrarily scattered locations, are the primary data obtained from terrestrial gravimetric surveys. One can use these data, together with detailed digital elevation information, to estimate area-mean values of gravity anomalies (denoted by $\overline{\Delta g}$), over cells equiangular in latitude and longitude. *Colombo* [1981] put forward very efficient numerical techniques that may be used to estimate a set of spherical harmonic coefficients, given a complete global set of data values, defined on an equiangular grid, over a surface of revolution e.g., an ellipsoid of revolution. It is therefore desirable, starting from the original Δg data, which may originate not only from terrestrial gravity surveys, but also from airborne, marine, and satellite altimetry-derived estimates, to form a complete global set of area-mean values of the quantities Δg^c (denoted by $\overline{\Delta g}^c$). If then these $\overline{\Delta g}^c$ values could be continued analytically, from their surface of reference (the Earth's topography), to the surface of an ellipsoid of revolution, then they could be used as input to the potential coefficient estimator, exploiting the efficiencies of Colombo's [*ibid*.] techniques. This procedure could yield a "terrestrial" estimate of the potential coefficients. This estimate, accompanied by its error covariance information, could then be combined in a least squares sense with a corresponding "satellite" estimate (obtained in the present study from GRACE data), to determine the potential coefficients of the combined solution.

The general procedure outlined above involves the application of several systematic corrections to the original data, as *Rapp and Pavlis* [1990] discuss in detail. Of these, the atmospheric correction, and the correction accounting for the second-order vertical gradient of normal gravity, are applied most conveniently during the pre-processing of the point gravity anomaly data Δg. Ellipsoidal corrections on the other hand, can be applied conveniently to the area-mean values $\overline{\Delta g}$, using some preliminary estimate of the potential coefficients (see also [*Pavlis*, 1988] for details). Finally, one may use some technique of analytical downward continuation [*Moritz*, 1980, p. 378], to compute from $\overline{\Delta g}^{c}$, a corresponding *fictitious* quantity $\overline{\Delta g}^{e}$, defined to reside on the surface of the reference ellipsoid. $\overline{\Delta g}^{e}$ is defined such that, when analytically continued in the opposite (upward) direction, it should reproduce $\overline{\Delta g}^{c}$. Apart from this requirement, $\overline{\Delta g}^{e}$ possesses no physical meaning and certainly does not represent the gravity anomaly inside the topographic masses. Let r_i^e be the geocentric distance to the center a cell residing on the *i*-th (*i=0, … N-1*) latitude belt ("row") and *j*-th (*j=0, … 2N-1*) meridional sector ("column"), within a global equiangular grid composed of *N* rows by *2N* columns of $\overline{\Delta g}_{ij}^{e}$ area-mean values, on the surface of the reference ellipsoid. For the small (5 arc-minute) equiangular cell size used in this study, the small and regular latitudinal variation of r^e within the cell can be safely ignored (see also [*Rapp and Pavlis*, 1990, p. 21,887]), so that we may approximate:

$$\overline{(r\Delta g)}_{ij}^{e} \approx r_i^e \cdot \overline{\Delta g}_{ij}^{e} \ . \tag{7}$$

The product $r_i^e \cdot \overline{\Delta g}_{ij}^{e}$, defined over the surface of the reference ellipsoid, can be expanded in surface ellipsoidal harmonic functions [*Heiskanen and Moritz*, 1967, section 1-20], as:

$$r_i^e \cdot \overline{\Delta g}_{ij}^{e} = \frac{1}{\Delta\sigma_i} \frac{GM}{a} \sum_{n=2}^{\infty} (n-1) \sum_{m=-n}^{n} \bar{C}_{nm}^{e} \cdot I\bar{Y}_{nm}^{ij} \ . \tag{8}$$

With δ denoting the reduced co-latitude [*Heiskanen and Moritz*, 1967, section 1-19], the terms in equation (8) are defined as:

$$\Delta\sigma_i = \Delta\lambda \int_{\delta_i}^{\delta_{i+1}} \sin\delta d\delta = \Delta\lambda \cdot \left(\cos\delta_i - \cos\delta_{i+1}\right) \tag{9}$$

$$I\bar{Y}_{nm}^{ij} = \int_{\delta_i}^{\delta_{i+1}} \bar{P}_{n|m|}(\cos\delta)\sin\delta d\delta \cdot \int_{\lambda_j}^{\lambda_{j+1}} \left\{ \begin{array}{c} \cos m\lambda \\ \sin|m|\lambda \end{array} \right\} d\lambda \qquad \begin{array}{l} \text{if } m \geq 0 \\ \text{if } m < 0 \end{array} \ . \tag{10}$$

The quantity $r^e \Delta g^e$ represents a harmonic function, and, under the approximation of equation (7), so does the quantity $r_i^e \cdot \overline{\Delta g}_{ij}^{e}$. This allows one to relate the *ellipsoidal* harmonic coefficients $\bar{C}^e_{nm}$ of equation (8), to the corresponding *spherical* harmonic coefficients $\bar{C}^s_{nm}$ appearing in equations (3) and (5), using the exact transformations derived by *Jekeli* [1988] and implemented and verified by *Gleason* [1988]. Note that our $\bar{C}^s_{nm}$ and $\bar{C}^e_{nm}$ coefficients are related to the corresponding $\bar{g}^s_{n,m}$ and $\bar{\bar{g}}^e_{n,m}$ coefficients of Gleason [*ibid.*] by:

$$\begin{Bmatrix} \bar{g}^s_{n,m} \\ \bar{\bar{g}}^e_{n,m} \end{Bmatrix} = \frac{GM}{a^2}(n-1)\cdot\begin{Bmatrix} \bar{C}^s_{nm} \\ \bar{C}^e_{nm} \end{Bmatrix} . \tag{11}$$

The transformation from spherical to ellipsoidal harmonic coefficients is given in [*Gleason*, 1988, equation 2.8]:

$$\bar{\bar{g}}^e_{n,m} = \bar{S}_{n|m|}\left(\frac{b}{E}\right)\cdot\sum_{k=0}^{s'}\lambda_{n,m,k}\cdot\bar{g}^s_{n-2k,\,m} , \tag{12}$$

and the transformation from ellipsoidal to spherical harmonic coefficients is given in [*Gleason*, 1988, equation 2.10]:

$$\bar{g}^s_{n,m} = \sum_{k=0}^{s'}\frac{1}{\bar{S}_{n-2k,\,|m|}\left(\frac{b}{E}\right)}\cdot L_{n,\,m,\,k}\cdot\bar{\bar{g}}^e_{n-2k,\,m} . \tag{13}$$

b is the semi-minor axis and E the linear eccentricity of the adopted reference ellipsoid [*Heiskanen and Moritz*, 1967, section 1-19], and the definition of the other terms appearing in equations (12) and (13) can be found in [*Gleason*, 1988]. It is important to note that both transformations are linear, and they both relate coefficients of the same type, order, and parity of *n-m*. Importantly, equations (12) and (13) imply that both transformations preserve the maximum order but *not* the maximum degree of a set of coefficients. As *Jekeli* [1988, p. 112] has pointed out, a finite number of spherical harmonic coefficients generate an infinite number of ellipsoidal harmonic coefficients and vice versa. The additional coefficients, of degree higher than the highest degree within the series being transformed, are linear combinations of lower degree terms. These "extra" terms may be negligible for expansions up to degree 360 or so, but become important for expansions up to degree 2159, as *Holmes and Pavlis* [2007] have demonstrated.

Considering also equation (11), the transformation (12) can be written in vector-matrix form as:

$$\left[\bar{\mathbf{C}}^{e}_{\mathrm{g}m}\right] = \mathbf{T}^{se}_{m} \cdot \left[\bar{\mathbf{C}}^{s}_{\mathrm{g}m}\right], \tag{14}$$

where $\mathbf{T}^{se}_{m}$ is the transformation matrix applicable to order m, whose elements are computed based on equation (12). Let $\mathbf{T}^{se}$ be the combined transformation matrix, composed of the $\mathbf{T}^{se}_{m}$ sub-matrices, for all orders within a set of spherical harmonic coefficients $\bar{\mathbf{C}}^{s}$, whose error covariance matrix is $\mathbf{\Sigma}_{\bar{\mathbf{C}}^{s}}$. Then the computation of the corresponding ellipsoidal harmonic coefficients, using the transformation of equation (12), can be written in the form:

$$\left[\bar{\mathbf{C}}^{e}\right] = \mathbf{T}^{se} \cdot \left[\bar{\mathbf{C}}^{s}\right] . \tag{15}$$

Error propagation implies that the error covariance matrix of $\bar{\mathbf{C}}^{e}$ is given by:

$$\Sigma_{\bar{\mathbf{C}}^{e}} = \mathbf{T}^{se} \cdot \Sigma_{\bar{\mathbf{C}}^{s}} \cdot \left(\mathbf{T}^{se}\right)^{T} , \tag{16}$$

where the superscript *"T"* denotes the transpose of a matrix. Similarly, the transformation from ellipsoidal to spherical harmonic coefficients can be written in the form:

$$\left[\bar{\mathbf{C}}^{s}\right] = \mathbf{T}^{es} \cdot \left[\bar{\mathbf{C}}^{e}\right] , \tag{17}$$

and the corresponding error covariance propagation formula is:

$$\Sigma_{\bar{\mathbf{C}}^{s}} = \mathbf{T}^{es} \cdot \Sigma_{\bar{\mathbf{C}}^{e}} \cdot \left(\mathbf{T}^{es}\right)^{T} , \tag{18}$$

where the elements of matrix $\mathbf{T}^{es}$ are computed from equations (11) and (13).

The formulation presented so far allows one to estimate a set of ellipsoidal harmonic coefficients $\bar{\mathbf{C}}^{e}$ from a global set of area-mean free-air gravity anomalies $\overline{\Delta g}^{e}$ that have been analytically continued to the surface of the reference ellipsoid. The estimation of $\bar{\mathbf{C}}^{e}$ is based on the (linear) mathematical model of equation (8), expressed as a finite series, truncated to some maximum degree Nmax that is commensurate with the size of the equiangular cells forming the global grid (e.g., Nmax=2159 for 5 arc-minute equiangular cells), i.e.,

$$r_i^e \cdot \overline{\Delta g}_{ij}^e = \frac{1}{\Delta\sigma_i} \frac{GM}{a} \sum_{n=2}^{\mathrm{Nmax}} (n-1) \sum_{m=-n}^{n} \bar{C}_{nm}^e \cdot I\bar{Y}_{nm}^{ij} \ . \tag{19}$$

Based on equation (19), one forms a system of observation equations that can be written as:

$$\mathbf{v} = \mathbf{A} \cdot \hat{\mathbf{x}} - \mathbf{L_b} \ , \tag{20}$$

where $\mathbf{L_b}$ is the vector of observations $\overline{\Delta g}^e$, $\mathbf{v}$ is the vector of corresponding residuals, $\mathbf{A}$ is the design matrix whose elements are formed based on equation (19), and $\hat{\mathbf{x}}$ represents the vector of estimated coefficients $\bar{\mathbf{C}}^e$. To be specific, $\hat{\mathbf{x}}$ represents estimated incremental changes to the coefficients. The actual coefficient values are obtained after the adjustment, by adding $\hat{\mathbf{x}}$ to the reference coefficient values that were used within the adjustment. The least squares solution, $\hat{\mathbf{x}}$, which satisfies the condition:

$$\mathbf{v^T P v} = \text{minimum} \ , \tag{21}$$

is given by [*Uotila*, 1986]:

$$\left.\begin{array}{ll} \hat{\mathbf{x}} = \mathbf{N}^{\mathbf{-1}}\mathbf{U} & \text{(a)} \\ \mathbf{N} = \mathbf{A}^{\mathbf{T}}\mathbf{P}\,\mathbf{A} & \text{(b)} \\ \mathbf{U} = \mathbf{A}^{\mathbf{T}}\mathbf{P}\,\mathbf{L}_{\mathbf{b}} & \text{(c)} \end{array}\right\} , \tag{22}$$

where $\mathbf{P}$ is the weight matrix associated with the observations $\overline{\Delta g}^{e}$. In this study, $\mathbf{P}$ was assumed to be diagonal, with each diagonal element equal to the reciprocal of the error variance of the corresponding gravity anomaly observation, i.e.:

$$\mathbf{P} = \sigma_0^2 \cdot \begin{pmatrix} \dfrac{1}{\sigma_1^2} & & 0 \\ & \ddots & \\ 0 & & \dfrac{1}{\sigma_K^2} \end{pmatrix} , \tag{23}$$

where K is the total number of observations, and σ_0^2 is the *a priori* variance of unit weight, taken equal to 1. For the complete global equiangular grid of 5 arc-minute area-mean gravity anomalies used here, K=2160×4320=9331200. The assumption that the gravity anomaly errors are uncorrelated is made out of necessity, rather than desire. It is extremely difficult to estimate error correlations between the gravity anomalies with any degree of accuracy. It is also practically impossible to handle numerically an arbitrary fully-occupied (symmetric) weight matrix of dimension 9331200. Even the estimation of realistic error variances for the gravity anomalies is a very challenging task.

With Nmax=2159, the expansion given in equation (19) involves exactly 4665596 unknown ellipsoidal harmonic coefficients. A weight matrix $\mathbf{P}$, with elements of arbitrary value on the diagonal, would result, in general, in a fully-occupied, symmetric, normal matrix $\mathbf{N}$ of dimension 4665596×4665596. The creation, storage, and inversion of such a matrix are impractical, if not altogether impossible, given the presently available computational capabilities. Therefore, we have approximated the normal matrix $\mathbf{N}$ with its "Type 1" Block-Diagonal form (BD1), whereby non-zero off-diagonal elements occur only between coefficients of the same type, order, and parity of *n-m*, as it is discussed in detail by *N. Pavlis* in [*Lemoine et al., 1998*, section 8.2.2]. This approximation requires also the careful "calibration" of the values of the weights used in $\mathbf{P}$, so that excessive weight ratios are avoided (see also *N. Pavlis* in [*Lemoine et al., 1998*, section 8.5]).

The residuals obtained from equation (20) represent merely a measure of "goodness of fit" of the truncated model (19) to the input data $\overline{\Delta g}^{e}$, i.e., they show how well the truncated series of ellipsoidal harmonics of equation (19) manages to reproduce the input data, but do not provide any information regarding the accuracy itself of the input data.

Two additional aspects of the above formulation are noteworthy:

(a) In equations (8) and (19) the summation starts from harmonic degree 2. However, there is no guarantee that real data will not possess any zero- and first-degree terms. These terms, meaningless as they may be, if left in the data and are not solved-for, could alias other low degree coefficients of the same order. Therefore, in this study, any zero- and first-degree

$\bar{\mathbf{C}}^e$ coefficients.

(b) The properties of the transformations (12) and (13) discussed previously are such that both transformations preserve the BD1 block-diagonal pattern in normal and error covariance matrices of coefficient sets. This has important implications in the combination solution estimation method used in this study, as we discuss in section 2.4.

2.3 Analytical Downward Continuation

The formulation presented in section 2.2 requires that the free-air gravity anomaly observations $\overline{\Delta g}^t$, originally referring to the Earth's topography, be continued analytically downward to the reference ellipsoid, to form the quantities $\overline{\Delta g}^e$. Here, $\overline{\Delta g}^t$ denotes area-mean values of the Molodensky surface free-air gravity anomalies defined in equation (6). The superscript "*t*" is used here to distinguish these values that refer to the topography, from their counterparts $\overline{\Delta g}^e$, which are values analytically continued to the ellipsoid. All the surface free-air gravity anomalies mentioned in this paper are "Molodensky surface free-air gravity anomalies". Such continuation requires knowledge of the vertical gradients of the area-mean free-air gravity anomalies, since:

$$\overline{\Delta g}^t = \overline{\Delta g}^e + \sum_{k=1}^{\infty} \frac{1}{k!} \frac{\partial^k \overline{\Delta g}^e}{\partial h^k} h^k \ . \tag{24}$$

h denotes here the area-mean value of the ellipsoidal height of the cell to which both $\overline{\Delta g}^{t}$ and $\overline{\Delta g}^{e}$ refer. h can be computed from the area-mean value of the orthometric height H that is available from a global Digital Topographic Model and from a geoid undulation estimate N that can be obtained from an existing gravitational model, as $h = H + N$. Under linear theory, and for quantities related to the disturbing potential, such as the gravity anomaly and its vertical gradients, we make no distinction here between the topographic surface and the telluroid. Truncation of the series in (24) to the linear term yields [*Rapp and Pavlis*, 1990]:

$$\overline{\Delta g}^{e} = \overline{\Delta g}^{t} + g_1 = \overline{\Delta g}^{t} - \overline{hL(\Delta g)} \ , \tag{25}$$

where:

$$L(\Delta g) = \frac{R^2}{2\pi} \iint_{\sigma} \frac{\Delta g - \Delta g_P}{\ell_0^3} \, d\sigma \ . \tag{26}$$

ℓ_0 is the distance between the variable point and the computation point *P*, and *R* is a mean-Earth radius (e.g., 6371 km). Equation (25) provides the so-called “gradient solution” to the downward continuation problem [*Moritz*, 1980, p. 387]. If, in addition, one assumes that the free-air gravity anomaly is linearly correlated with elevation, i.e.,

$$\Delta g = a + bh \ , \tag{27}$$

where $b = 2\pi G\rho$ and ρ is the crustal density, then the g_1 terms in equation (25) become [*Rapp and Pavlis,* 1990]:

$$g_1 = -G\rho R^2 h_P \iint_\sigma \frac{h - h_P}{\ell_0^3} d\sigma \ . \tag{28}$$

Use of a constant value of $\rho = 2670$ kg/m^3, implies that the g_1 terms may be computed based purely on elevation information, under the above assumptions and approximations. *Wang* in [*Lemoine et al., 1998*, section 8.4] implemented and compared three different techniques for the analytical continuation of the 30 arc-minute area-mean gravity anomalies used to develop EGM96: equation (28), Poisson's integral [*Heiskanen and Moritz*, 1967, p. 318], and computation of the g_1 terms using the first-order free-air gravity anomaly gradient implied by a pre-existing gravitational model complete to degree 360. The final computation of EGM96 employed the g_1 terms computed based on equation (28).

In this study we implemented two methods for the analytical continuation of the 5 arc-minute area-mean gravity anomalies:

Method A: We computed g_1 terms based on equation (28), using the 30 arc-second elevation data of the DTM2006.0 database that we describe in section 3.2. Global equiangular grids of area-mean values of these g_1 terms were formed, in both 2 and 5 arc-minute grid sizes.

Method B: Computation based on an iterative implementation of equation (24). We rewrite equation (24) as:

$$\overline{\Delta g}^{e} = \overline{\Delta g}^{t} - \sum_{k=1}^{M} \frac{1}{k!} \frac{\partial^k \overline{\Delta g}^{e}}{\partial h^k} h^k \ , \tag{29}$$

and employ an iterative approach to estimate the free-air gravity anomaly gradients. We initialize this approach by setting:

$$\overline{\Delta g}_0^{e} = \overline{\Delta g}^{t} \ . \tag{30}$$

We use these $\overline{\Delta g}_0^{e}$ values to estimate an initial set of ellipsoidal harmonic coefficients complete to degree and order 2159. From these coefficients, we compute an initial set of gradients, and from these, using equation (29), an updated set of downward continued anomalies $\overline{\Delta g}_1^{e}$. We iterate this process until we achieve convergence. Through numerical tests we determined that a value of M=10 and seven iterations, were sufficient to achieve convergence. Larger values of M neither improved the results, nor reduced the number of iterations necessary to achieve convergence.

Method B has significant advantages compared to Method A. It is self-consistent and relies only on the available area-mean gravity anomalies to be continued for its implementation, it avoids the truncation of gradients to first-order terms only, and it is free of the assumption (27). Through numerical tests with simulated data, we verified that Method B performs particularly well when the 5 arc-minute data are band-limited to degree 2159 in ellipsoidal harmonics; we found however that its performance degrades considerably when the frequency content of the data exceeds this degree. Therefore, part of our efforts related to the compilation of the global 5 arc-minute area-mean gravity anomaly data set, focused on the development of a technique for the estimation of $\overline{\Delta g}^t$ that would yield values band-limited to degree 2159, to a high degree of approximation. We discuss this estimation technique in section 3.4.

2.4 Combination Solution Estimation

The solution obtained from equation (22a) represents an estimate of the ellipsoidal harmonic gravitational potential coefficients, $\bar{\mathbf{C}}^e$, obtained *solely* on the basis of the "terrestrial" $\overline{\Delta g}^e$ gravity anomaly data. It is well known (see, e.g., [*Pavlis*, 1998]) that these data suffer from significant long wavelength errors. In contrast, the gravitational information obtained from satellite tracking data is highly accurate at long wavelengths, but lacks short wavelength details, due to the attenuation of the gravitational signal with altitude. We have exploited the complementary character of terrestrial and satellite data, and developed the global gravitational model from the least squares combination of these two sources of gravitational information. Two specific aspects of this least squares adjustment are discussed next.

Satellite-only models like the ITG-GRACE03S model used here are conveniently developed in terms of *spherical* harmonic coefficients. In contrast, the $\bar{\mathbf{C}}^e$ estimates represent *ellipsoidal* harmonic coefficients. Therefore, before any least squares combination of the two estimates is made, the two estimates, as well as their associated error covariance information, must be converted to a common type of coefficients. This is done most conveniently by first converting the satellite-only model and its associated error covariance matrix from the spherical to the ellipsoidal harmonics representation, using equations (15) and (16) respectively. Then, the least squares adjustment is performed in terms of ellipsoidal harmonic coefficients. Finally, the adjusted coefficients and their error estimates are converted to the spherical harmonics representation, since this representation is most commonly used in geodetic and geophysical applications, using equations (17) and (18) respectively. This last conversion, produces the "extra" coefficients, beyond degree 2159 and up to degree 2190.

As noted in section 2.2, the conversions between the ellipsoidal and spherical harmonic coefficients preserve the BD1 block-diagonal pattern. This implies that the error covariance matrix of the ITG-GRACE03S model, which in its original spherical harmonics representation can be approximated very closely with this block-diagonal pattern, maintains this form also after the conversion to ellipsoidal harmonics. Therefore, the least squares combination solution can be performed efficiently by "overlaying" and adding together the diagonal blocks of the BD1-approximated ITG-GRACE03S normal equations, expressed in ellipsoidal harmonics, over the larger corresponding blocks of the "terrestrial" normal equations. To illustrate the process, in Figure 1 we use a hypothetical combination of satellite-only normal equations from an expansion complete from degree 2 to degree 4, with "terrestrial" normal equations from an expansion complete from degree 2 to degree 6. Figure 1 shows in gray the blocks of the normal matrix $\mathbf{N}$ and the elements of the right-hand-side vector $\mathbf{U}$ of the "terrestrial" normal equations, overlaid with the corresponding elements of the satellite-only normal equations, which are shown in black. The ordering of the unknown coefficients is according to the ordering pattern "V" in [*Pavlis, 1988*, Table 3].

The optimal combination of the satellite-only normal equations with their "terrestrial" counterparts depends critically on the relative weights used for the two sources of gravitational information, as we discuss in section 4. The combined normal equations can then be solved, one diagonal block at a time. In this fashion, the largest symmetric matrix that needs to be inverted in our case has dimension 1080, which does not present a computational challenge nowadays. This approximation simplifies significantly the combination solution adjustment, compared to the approach that was implemented in the development of the degree 71 to 359 portion of EGM96. As *N. Pavlis* in [*Lemoine et al., 1998*, section 8.2.4] discusses in detail, the combination solution normal equations there acquired the "falling kite" pattern of Figure 8.2.4-1(f) [*ibid.*]. This was primarily due to the inability to approximate the EGM96S normal equations with any block-diagonal pattern, without compromising significantly the quality of the results. The error characteristics of the GRACE information enabled here this block-diagonal approximation to be made, which allowed the combination solution to be performed in a highly efficient and rather elegant fashion, without compromising the quality of the results.

3. DATA USED IN THE ANALYSIS

The essential "ingredients" necessary for the estimation of the present high resolution global gravitational model are a solution based on GRACE data, accompanied by its complete error covariance matrix, and a complete global set of 5 arc-minute area-mean free-air gravity anomalies. The estimation of these gravity anomalies, as well as other aspects of the solution, also require a very high resolution global Digital Topographic Model (DTM). The estimated gravity anomalies need to be analytically downward continued to the surface of the reference ellipsoid. Ideally, these gravity anomalies should have uniform and high accuracy, and should only contain spectral information associated with the solved-for harmonic coefficients. Since the 5 arc-minute equiangular grid of the gravity anomalies on the reference ellipsoid permits the unbiased estimation of a set of ellipsoidal harmonic coefficients, complete to degree and order 2159 [*Colombo*, 1981], in order to minimize aliasing effects it is desirable to filter out of the 5 arc-minute data any spectral contributions beyond ellipsoidal harmonic degree and order 2159 (see also [*Pavlis*, 1988] and [*Jekeli*, 1996]). In the following sections we describe the data that were used to compile the essential "ingredients" necessary to develop this combination solution.

3.1 The ITG-GRACE03S Model

The ITG-GRACE03S [*Mayer-Gürr*, 2007] satellite-only model that was used in the development of EGM2008 was computed at the Institute of Theoretical Geodesy of the University of Bonn in Germany. ITG-GRACE03S is based on GRACE Satellite-to-Satellite Tracking (SST) data acquired during the 57-month period from September 2002 to April 2007. No other data were used in its development, which followed the short-arc analysis approach described by *Mayer-Gürr et al.* [2007]. ITG-GRACE03S is complete to spherical harmonic degree and order 180, and was made available accompanied by its fully-occupied error covariance matrix. The model was developed without application of any *a priori* information or any other regularization constraint. Therefore, the model ($\hat{\mathbf{x}}$) itself and its complete error covariance matrix ($\Sigma_{\mathbf{x}}$) are sufficient to recreate exactly the normal equation system that produced it, recalling that the error covariance matrix is the inverse of the normal equation matrix, i.e. from:

$$\left.\begin{array}{ll}\mathbf{N} = \Sigma_{\mathbf{x}}^{\mathbf{-1}} & \text{(a)} \\ \mathbf{U} = \mathbf{N}\hat{\mathbf{x}} & \text{(b)}\end{array}\right\} . \tag{31}$$

3.2 The Digital Topographic Model DTM2006.0

The pre-processing and analysis of the detailed surface gravity data necessary to support the development of an EGM to degree 2160, depends critically on the availability of accurate topographic data, at a resolution sufficiently higher than the 5 arc-minute resolution of the area-mean gravity anomalies that will be used eventually to develop the EGM. *J.K. Factor* in [*Lemoine et al.*, 1998, Section 2.1] discusses some of the uses of such topographic data within the context of a high resolution EGM development. These include the computation of Residual Terrain Model (RTM) effects [*Forsberg*, 1984], the computation of analytical continuation terms, the computation of Topographic/Isostatic gravitational models that may be used to "fill-in" areas void of other data, and the computation of models necessary to convert height anomalies to geoid undulations [*Rapp*, 1997]. For these computations to be made consistently, it is necessary to first compile a high-resolution global Digital Topographic Model (DTM), whose data will support the computation of all these terrain-related quantities.

For EGM96 [*Lemoine et al.*, 1998], which was complete to degree and order 360, a global digital topographic database (JGP95E) at 5 arc-minute resolution was considered sufficient. JGP95E was formed specifically to support the development of EGM96, by merging data from 29 individual sources, and, as acknowledged by its developers, left a lot to be desired in terms of accuracy and global consistency. Since that time, and thanks primarily to the Shuttle Radar

Topography Mission (SRTM) [*Werner*, 2001], significant progress has been made on the topographic mapping of the Earth from space. During approximately 11 days in 2000 (February 11-22), the SRTM collected data within latitudes 60°N and 56°S, thus covering approximately 80 percent of the total land area of the Earth with elevation data of high, and fairly uniform, accuracy. *Rodriguez et al.* [2005] discuss in detail the accuracy characteristics of the SRTM elevations. Comparisons with ground control points whose elevations were determined independently using kinematic GPS positioning, indicate that the 90 percent absolute error of the SRTM elevations ranges from ±6 to ±10 meters, depending on the geographic area [*ibid.*, Table 2.1]. Additional information regarding the SRTM can be obtained from the web site of the United States' Geological Survey (USGS) (http://srtm.usgs.gov/), and from the web site of NASA's Jet Propulsion Laboratory (http://www2.jpl.nasa.gov/srtm).

In preparation for the development of the EGM2008 model, we compiled DTM2006.0 by overlying the SRTM data over the data of DTM2002 [*Saleh and Pavlis*, 2003]. In addition to the SRTM data, DTM2006.0 contains ice elevations derived from ICESat laser altimeter data over Greenland [*Ekholm, personal communication*, 2005] and over Antarctica [*DiMarzio, personal communication*, 2005]. Over Antarctica, data from the "BEDMAP" project (http://www.antarctica.ac.uk/aedc/bedmap/) were also used to define ice and water column thickness. Over the ocean, DTM2006.0 contains essentially the same information as DTM2002, which originates in the estimates of bathymetry [*Smith and Sandwell*, 1997] from altimetry data and ship depth soundings. DTM2006.0 was compiled in 30 arc-second resolution, providing height and depth information only, and in 2, 5, 30 and 60 arc-minute resolutions, where lake depth and ice thickness data are also included. DTM2006.0 is identical to DTM2002 in terms of database structure and information content. We used the DTM2006.0 data to compute the following quantities:

(a) Fully-normalized spherical harmonic coefficients of the elevations ($\overline{H}_{nm}$). These are consistent with the model:

$$\overline{H}_{ij} = \overline{H}(\theta_i, \lambda_j) = \frac{1}{\Delta\sigma_i} \sum_{n=0}^{K} \sum_{m=-n}^{n} \overline{H}_{nm} \cdot I\overline{Y}_{nm}^{ij} , \tag{32}$$

where $\overline{H}_{ij}$ represents the area-mean value of an elevation-related quantity, such as heights above and depths below Mean Sea Level (MSL), over a cell located on the *i*-th "row" and *j*-th "column" of the global equiangular grid. The terms $\Delta\sigma_i$ and $I\overline{Y}_{nm}^{ij}$ are defined exactly as in equations (9) and (10), but evaluated here using the geocentric co-latitude θ, instead of the reduced co-latitude δ. We used the 2 arc-minute version of DTM2006.0 to evaluate a set of coefficients $\overline{H}_{nm}$ complete to degree and order *K*=2700. These coefficients, up to degree and order 2160, were used to form the terms necessary to convert height anomalies to geoid undulations, as described by *Rapp* [1997]. The same coefficients, to degree and order 360, we used to form the reference surface, with respect to which we computed the RTM-implied gravity anomalies, as we discuss below.

(b) We used the 30 arc-second version of DTM2006.0 to evaluate the g_1 analytical continuation terms according to equation (28), over all land areas, on the same 30 arc-second grid. We then formed global equiangular grids of the area-mean values of these terms in both 2 and 5

(c) We used the 30 arc-second version of DTM2006.0 and computed on the same 30 arc-second grid extending over all of the Earth's land areas, including a 10 km margin protruding into the ocean, gravity anomalies implied by a Residual Terrain Model (RTM). This RTM was referenced to a topographic surface, created from the elevation harmonic coefficients described under (a) above, to degree and order 360. We computed the RTM-implied gravity anomalies Δg_{RTM} as described in detail by *Forsberg* [1984]. We then formed 2 arc-minute area-mean values of these anomalies and supplemented this (primarily) land dataset with zero values for the cells that are located over ocean areas, excluding the margin mentioned above. We analyzed harmonically this 2 arc-minute Δg_{RTM} grid to compute a set of ellipsoidal harmonic coefficients complete to degree and order 2700. As we also discuss in sections 3.4 and 3.5, the computation of the RTM-implied gravity anomalies globally and on a regular grid enables their spectral decomposition, and so is of critical importance both to the estimation of a band-limited set of 5 arc-minute mean anomalies from terrestrial gravity data, and to the computation of "fill-in" anomalies in areas covered with proprietary data.

(d) We have used the formulation described by [*Pavlis and Rapp*, 1990] to determine spherical harmonic coefficients of the Topographic/Isostatic (T/I) potential implied by the Airy/Heiskanen isostatic hypothesis, with a constant 30 km depth of compensation. We evaluated these coefficients up to degree and order 2160, employing the DTM2006.0 database, in two ways: (i) using 5 arc-minute data, and, (ii) using 2 arc-minute data. We intended originally to use these coefficients, in combination with the satellite-only model, to compute "fill-in" gravity anomalies. This was not done however, as we opted instead for the

Pavlis et al. [2007b] provide additional details about the DTM2006.0 database and its use towards the development and implementation of the EGM2008 model. We should emphasize here that a single DTM should be used consistently in all the processes related to the development and the subsequent use of an EGM. This DTM is in fact inextricably connected to the resulting EGM. For example, elevation errors will propagate into errors in the downward continuation of gravity anomalies from the topography to the ellipsoid. However, one expects these propagated errors to cancel out to a large extent, when the resulting EGM is used to compute quantities such as height anomalies or gravity anomalies back on the topography, as long as the same DTM is used consistently in both operations. Otherwise, the use of different elevation information in these operations could create inconsistencies that may degrade the results. Therefore, the availability of a global DTM of the highest possible accuracy and resolution is an important prerequisite of any high resolution EGM development effort and use.

3.3 The Gravity Anomalies Derived From Satellite Altimetry

The altimetry-derived gravity anomalies cover approximately 70 percent of the globe, and so are crucial to the formation of a complete, global 5 arc-minute gravity anomaly grid that can support the determination of a marine geoid with long wavelength integrity and very high resolution. The global 5 arc-minute merged gravity anomaly files, which supported the development of several Preliminary Gravitational Models (PGM) during the course of this project, employed altimetry-derived gravity anomalies from a variety of sources. We discuss next only those two sources that were used in the development of the final EGM2008 solution. One of them was computed at the Danish National Space Center (DNSC), the other at Scripps Institution of Oceanography, in collaboration with the National Oceanic and Atmospheric Administration (SIO/NOAA).

In order to ensure compatibility of the anomalies produced by both teams, we provided both with a common set of reference values computed using the PGM2007B model, to spherical harmonic degree 2190, and its associated DOT model, designated DOT2007A, to spherical harmonic degree 50. These reference files contained values of height anomalies and DOT, necessary in the "remove" step of Least Squares Collocation (LSC) prediction algorithms, and free-air gravity anomalies, necessary in the "restore" step of such algorithms. In section 3.4, we provide additional information about the LSC technique and the "remove-compute-restore" methodology. The reference files were all provided at 1 arc-minute grid size, in terms of point values, and covered all ocean areas plus an inland coastal swath of 75 km width. The gravity anomaly and height anomaly files covered also the Caspian Sea. Both teams used these reference files and estimated point values of free-air gravity anomalies at 1 arc-minute grid size, which they provided back to the EGM project.

The DNSC set that was provided back to this project is designated DNSC07 and is a predecessor of the DNSC08GRA set described in [*Andersen et al.*, 2010]. Details regarding the data used to produce DNSC07 and the estimation algorithm employed can be found in [*Andersen et al.*, 2010], since these are the same for both DNSC07 and DNSC08GRA. The essential difference between the two sets is that DNSC08GRA was produced after EGM2008 was finalized and released, and thus benefited from reference values computed using the final EGM2008 model.

The SIO/NOAA set that was provided back to this project is designated here SS v18.1. This set is a predecessor of the gravity anomaly set described in [*Sandwell and Smith*, 2009]. The latter was also produced after EGM2008 was released, as in the case of DNSC08GRA discussed previously.

The main difference between the estimation algorithms employed by DNSC and SIO/NOAA is the form in which the altimeter data enter the estimation of gravity anomalies. DNSC uses (residual) Sea Surface Heights (SSH), while SIO/NOAA use (residual) slopes of the SSH, determined from the numerical differentiation of neighboring altimeter data. There are advantages and disadvantages associated with either of the two estimation techniques, as these were actually implemented by DNSC and SIO/NOAA respectively. In particular, for a given reference gravitational model whose resolution is always finite, the use of residual SSH is affected less by the lack of data on the side of land in near-coastal areas, as compared to the use of residual SSH slopes. The use of residual SSH slopes on the other hand, tends to produce gravity anomalies that are noticeably "richer" in high frequency content, as compared to the use of residual SSH. The difference between the results from the two estimation techniques are of course reduced as the common reference models used in both become more accurate and of higher resolution. Table 1 summarizes the essential statistics from the comparison of the DNSC07 and SS v18.1 altimetry-derived gravity anomaly datasets, among themselves, as well as with the PGM2007B reference values used in both estimations.

In terms of 5 arc-minute area-mean gravity anomalies, the two independently computed altimetry-derived datasets differ by less than ±2 mGal (1 mGal=10^{-5} m/s^2) on an ocean-wide basis. We performed additional comparisons with independent marine gravity anomaly data available to NGA and verified that DNSC07 performed consistently better than SS v18.1 in near coastal areas. On the basis of these results, we combined PGM2007B, DNSC07, and SS v18.1, in terms of 1 arc-minute gravity anomalies, in the following fashion:

(a) We used PGM2007B values on land to within 65 km from the coastline.

(b) We used a tapered transition from PGM2007B to DNSC07, over the remaining 65 km to the coastline.

(c) We used DNSC07 values over the ocean, within 195 km from the coastline.

(d) We used a tapered transition from DNSC07 to SS v18.1 over the ocean from 195 km to 280 km distance from the coastline.

(e) For all ocean cells beyond 280 km from the coastline, we used SS v18.1 values.

In this fashion we created a global 1 arc-minute gravity anomaly dataset, which we analyzed harmonically. We used the estimated ellipsoidal harmonics, from degree 2 to degree 2159, to create a 5 arc-minute band-limited version of this dataset. The latter dataset served as the foundation file upon which other data files were overlaid, in order to produce the final merged 5 arc-minute gravity anomaly file that supported the development of EGM2008, as we discuss in the following sections.

3.4 The Gravity Anomalies Estimated From Terrestrial Data

The estimation of the 5 arc-minute area-mean free-air surface gravity anomalies $\overline{\Delta g}_{ij}^{t}$ from the corresponding point values was performed using a LSC prediction algorithm [*Moritz*, 1980], in a "remove-compute-restore" fashion. LSC is a mathematical technique for determining the Earth's figure and gravitational field by a combination of heterogeneous data of different kinds. Its formulation may be interpreted in very different ways: as the solution of a geophysical inverse problem, as a statistical estimation method combining least squares adjustment and least squares prediction, and as an analytical approximation to the Earth's potential by means of harmonic functions [*Moritz*, 1978]. LSC is a form of linear regression – estimating stochastic quantities from other stochastic quantities by using their statistical correlations – that is formally identical to objective mapping. Our implementation of LSC used the remove-compute-restore computational methodology that is well known to geodesists. Thereby, long wavelength trends are removed from the observations using some *a priori* known reference model(s), the LSC prediction is applied to the residuals after the removal of the values of the reference model(s) from the observations, and finally the effects of the reference model(s) are restored back to the estimated quantities. *Moritz* [1980] provides a comprehensive treatise of LSC and the remove-compute-restore computational methodology. The main elements of our formulation are:

$$\overline{\Delta g}_{ij}^{t} = \mathbf{C}_{\overline{\Delta g}_{ij}^{t},\, \Delta g_k^t} \cdot \left(\mathbf{C}_{\Delta g_k^t,\, \Delta g_k^t} + \mathbf{V}\right)^{-1} \cdot \mathbf{L} + r_{ij} \ , \tag{33}$$

where $\mathbf{C}_{\overline{\Delta g}_{ij}^{t},\, \Delta g_k^t}$ is the signal cross-covariance matrix between the area-mean value to be predicted and the point values of the observations Δg_k^t, and $\mathbf{C}_{\Delta g_k^t,\, \Delta g_k^t}$ is the auto-covariance matrix of the observations involved in the prediction. $\mathbf{V}$ is the noise covariance matrix of these observations, which was taken in this study to be diagonal, and $\mathbf{L}$ is the vector of observations. From the observations, quantities that can be modeled have been removed, so that an element ℓ_k of the vector $\mathbf{L}$ is given by:

$$\ell_k = x_k - y_p \ , \tag{34}$$

where:

$$x_k = \Delta g_k^t - \Delta g_k^t(SH) - \Delta g_k^t(RTM) \ , \tag{35}$$

and y_p is the mean value of x_k over the area involved in the prediction, so that the residual observations involved in the prediction are centered. $\Delta g_k^t(SH)$ and $\Delta g_k^t(RTM)$ are point values of the free-air gravity anomalies implied by the reference spherical harmonic model used and by the RTM computation. In our notation, "*SH*" abbreviates "*Spherical Harmonics*" and refers to the computational method used to evaluate these gravity anomalies. It does not represent a variable, to which these gravity anomalies depend, but rather a computational method. The same applies to our "*RTM*" notation. Finally, in equation (33), r_{ij} is given by:

$$r_{ij} = \overline{\Delta g}_{ij}^t(SH) + \overline{\Delta g}_{ij}^t(RTM) + y_p \ , \tag{36}$$

and represents the sum of the area-mean values of the reference terms that have to be "restored" to the predicted quantity $\overline{\Delta g}_{ij}^{t}$. In equations (35) and (36), proper care should be exercised, so that the reference gravitational model and the RTM effects neither overlap nor leave any "gaps" in terms of spectral content. This LSC prediction algorithm was used to estimate the terrestrial gravity anomalies that supported the PGM2004A, PGM2006A/B/C, and PGM2007A/B preliminary models. For the final EGM2008 model however, we implemented a modification to this algorithm, which results in predicted gravity anomalies $\overline{\Delta g}_{ij}^{t}$ that are band-limited to a high degree of approximation.

Consider the frequency content of the various terms appearing in equations (35) and (36). The point value of the surface free-air gravity anomaly Δg_k^t contains the full spectrum of the gravity field. The reference values $\Delta g_k^t(SH)$ and $\overline{\Delta g}_{ij}^{t}(SH)$ contain only the bandwidth of the reference model used in the estimation. Due to the use of a reference topographic surface created from an expansion to degree 360 of the topography (see section 3.2), the point values $\Delta g_k^t(RTM)$, contain spectral power from degree ~360, up to a degree commensurate with the grid size of the DTM used in their computation, which, in this case, was 30 arc-seconds. The corresponding 5 arc-minute area-mean values $\overline{\Delta g}_{ij}^{t}(RTM)$, which are the result of averaging, are certainly not band-limited, and contain spectral power beyond degree 2159. A simple modification of the quantities used in the LSC algorithm discussed before, can provide much better control on the frequency content of the predicted mean anomalies. Specifically, we modify equations (35) and (36) as:

$$x_k = \Delta g_k^t - \left[\Delta g_k^t (SH,\ n = 2159) - \Delta g_k^t (RTM,\ n = 2159) \right] - \Delta g_k^t (RTM)\ , \tag{37}$$

and

$$r_{ij} = \overline{\Delta g}_{ij}^t (SH,\ n = 2159) + y_p\ , \tag{38}$$

where we have assumed here that the reference gravitational model extends to degree 2159, in ellipsoidal harmonics. Use of equations (37) and (38), instead of (35) and (36), results in predicted gravity anomalies $\overline{\Delta g}_{ij}^t$ that are band-limited to a high degree of approximation. The key element that enables the implementation of this new approach is the availability of the RTM-implied Δg, globally and in the form of a regular grid. Without such a file, there can be no spectral decomposition of the RTM-implied Δg, which produces the coefficients necessary to synthesize the terms $\Delta g_k^t (RTM,\ n = 2159)$ with the required spectral content. In our implementation, the quantity within the brackets in equation (37), referring to the Earth's topography, was evaluated as point values on a global 30 arc-second grid. This grid was then used to interpolate the values to the locations of the point gravity data. Figure 2 demonstrates the effectiveness of the technique, by comparing the gravity anomaly degree variances c_n, computed according to equation (39), as obtained from two versions of the global database, one without (v050707a) and one with (v021408a) the application of this technique.

$$c_n = \left[\frac{GM}{a^2} \cdot (n-1) \right]^2 \sum_{m=-n}^{n} \left(\bar{C}_{nm}^e \right)^2\ . \tag{39}$$

Notice the “jump” between degrees 2159 and 2160. This jump is of course a consequence of limiting also the bandwidth of the altimetry-derived anomalies to degree 2159, as we described in section 3.3.

The LSC prediction algorithm implemented here also provides estimates of the error variances of the predicted $\overline{\Delta g}_{ij}^{t}$. These are given by:

$$\sigma^2_{\overline{\Delta g}_{ij}^{t}} = \mathbf{C}_{\overline{\Delta g}_{ij}^{t},\,\overline{\Delta g}_{ij}^{t}} - \mathbf{C}_{\overline{\Delta g}_{ij}^{t},\,\Delta g_k^t} \cdot \left(\mathbf{C}_{\Delta g_k^t,\,\Delta g_k^t} + \mathbf{V}\right)^{-1} \cdot \mathbf{C}_{\Delta g_k^t,\,\overline{\Delta g}_{ij}^{t}} \quad . \tag{40}$$

These error variances do not necessarily account for systematic errors in the gravity data, and have to be modified carefully, in order to provide realistic measures of the errors associated with these data. We discuss the calibration of the gravity anomaly error estimates in section 4.3.

For the final iteration of the estimation of the 5 arc-minute gravity anomalies, which supported the development of EGM2008, the PGM2007B solution was used as reference gravitational model (to spherical harmonic degree 2190), consistent with the estimation of the altimetry-derived values. Additional details on the implementation of the gravity anomaly prediction algorithm can be found in [*Factor*, 2006].

3.5 Fill-in Gravity Anomalies Using RTM Forward Modeling

In terms of their availability, the gravity anomaly data that were necessary for this project divide the Earth into three distinct sub-divisions, as we show in Figure 3a.

(a) Areas where gravity anomaly data exist, and were made available for the computation of the 5 arc-minute area-mean values necessary for this project, without any restrictions. Thanks primarily to the altimetry-derived gravity anomalies, the majority of ocean areas fall into this category. These areas are colored green in Figure 3a.
(b) Areas where gravity anomaly data are either unavailable, or too sparse, or too inaccurate, to support the estimation of 5 arc-minute area-mean values of meaningful quality. These areas are colored red in Figure 3a.
(c) Areas where the gravity anomaly data available to this project were of proprietary nature. In agreement with the co-owners of these data, within this project, their use was only permitted at a resolution corresponding to 15 arc-minute area-mean values. The domain of these data covers approximately 42.9 percent of the Earth's total land area, and is colored gray in Figure 3a.

In order to compile a global gravity anomaly dataset with as uniform spectral content as possible, capable of supporting the estimation of potential coefficients to degree 2159, the spectral content of the gravity anomalies in category (c) above, beyond degree 720 that corresponds to the 15 arc-minute resolution, and up to degree 2159, was supplemented with the gravitational information obtained from the global set of RTM-implied gravity anomalies discussed in section 3.2. The specific details of the implementation of this approach can be found in [*Pavlis et al.*, 2007b]. We tested and verified this approach locally, over extended areas where high quality gravity anomaly data are available (USA, Australia), as *Pavlis et al.* [2007b]

discuss in detail. In addition, we compared the gravity anomaly degree variances obtained from the analysis of a global 5 arc-minute dataset that included the proprietary data with the degree variances obtained from the use of the RTM-implied gravity information. Figure 5 in [*Pavlis et al.*, 2007b] demonstrates that the two spectra are in excellent agreement. Only after degree ~1650 the use of the RTM-implied gravity information provides a somewhat underpowered gravity anomaly spectrum. With this approach, we managed to circumvent the proprietary data issues without degrading the gravitational solution significantly, at least in terms of the recovered power spectrum. An obvious shortcoming of our RTM-based forward modeling approach is that it can only improve the modeling of short wavelength gravitational signals (beyond degree 720), to the extent that these signals are generated by topographic masses.

Finally, we needed to provide an estimate for each of the 5 arc-minute area-mean gravity anomalies under (b) above. These cover approximately 12.0 percent of the Earth's land area, and are located in Africa, South America, and Antarctica. Over Africa and South America, we originally synthesized 5 arc-minute values using the GGM02S coefficients [*Tapley at al.*, 2005] for degrees 2 to 60, augmented with the EGM96 coefficients [*Lemoine et al.*, 1998] for degrees 61 to 360, and further augmented with the coefficients from the analysis of the RTM-implied anomalies for degrees 361 to 2159. Over Antarctica, we synthesized 5 arc-minute values using only the ITG-GRACE03S [*Mayer-Gürr*, 2007] model coefficients, up to degree and order 180, as we discuss in the next section.

3.6 The 5 Arc-Minute Global Merged Gravity Anomaly File

The estimation of the $\bar{\mathbf{C}}^e$ ellipsoidal harmonic coefficients implied by the "terrestrial" data up to degree and order 2159 requires a global, complete file of 5 arc-minute area-mean gravity anomalies $\overline{\Delta g}_{ij}^{t}$. Since this estimator does not allow for any gaps or overlapping duplicate data input, one has to select for each 5 arc-minute cell on the ellipsoid, the most accurate anomaly estimate out of multiple data that may be available for that cell (e.g., marine and altimetry-derived values). *Rapp and Pavlis* [1990] discuss such kind of data selection and merging algorithm. In the development of EGM2008, a similar algorithm was used. This process resulted in a complete global grid (9331200 values) of 5 arc-minute $\overline{\Delta g}_{ij}^{t}$, which were used in the model's estimation. Table 2 summarizes the overall statistics of this merged file.

It is interesting to note that the areas covered with the poorest quality gravity data, the "fill-in" values, are also characterized by the "roughest" gravity anomalies, with a ±46.8 mGal RMS gravity anomaly value, compared to ±34.5 mGal, which is the global RMS value of our present data. This should come as no surprise, since the areas occupied with "fill-in" data cover some of the most mountainous areas of the Earth, like the Himalaya and the Andes. We should also point out here that the RMS values of the error standard deviations of the data given in Table 2 represent the error estimates obtained from the LSC estimator of equation (40), before applying any error "calibration". These noise-only error estimates many times are rather optimistic. Figure 3b displays geographically the source identification of the 5 arc-minute area-mean gravity anomalies in the merged file used to develop the EGM2008 model.

Some noteworthy aspects of this merged file include the extensive use of 5 arc-minute area-mean gravity anomalies from the Arctic Gravity Project (ArcGP) [*Kenyon and Forsberg*, 2008], and the avoidance of use of any Topographic/Isostatic mean anomalies [*Pavlis and Rapp*, 1990]. Over Antarctica, the 5 arc-minute area-mean gravity anomalies were synthesized purely on the basis of the ITG-GRACE03S [*Mayer-Gürr*, 2007] model. This makes the EGM2008 model completely free of any isostatic hypothesis, at the cost of producing a smoother field over Antarctica, since ITG-GRACE03S is complete only up to degree and order 180. Over parts of Siberia, as well as over France, Poland, and Colombia, the 5 arc-minute values used in the merged file were contributed to NGA by external organizations or individuals. The "splicing" of the SS v18.1 altimetry-derived anomalies from SIO/NOAA with the DNSC07 values is also shown in Figure 3b. Over the Gulf Stream and Kuroshio areas, where the increased sea surface variability makes the altimetry-derived anomalies less reliable, we made some use of marine gravity anomalies, after their quality was verified through comparisons with other independent marine gravity data.

4. SOLUTION DEVELOPMENT AND EVALUATION

4.1 Preliminary Solutions

During the course of this project, we developed several Preliminary Gravitational Models (PGM) in order to test various aspects of the solution and evaluate alternative modeling and estimation approaches. The progression of our PGM developments also paralleled the availability of improved satellite-only solutions from the GRACE mission, as well as improved versions of the terrestrial and the altimetry-derived gravity anomaly data. Three of these PGM development efforts constituted significant milestones for this project. We summarize the main modeling gains achieved in these PGM developments next.

PGM2004A

PGM2004A [*Pavlis et al.*, 2005] was the first gravitational model *ever* developed that extended to degree 2160. Prior to PGM2004A, the GPM98A, B, and C solutions of *Wenzel* [1998], which extended to degree 1800, were the highest degree gravitational models available. However, beyond degree 1400, the GPM98A, B, and C models produced unrealistic gravity anomaly signal degree variances. With the development of PGM2004A we demonstrated our technical capability to meet the challenges associated with this project. PGM2004A used the GGM02S GRACE-only model [*Tapley et al.*, 2005], which extends to degree and order 160, and whose spherical harmonic coefficients were accompanied by their error estimates. The complete error covariance matrix of the GGM02S coefficients was not made available to this project. A very preliminary version of the 5 arc-minute merged gravity anomaly file supported the development of PGM2004A. Among other shortcomings, PGM2004A was developed based on gravity anomalies that had not been downward continued. Nevertheless, as discussed by *Pavlis et al.* [2005], comparisons with TOPEX altimeter data, astrogeodetic deflections of the vertical, geoid undulations and/or height anomalies obtained from GPS positioning and spirit leveling, demonstrated clearly that PGM2004A performed quite well. The results from these comparisons also indicated that the ±15 cm global RMS geoid undulation commission error goal set by NGA was well within reach. Furthermore, in PGM2004A the error propagation approach of *Pavlis and Saleh* [2005], which we discuss in some detail in section 5, was successfully implemented for the first time.

PGM2006A, B, and C

After the successful development of PGM2004A, the effort focused on the compilation and verification of several datasets that were deemed critical to the success of the project. These included the compilation of the DTM2006.0 global topographic database, the computation of RTM-implied gravity anomalies, globally and on a regular grid, and the computation of the g_1 analytical continuation terms, as we discussed in section 3.2. In parallel, we continued to evaluate the candidate Mean Sea Surface (MSS) datasets that were produced at DNSC and were provided to us for evaluation.

We compiled an updated global 5 arc-minute merged gravity anomaly file, where we incorporated for the first time fill-in anomalies computed as we discussed in section 3.5. The estimation of the gravity anomalies within this file employed a refined LSC prediction approach near the coastlines, where the available land, marine, and altimetry-derived gravity data were combined. Using this global 5 arc-minute gravity anomaly file and the available g_1 analytical continuation terms, we then created two combination solutions [*Pavlis et al.*, 2006a]: PGM2006A, where the gravity anomalies were not downward continued, and, PGM2006B, where the gravity anomalies were downward continued. Apart from the downward continuation, PGM2006A and B were identical in all other aspects of their development. In both combination solutions the GGM02S GRACE-only model was used.

Examination of the residual 5 arc-minute gravity anomalies from the respective least squares adjustments that produced PGM2006A and B demonstrated clearly the importance of the downward continuation corrections. These residuals are a measure of the difference between the gravity anomaly information implied by the GRACE-only model and the corresponding information contained in the merged 5 arc-minute gravity anomaly file that is input to the adjustment. In Figure 4 we display side-by-side the residuals from PGM2006A and B, over an area in southern Alaska and western USA and Canada, where the effect of the use of the downward continuation terms was particularly pronounced. It is clear that the GRACE-only model, which is independent of the terrestrial gravity information, "prefers" the downward continued anomalies, and, as expected, the discrimination is more pronounced over the rugged mountainous areas, where the downward continuation terms are more significant.

We evaluated the solutions PGM2006A, and B, using various independent data [*Pavlis et al.*, 2006a], as we had also done with PGM2004A. It was reassuring to see that both 2006 solutions were performing considerably better than the 2004 solution, and furthermore that PGM2006B was outperforming PGM2006A. This gave us confidence that our gravity anomaly prediction and processing methods were working well. We also recognized however that improvements could be made to some aspects of the solution, particularly those aspects that could influence the estimation of a DOT model, as a by-product of the solution, by differencing the solution's geoid from a MSS. We discuss these aspects next.

A problem associated with GRACE-only gravitational models is the presence of systematic errors that manifest themselves as "stripes", oriented in a more-or-less north-south direction, in model-derived quantities, such as geoid undulations and gravity anomalies. These systematic errors were particularly evident in the early GRACE-only gravitational models. To our knowledge, the specific origin(s) of these stripes remains unknown. Imperfections in the modeling and/or the estimation approach used for the recovery of the "static" gravitational signal, which requires the pre-filtering or the simultaneous estimation of all time-variable gravitational signals affecting the measurements, as well as sampling problems, both spatial and temporal, certainly contribute to the creation of these stripes. The treatment of some of these problems still keeps improving as newer GRACE models are being developed, which are also based on data with ever improving spatial distribution. The presence of these stripes creates significant problems especially in DOT modeling that uses GRACE-based geoid models. Post-processing approaches for "de-striping" GRACE products employ various filtering and smoothing techniques, as discussed, e.g., by *Chambers and Zlotnicki* [2004] and *Swenson and Wahr* [2006]. These techniques, in general, reduce the systematic errors at the expense of the spatial resolution of the GRACE products. Some of them have the advantage that they involve only GRACE information therefore the post-processed product remains a "GRACE-only" estimate. This is of little importance in our case, since we intend to combine the GRACE information with surface gravity and satellite altimetry data anyway. Furthermore, in our case, any such filtering and/or smoothing procedure applied to the GRACE-only gravitational model coefficients would have to be reflected also on the error covariance matrix associated with these coefficients, through rigorous error propagation. One may avoid these complications altogether, by recognizing that the combination of the GRACE information with surface gravity and satellite altimetry data, which are free of any stripe artifacts, could conceivably minimize the effects of the GRACE stripes, provided that the relative weights used in the combination solution for the different data are selected appropriately. We opted for this latter approach, and considered the development for a "stripe-free" DOT model by-product of our solution, a primary optimization requirement for our EGM modeling effort. To assist in our evaluation of the DOT models resulting from the subtraction of our PGM geoid models from various MSS models, we acquired

[*Wunsch, personal communication*, 2006] the DOT output for the 12-year period [1993, 2004], of the Massachusetts Institute of Technology (MIT) version of the general circulation model known as ECCO (Estimating the Circulation and Climate of the Ocean). This product is also described in [*Wunsch and Heimbach*, 2007].

In October 2006, the DNSC team provided us with the fifth version of their MSS model, designated DNSC06E [*Andersen, personal communication*, 2006]. DNSC06E was delivered to us in 1, 2, and 5 arc-minute versions. DNSC06E is an antecedent of the DNSC08 MSS discussed by *Andersen and Knudsen* [2009]. DNSC06E was based on TOPEX/Poseidon and Jason-1 altimeter data from the 12-year period [1993, 2004]. Building upon our experience with PGM2006A, and B, we developed the PGM2006C solution paying special attention to the DOT that it implied. Over land areas, the differences between PGM2006B and PGM2006C are marginal. Three DOT models were computed by subtracting 2 arc-minute area-mean values of height anomalies computed from three gravitational models, from the 2 arc-minute version of the DNSC06E MSS. These 2 arc-minute DOT estimates were then averaged to 1°×1° cells, without applying any other smoothing or filtering. The gravitational models that we used in these comparisons were: (a) the GGM02C model [*Tapley et al.*, 2005], which is complete to degree and order 200, augmented by the EGM96 model [*Lemoine et al.*, 1998], from degree 201 to degree 360, (b) the EIGEN-GL04C model [*Förste et al.*, 2008], which is complete to degree and order 360, and (c) the PGM2006C solution to spherical harmonic degree 2190. In addition to these three models, we considered the 1°×1° DOT model of [*Chambers and Zlotnicki*, 2004], which is also based on GGM02C augmented with EGM96, but results from the application of their iterative filtering approach. Table 3 shows the standard deviation of the differences between the ECCO DOT estimate and the various MSS minus geoid model DOT estimates.

Geographic plots similar to those that we show in Figure 10 demonstrated that the use of the GGM02C model resulted in significant stripe artifacts in the DOT, as did the use of EIGEN-GL04C but to a lesser extent. EIGEN-GL04C on the other hand created some "ringing" artifacts, similar to those that can be seen in Figure 10. PGM2006C was largely free of any artifacts. At the same time, the DOT implied by PGM2006C was not overly smooth, as was the DOT model of *Chambers and Zlotnicki* [2004]. The latter DOT model shows the smallest standard deviation difference with the ECCO model, but this may be because the ECCO model itself used the GGM02C_EGM96 geoid in its development [*Wunsch, personal communication*, 2006]. These results, which were reported in [*Pavlis et al.*, 2006b], gave us confidence that our relative data weighting approach was performing quite well, as far as the DOT estimation was concerned.

PGM2007A and B

The development of PGM2007A and B constitutes the first of the two iterations of the general estimation approach that we discussed in section 2.1. The computation of these two solutions incorporated all the essential elements of our estimation approach. In addition, by the time of the development of these two solutions, we had assembled and prepared several sets of independent data, and we had set up standard procedures for the objective evaluation of our test solutions on a routine basis. These test data also included the latest high resolution regional geoid models for the United States of America and for Australia that were available at that time. For the USA, we acquired from http://www.ngs.noaa.gov/GEOID/USGG2003 the gravimetric geoid model USGG2003, which is also discussed in [*Wang and Roman*, 2004]. We also acquired from http://www.ngs.noaa.gov/GEOID/DEFLEC99/ the deflections of the vertical that accompany the GEOID99 product of the US National Geodetic Survey (NGS), since a corresponding file for the deflections of the vertical accompanying the USGG2003 was not available. For Australia, we acquired from http://www.ga.gov.au/geodesy/ausgeoid/files.jsp the AUSGeoid98 geoid model of [*Featherstone et al.*, 2001]. Upon our request, we also received a set of astrogeodetic deflections of the vertical scattered over Australia [*Featherstone, personal communication*, 2006]. This set augmented our astrogeodetic deflection of the vertical test data holdings, which until that time contained only the set of values scattered over the Conterminous US area (CONUS), which are discussed also in [*Jekeli*, 1999]. These newly acquired test data sets allowed us to perform additional comparisons, resulting in a more thorough evaluation of our test solutions. In section 4.4, we provide more detailed discussion regarding our testing and evaluation procedures, including the results that we obtained from these tests for our final EGM2008 model.

Upon our request, in late January 2007 we received from NASA's Jet Propulsion Laboratory the JEM01-RL03B GRACE-only gravitational model along with its complete error covariance matrix [*Watkins, personal communication*, 2007]. This model is complete to degree and order 120. The availability of its complete error covariance matrix, which we converted from spherical to ellipsoidal harmonic representation as we discussed in section 2.2, also gave us the capability to test and verify our combination solution estimation algorithm that uses the block-diagonal approximation of this matrix, as we discussed in section 2.4.

In May of 2007, the DNSC provided to us the eighth version of their MSS, which was designated DNSC07C [*Andersen, personal communication*, 2007]. This version of the DNSC MSS had addressed several problems associated with the previous versions. We evaluated the

DNSC07C MSS, and found that it could be used to develop a preliminary DOT model. This DOT model, along with the PGM, would form the reference models necessary for the re-iteration of the estimation of the altimetry-derived gravity anomalies.

In preparation for the PGM2007A and B solutions, an updated set of 5 arc-minute area-mean gravity anomalies was formed. This set used the PGM2006B solution as the reference model in the LSC estimation algorithm, in place of the EGM96 solution [*Lemoine et al.*, 1998] that had been used in all previous gravity anomaly estimations. Over the areas occupied by proprietary data, the spectral content of the 5 arc-minute area-mean gravity anomalies from ellipsoidal harmonic degree 721 and up to degree 2159 was supplemented by the spectral information extracted from the RTM-implied gravity anomalies, as we discussed in section 3.5 (see also [*Pavlis et al.*, 2007b] for additional details).

Careful examination of the DOT field implied by the DNSC07C MSS and the geoid undulations of the PGM2006C solution, which is shown in Figure 5a, indicated that over certain coastal areas, errors in the gravity anomaly data were corrupting the geoid solution in ways that primarily manifested themselves as broad-scale "ringing" patterns in the model-implied DOT field. These patterns slowly dissipate into the ocean away from the coast, and are most clearly visible over the western coast of South America, as well as over some coastal areas of Africa and Indonesia.

These patterns were also correlated geographically with areas of large residual gravity anomalies from the least squares combination solution adjustment, indicating large discrepancies between the gravity information obtained from GRACE and the corresponding information obtained from the terrestrial data. To address this problem, we excluded the use of near-coastal marine gravimetric data from the estimation of the 5 arc-minute area-mean gravity anomalies over selected problematic coastal areas. Over these areas, we used instead purely altimetry-derived gravity anomalies to the maximum extent possible. This approach improved the situation considerably from the ocean side of the coastline. However, we soon recognized that these "ringing" patterns originated, for the most part, from the inland side of the coastline. There, long wavelength errors in the terrestrial near-coastal data were causing problems in the combination solution, which were "propagating" into the ocean, thus corrupting the marine geoid and manifesting themselves most prominently in the implied DOT model. We addressed this problem by replacing the degree 2 through 120 ellipsoidal harmonic spectral components of the terrestrial 5 arc-minute area-mean gravity anomaly data, with the corresponding component of the JEM01-RL03B GRACE-only gravitational model. This data editing was implemented selectively over the most problematic land areas, such as the western coast of South America, some coastal areas of Africa and Indonesia, as well as over a few near-coastal ocean areas. Several iterations were required to "tune" this editing procedure towards yielding the "cleanest" possible DOT, i.e., the DOT least corrupted by artifacts associated with geoid model errors. Over Antarctica, we replaced the 5 arc-minute area-mean gravity anomaly data over the entire continent with values synthesized from the JEM01-RL03B GRACE-only gravitational model coefficients, from degree 2 to degree and order 120. This replacement tapered some 300 km into the Southern Ocean.

During the development of the PGM2007A and B solutions we also tested and inter-compared the results from the implementation of the two alternative downward continuation approaches that we discussed in section 2.3. We found that the downward continuation corrections computed using Method A, the approach based on the g_1 terms, contained considerably higher spectral power at the longer wavelengths (approximately up to degree 380) compared to those computed using Method B. Beyond degree 380, the corrections computed using Method B exhibited higher power than those computed using Method A. Examination of the respective residuals from two least squares adjustment combination solutions with the JEM01-RL03B GRACE-only model, each using either one of the two alternative downward continuation approaches, showed smaller residuals when Method A was used. Although the specific reason for this observed behavior is still not clear to us, after careful examination of the results, we adopted a “hybrid” method for the downward continuation of the 5 arc-minute area-mean gravity anomalies. Thereby, we form the downward continuation correction terms by spectrally combining the ellipsoidal harmonic coefficients from an analysis of the downward continuation correction terms obtained using Method A up to degree 380, with the corresponding coefficients from the analysis of the downward continuation correction terms obtained using Method B, from degree 381 to degree 2159. Although this approach gave us the best overall results, in terms of reduced residuals with respect to GRACE and also validation with independent data, additional work should be done in the future, to better understand the exact reasons for this observed behavior.

With the above considerations in mind, we performed the least squares combination of the JEM01-RL03B GRACE-only model with the gravitational information contained within the 5 arc-minute area-mean gravity anomaly data, to create the PGM2007A solution [*Pavlis et al.*, 2007a]. We evaluated the PGM2007A solution thoroughly by examining the residual gravity anomalies from the least squares combination solution and by performing all the comparisons with independent data that were available to us. Comparisons with GPS/Leveling data showed that PGM2007A performed only slightly better than PGM2006B over CONUS and Australia, but demonstrated a more noticeable improvement over PGM2006B globally. This was to be expected, as the terrestrial gravity data over the well-surveyed areas of CONUS and Australia, as well as their modeling, had changed only marginally between these two solutions. More importantly, over two specific areas where our previous solutions were giving poor comparison results, PGM2007A demonstrated clear and substantial improvements. Over France, with 167 points compared, the standard deviation of the differences between GPS/Leveling geoid undulations and model-implied values dropped from ±13.1 cm for PGM2006B, to ±8.7 cm for PGM2007A. Over Switzerland, with 115 points compared, the corresponding value dropped from ±27.0 cm for PGM2006B, to ±7.1 cm for PGM2007A. Also noteworthy is the fact that PGM2007A outperformed both the USGG2003 and the AUSGeoid98 high resolution regional geoid models for CONUS and Australia, respectively, in the comparisons with GPS/Leveling data. PGM2007A and PGM2006B showed roughly equivalent performance in the respective comparisons with the astrogeodetic deflections of the vertical over CONUS and Australia, with standard deviation differences of ±1.1 arc-seconds ($\Delta\xi$) and ±1.2 arc-seconds ($\Delta\eta$) over CONUS, and ±1.2 arc-seconds ($\Delta\xi$) and ±1.3 arc-seconds ($\Delta\eta$) over Australia. PGM2007A performed slightly better than AUSGeoid98 in the comparisons with the astrogeodetic deflections over Australia, while DEFLEC99 performed best against the CONUS astrogeodetic deflections. Most importantly, the DOT computed by subtracting the PGM2007A geoid from the DNSC07C MSS model showed significantly reduced "ringing" and other distortions near the coastlines, compared to the corresponding DOT from the previous PGM2006C model, as it can be seen by comparing panels (a) and (b) of Figure 5. The improvements gained in our DOT modeling with the PGM2007A solution, and to a lesser extent with the DNSC07C MSS instead of the DNSC06E, were also evident from comparisons to the ECCO DOT output, similar to those summarized in Table 3. The standard deviation of the differences between the ECCO DOT model and the DOT model implied by the DNSC07C MSS minus the PGM2006C geoid model, over 33016 1°×1° ocean cells, was ±8.7 cm. This value dropped to ±7.7 cm for the DNSC07C MSS minus the PGM2007A geoid model. We presented these results at the XXIV General Assembly of the International Union of Geodesy and Geophysics (IUGG) that was held in Perugia, Italy on July 2-13, 2007 [*Pavlis et al.*, 2007a].

At this juncture of the EGM project, we envisioned one additional re-iteration of the entire model estimation process before finalizing our EGM solution. The timing, as well as the modeling gains that we had achieved with the development of PGM2007A, warranted therefore the release of that solution to the Special Working Group (SWG) of the IAG/IGFS for their independent evaluation. At the XXIV IUGG General Assembly we released to this group the PGM2007A model, expressed both in the Tide-Free and in the Zero Tide system [*Lemoine et al.*, 1998, chapter 11], in spherical harmonic coefficients extending to degree 2190 and order 2159. In addition, we released to the same group the spherical harmonic coefficients of the DTM2006.0 heights and depths, computed using equation (32), complete from degree 0 to degree and order 2160. Anticipating that most of the members of this group would need to have well-tested and verified computer software, capable of evaluating various gravitational field functionals from such high degree and order expansions, already in 2006 we had publicly released to this SWG the FORTRAN program HARMONIC_SYNTH [*Holmes and Pavlis*, 2006], along with test input and output data and associated documentation. Nevertheless, recognizing that these computations could have been challenging for some members of the SWG, we also provided the SWG with global, 2 arc-minute grids of gravity anomalies, height anomalies, and geoid undulations computed from PGM2007A, as well as software to read the values off these grids.

After the release of PGM2007A to the SWG, we continued with some refinements to the gravitational model, aiming specifically at the development of an optimal DOT model. To this end, we refined the PGM2007A solution, and developed a model designated PGM2007B. PGM2007B incorporated some refinements over those near-coastal areas where the degree 2 through 120 ellipsoidal harmonic spectral components of the terrestrial 5 arc-minute area-mean gravity anomaly data could be usefully replaced with the corresponding components of the JEM01-RL03B GRACE-only gravitational model. Also, PGM2007B was developed using a slightly different weighting scheme, compared to PGM2007A.

We computed a DOT surface grid by subtracting the PGM2007B geoid undulations from the DNSC07C MSS model. We then edited this DOT surface, in order to minimize the effect of some obvious artifacts, present in the DNSC07C MSS. This process included the smoothing-over of a linear "step" feature running along the parallel at 64°S, and the masking of certain cells containing suspiciously high DOT spikes, gradients or roughness, particularly near the coast. Using harmonic analysis, we developed a spherical harmonic model of this edited DOT surface, designated DOT2007A. We truncated this model to degree and order 50, in order to avoid any remaining stripe artifacts resulting from the influence of the GRACE information in our combination solution. PGM2007B and DOT2007A were the fields that we used consistently as reference models for the estimation of both the altimetry-derived gravity anomalies (section 3.3), and for the terrestrial LSC predictions (section 3.4), for the second and final re-iteration of our model estimation process. PGM2007B was not released to the SWG for evaluation, as this solution was only a slight variant of PGM2007A, and the two solutions were essentially identical over land areas.

4.2 Evaluation of PGM2007A by the IAG/IGFS SWG

By the end of October 2007, thanks to the prompt response by several members of the IAG/IGFS SWG, 19 reports from the evaluation of PGM2007A were made available to us. The majority of these reports involved comparisons with locally available gravity anomaly data and with geoid undulations or height anomalies from GPS positioning and leveling data, as well as comparisons with local and regional high resolution geoid models. Minkang Cheng (Univ. of Texas, Center for Space Research – UT/CSR) reported orbit fit comparison results, using satellites tracked by Satellite Laser Ranging (SLR). These comparisons are particularly sensitive to long wavelength errors in the gravitational model. However, after the inclusion in combination solutions of the highly accurate long wavelength gravitational information from GRACE, acceptable results from these comparisons constitute a *necessary but not sufficient* condition for the long wavelength accuracy of the model, as we also discuss in section 4.4 (see also Table 4). *Newton's Bulletin Issue n°4* [2009], contains 25 reports from the evaluation of our final solution EGM2008. Most of these reports include also the results from the evaluation of PGM2007A.

We carefully studied the reports from the evaluation of PGM2007A, and made an effort to address any comments indicating that the performance of the model could be improved, at least over those areas where we had available the data necessary to address such comments. Two specific cases where the information that we received from the SWG proved beneficial to the development of the final model involve the report that we received from Jonas Ågren (Swedish mapping, cadastre and registry authority) for the evaluation of PGM2007A over Sweden, and the report from Heiner Denker (Institut für Erdmessung, Hannover, Germany) who evaluated PGM2007A over Eurasia. The former indicated that some of the gravity anomaly data from the Arctic Gravity Project over Scandinavia, north of the 64°N parallel, could be improved; the latter revealed some problems over Eurasia, the most severe of which involved the data over Turkey. There, after some comparisons we determined that the data, contrary to their documentation, had the terrain corrections included in their values. We re-examined carefully our data over these problematic areas and corrected these problems in the next, and final, re-iteration of our LSC estimation of gravity anomalies, which produced the data used in the final EGM2008 model.

The independent evaluation of PGM2007A from the IAG/IGFS SWG verified for us the significant modeling gains, which we had achieved with that solution. In addition to the feedback that we received from the SWG, we contacted Michael Watkins and Dah-Ning Yuan (NASA's Jet Propulsion Laboratory – JPL) and asked whether they could perform comparisons involving fits to K-band range-rate data from GRACE. Yuan [*personal communication*, 2007] provided us with the RMS fits to K-band range-rate data, computed from 30 daily arcs spanning the month of November 2005, using PGM2007B, as well as two contemporary GRACE-only solutions, one computed at UT/CSR, the other at JPL. These results indicated to us a critical shortcoming of the PGM2007B model, which was not identified in any of the other comparisons reported by the SWG. Namely, the weight of the GRACE information was too low, compared to the weight used for the terrestrial data in PGM2007B (and PGM2007A). This tended to "favor" comparisons with GPS/Leveling data, but resulted in unacceptable performance in the GRACE

K-band range-rate data comparisons. This critical issue was resolved in the development of the final EGM2008 model, as we discuss in section 4.4 (see also Figure 8).

4.3 The Final EGM2008 Solution

In late October 2007, we acquired the ITG-GRACE03S GRACE-only model [*Mayer-Gürr*, 2007], complete to degree and order 180, along with its complete error covariance matrix, as we discussed in section 3.1. We compared this solution to the JEM01-RL03B GRACE-only model, and verified that ITG-GRACE03S represented a substantial improvement over JEM01-RL03B, both in terms of reduced stripe artifacts, and it terms of higher resolution. By early February 2008, we had at our disposal all the "ingredients" necessary for the development of the next and final model, including the latest 5 arc-minute area-mean terrestrial gravity anomalies obtained from the last re-iteration of our LSC estimation algorithm, using the procedure that we described in section 3.4, and the two sets of altimetry-derived gravity anomalies, DNSC07 and SS v18.1, which we combined as we discussed in section 3.3. We then compiled our merged 5 arc-minute area-mean global gravity anomaly file by combining the terrestrial and altimetry-derived data files, as we discussed in section 3.6. Wherever we were using gravity anomaly information from the JEM01-RL03B GRACE-only model in the previous merged file that supported the development of PGM2007B, we replaced that information with the corresponding one obtained from the ITG-GRACE03S model, complete to degree and order 180, in the current merged file. We carefully examined the data within this merged file, paying special attention to smooth out any existing discontinuities over the boundaries between data from different sources. We also applied to the 5 arc-minute area-mean gravity anomalies of this merged file the ellipsoidal corrections (see also [*Pavlis*, 1988] and [*Rapp and Pavlis*, 1990]) and the correction associated with the use of orthometric instead of normal heights in the numerical evaluation of the Molodensky free-air gravity anomalies [*Pavlis*, 1998]. We used the reference PGM2007B solution to evaluate these corrections. Finally, we analytically continued downward the 5 arc-minute gravity anomalies, from the Earth's topography where they refer, to the reference ellipsoid, using the same "hybrid" method that we had used in the development of the PGM2007A and B solutions. Over Taiwan and Hawaii, we used the downward continuation approach of the elevation-based g_1 terms (Method A of section 2.3), and over Antarctica we used the iterative gradient approach (Method B of section 2.3), since in our merged file all gravity anomaly values over Antarctica were synthesized from the ITG-GRACE03S model to degree and order 180.

Using our complete global 5 arc-minute grid of merged gravity anomalies we estimated an initial set of "terrestrial" ellipsoidal harmonic coefficients complete from degree 2 to degree and order 2159, according to the formulation of equations (19) through (22), employing the "BD1" block-diagonal approximation of the normal equations, and a preliminary set of gravity anomaly weights. The residuals from this fit of ellipsoidal harmonic coefficients to the 5 arc-minute data had an area-weighted mean value of 0.000 mGal, as expected, since we had already removed from the data the contributions from degrees zero and one, and an area-weighted standard deviation of ±0.452 mGal. Weighting of area-mean values by the area of the corresponding equiangular cell accounts for the variable area represented by such values at different absolute latitudes. This weighting is a function of the cosine of latitude. Recall that these residuals are only a measure of "goodness of fit". For comparison purposes, we mention here that *Wenzel* [1998] reported corresponding RMS residual misfits of ±5.3 mGal, ±5.1 mGal, and ±7.9 mGal for GPM98A, B, and C, respectively, considering in all cases expansions to degree 1799. Albeit preliminary, the weights used for the 5 arc-minute area-mean gravity anomalies enabled us to perform a meaningful initial combination solution with the ITG-GRACE03S model, using also the BD1 approximation of its error covariance matrix. This initial solution was the starting point for the "calibration" of the 5 arc-minute gravity anomaly error estimates and the relative weighting of the GRACE information versus the gravity anomaly information in the combination solution, as we discuss next.

One expects the errors associated with the 5 arc-minute area-mean gravity anomalies used in the combination solution to be correlated. These error correlations may arise from the LSC algorithm used to estimate the 5 arc-minute area-mean values, since this algorithm employs overlapping point-value data to simultaneously estimate all the 5 arc-minute area-mean values residing within each 1°×1° cell. In addition, the presence of un-modeled regional systematic biases in the terrestrial data [*Pavlis*, 1998] may also correlate the errors regionally. The problem is that these error correlations are very difficult to estimate accurately, and furthermore, the size of the error covariance matrix associated with the 9331200 5 arc-minute data is so large, that the presence of off-diagonal elements attaining arbitrary values in this matrix makes the solution extremely demanding computationally. To overcome these problems, a common practice in the development of combination solutions has been to consider a diagonal weight matrix for the gravity anomalies, and modify in some fashion the weights of the gravity anomaly data in an attempt to compensate for the omission of the error correlations. These weight modifications, which generally increase the standard deviations of the gravity anomalies, are designed with the objective to yield an optimal least squares adjustment combination of the gravity anomaly information with the long-wavelength satellite-only information. Such approaches have been used in the development of, e.g., OSU89A/B [*Rapp and Pavlis*, 1990] and EGM96 [*Lemoine et* *al.*, 1998]. An undesirable side effect of these weighting approaches is that they generally yield unrealistically large gravity anomaly error spectra at the higher degrees, as it can be seen in [*Rapp and Pavlis*, 1990, Figure 12], where the signal degree variances dip below the noise near degree 260, although the OSU89B model contained significant reliable signal information up to its maximum degree 360. In EGM96, to compensate for this side effect, an *a priori* constraint was applied at the higher degrees of the solution, as *N. Pavlis* in [*Lemoine et al.*, 1998, section 8.5.6] discusses in detail. However, this approach, besides the noise, slightly dampens the signal spectrum as well, which could have contributed to EGM96 being slightly underpowered at the higher degrees [*cf. Jekeli*, 1999].

In order to avoid the limitations of the weighting approaches used in the past, we designed and implemented a different method for the calibration of the gravity anomaly weights and the propagated error properties of the solution. The two essential elements of our approach are: (a) the comparison of gravimetric quantities obtained from a test solution to independent data, and, (b) the error propagation method that we describe in section 5.1. The comparison of various gravimetric quantities implied by a test combination solution to independent data, such as geoid heights obtained from GPS positioning and leveling data, astrogeodetic deflections of the vertical, and TOPEX altimeter data, provides estimates of the total, i.e., commission plus omission errors associated with the model. These estimates reflect actual performance of the model, and vary as a function of data source, terrain roughness, geographic area, and gravimetric functional in question. Of course, due care should be given to the fact that the independent data are not perfect either. On the other hand, the propagation of gravity anomaly errors and formal error estimates derived from the covariance matrix of the ITG-GRACE03S model provide statistical estimates of the error properties of the combination solution. We consider that the data weights have been properly calibrated, when the actual performance of the model matches its estimated error properties to a satisfactory degree. This can be achieved in an iterative fashion, by appropriate modifications of the data weights.

We initialized this iterative approach by partitioning the global set of 5 arc-minute gravity anomalies into 23 distinct "classes" representing various geographic regions and/or data types, such as terrestrial or altimetry-derived values. We assigned, more or less empirically, to each class a unique initial overall "error profile" consisting of the values corresponding to the minimum, maximum, and RMS gravity anomaly error over that class. We then generated initial error estimates for each individual 5 arc-minute gravity anomaly within the class, consistent with the overall error profile of each class. In this process, we accounted for the variation of the gravity anomaly error within each class using several proxy metrics, including the individual 5 arc-minute gravity anomaly error estimates obtained from the LSC estimator, gravimetric and topographic roughness information, and observed discrepancies of our test solution with independent data. Using the methodology described in section 5.1, we then propagated the error estimates of the 5 arc-minute gravity anomalies onto gravimetric quantities such as geoid heights and deflections of the vertical, accounting also for the contribution of the ITG-GRACE03S model errors in the formation of the entire commission error budget. We then compared these geographically specific commission error estimates with the performance of our test solution against independent data. The latter represents actual performance when the independent data cover the geographic area in question in a fairly uniform fashion, e.g., as altimeter data do over most of the ocean. Over areas where the independent data coverage is inadequate or where such data are missing altogether, the performance of our test solution was gauged by extrapolating its actual performance from areas where the gravity data quality and the terrain characteristics are similar. For each class, the discrepancies between the estimates of the test solution errors and the actual performance of the test solution itself, guided the refinements necessary to the error assignment scheme used for that class. This process was iterated several times, until the geographically specific error estimates obtained through error propagation, satisfactorily matched the observed or extrapolated performance of the actual solution, as reflected in the comparisons with independent data. Within this error calibration process, we also employed a set of spectral weights. Their purpose was to ensure that the error degree variances associated with the solution, particularly at the very high degrees, would be meaningful, i.e., the signal spectrum would not dip "prematurely" below the propagated error spectrum, and would also correspond to the results obtained from the geographically specific error propagation. In this fashion, the error properties of the solution examined either spectrally, or geographically, or gauged through comparisons with independent data are all to be consistent, as we discuss in more detail in section 5.

Of critical importance for the optimality of the combination solution, is the weight of the satellite information relative to that of the "terrestrial" gravity anomaly information in the least squares adjustment. Terrestrial is in quotes, since the global gravity anomaly dataset used here contains also altimetry-derived and airborne gravity anomalies, whose acquisition relies on methods that are not confined to the surface of the Earth. This notation, with this meaning, is used throughout our paper. In the development of our final EGM2008 combination solution, the original ITG-GRACE03S normal equations were down-weighted by a factor of 25. This corresponds to an increase of the formal errors of the ITG-GRACE03S solution by a factor of five. This down-weighting was established empirically, through an iterative process aiming to produce a combination solution with the minimal amount of GRACE stripe artifacts, which would perform satisfactorily in orbit fit comparisons, including those involving GRACE K-band range-rate data, which were problematic in our previous, PGM2007B, solution.

Using the calibrated error estimates both for the "terrestrial" data and for the ITG-GRACE03S normal equations, we combined these two sources of gravitational field information in a least squares adjustment that yielded the final EGM2008 solution. Figure 6a shows the 5 arc-minute residual gravity anomalies from this adjustment. These residuals represent differences between the gravity anomaly information obtained from GRACE and the corresponding information contained in the "terrestrial" data, up to ellipsoidal harmonic degree 180. Up to this degree the RMS gravity anomaly residual is approximately ±2.3 mGal. As expected, large residuals occur over areas where the gravity anomaly data are of poor quality, such as over certain regions in Africa, Asia, South America and some mostly coastal areas of Greenland. Due to the high quality of altimetry-derived gravity anomalies, over most ocean areas the residuals are within ±2 mGal. Notice however that over areas with increased sea surface variability, such as over some regions in the Southern Ocean, the residuals faithfully reflect the increased noise in the altimetry-derived gravity anomalies. Of course, the small residuals over Antarctica simply reflect the fact that the ITG-GRACE03S gravitational model itself was used to fill-in the entire continent with synthetic gravity anomaly values.

Figure 6b shows the 5 arc-minute gravity anomaly differences ITG-GRACE03S minus EGM2008. These have a global RMS value of approximately ±1.0 mGal. Ideally, if our relative weighting scheme was perfect, the "terrestrial" data within our combination solution should have enabled us to filter out of the ITG-GRACE03S model *all* the stripe artifacts. In turn, the ITG-GRACE03S model should have enabled us to filter out of the "terrestrial" gravity anomalies all errors with wavelengths longer than the shortest wavelength that is represented accurately within ITG-GRACE03S. In such an ideal case, Figure 6a should have contained *only* errors associated with the "terrestrial" data, while Figure 6b should have contained *only* stripe artifact features (regarding stripe artifacts, see also Figures 9a and 10a). Examination of Figures 6a and 6b indicates that this is indeed the case in general, certainly over all areas covered with high quality gravity data. Only over some areas with gravity data of poor quality, such as some regions in Africa, Asia, and South America, the imperfections in our weighting scheme seem to have caused a small part of the gravity anomaly error to creep into the EGM2008 combination solution, as it can be seen from Figure 6b.

Figure 7a displays the gravity anomaly degree variances, computed based on equation (39), for the signal and error spectra associated with the final EGM2008 model up to ellipsoidal degree 2159. As expected, due to the extremely high accuracy of the GRACE information, the ITG-GRACE03S model dominates the low degree part of the EGM2008 solution. Therefore the EGM2008 error up to degree approximately 70 is practically identical to the calibrated error of ITG-GRACE03S. The "terrestrial" gravity information dominates at the higher degrees, and the error spectrum of EGM2008 beyond degree approximately 120 is practically identical to the calibrated error of its surface gravity component. The transition from GRACE to surface gravity information takes place within the degree range 70 to 120, as it is shown more clearly in the enlargement of Figure 7b. The EGM2008 signal spectrum dips below its estimated noise spectrum at ellipsoidal harmonic degree 2090 or so.

4.4 Evaluation of EGM2008 Using Independent Data

The residual gravity anomalies and the error spectra associated with a combination solution represent internal consistency indicators of the quality of a solution. Comparisons with data independent from the solution offer a verification and validation capability, necessary to assess the actual performance of a model. A single, global set of independent data with the spectral sensitivity and accuracy necessary to test the entire spectral bandwidth of the model and its complete global coverage does not exist. Even if it did, it would probably have been used for the estimation of a solution in the first place, thus eliminating its independence from the model being tested. Therefore, one is forced to use independent data of different spectral sensitivities and/or geographic extent, in an effort to evaluate a solution based on tests of complimentary spectral and/or geographic character [*cf. Pavlis et al.*, 1999].

Orbit Fit Tests

Table 4, which was kindly provided by Minkang Cheng (UT/CSR), shows the average RMS residual from 3-day orbit fits, spanning the year 2003, for six SLR-tracked spacecraft. The fits were performed without, as well as with, the adjustment of one cycle-per-revolution (1-cpr) empirical accelerations (see also [*Colombo*, 1986, 1989]), and are reported for EGM96 [*Lemoine et al.*, 1998], GGM02C [*Tapley et al.*, 2005], and the EGM2008 solution. The models that include GRACE information, GGM02C and EGM2008, show practically the same performance in these tests, and an improvement over the pre-GRACE EGM96 solution, especially for the lower altitude spacecraft Stella, Starlette, and BE-C. Additional details about these orbit fit tests can be found in [*Cheng et al.*, 2009].

As we noted previously, satisfactory results from the orbit fit tests shown in Table 4 represent a necessary but not sufficient condition for the quality of any GRACE-based model. The results from these tests are very similar for any GRACE-based model, due to the limited spectral sensitivity of these tests, in conjunction with the very high accuracy of the long wavelength gravitational information provided by the GRACE data [*Cheng et al.*, 2009]. These tests may be capable of revealing serious flaws in the long wavelength component of a combination solution. No such flaws were revealed when testing the PGM2007A solution [*ibid.*, 2009]. But the tests did not prove to be sensitive enough to reveal the imperfections of our weighting of the GRACE information in the PGM2007A and B solutions, which were only revealed when the GRACE K-band range-rate data fits were examined. Given our experience with the PGM2007B solution, a test critical for the adoption of the final EGM2008 solution was the performance of the model in these GRACE K-band range-rate data fits. The precision of these data is higher than ±0.2 μm/s, probably reaching ±0.1 μm/s [*Rowlands, personal communication*, 2012]. These data cannot be considered independent from EGM2008, since our model has already incorporated the ITG-GRACE03S information. RMS fits to GRACE K-band range-rate data computed from 26 daily arcs spanning the month of November 2005 were performed at NASA's Goddard Space Flight Center [*Luthcke, personal communication*, 2008]. The results are shown in Figure 8. It is obvious that the weighting scheme used in EGM2008 has resolved the problem of PGM2007B in this test. The EGM2008 performance of ±0.385 μm/s average RMS over the 26 arcs, is only marginally inferior to GGM02C's performance of ±0.312 μm/s, reflecting most likely a different philosophy in the relative weighting of the GRACE versus the surface data information in the two solutions.

Comparisons with GPS/Leveling Data

Over several years we have maintained a global database of GPS/Leveling (GPS/L) data, generously contributed by various colleagues. These data remain independent from any of our gravitational models. Currently, the "thinned" version of our database contains a total of 12387 points, distributed over 52 countries or territories. The thinning of the points, which we apply after careful inspection of the geographic distribution of the original GPS/L data within each of our data sources, aims to avoid clusters of points located extremely close to each other. Such clusters may affect significantly the statistics of our comparisons and may produce misleading results. Our thinning algorithm considers *only* the geographic location of the data, and does not discard any points based on comparisons with any gravitational model. The global distribution of our GPS/L stations is uneven, with the majority of our data located over North America, Europe, and Australia, and considerably fewer points over South America, Asia, and Africa. 4201 of our thinned data were located over CONUS. These data originated from an update of the file documented in [*Milbert*, 1998] and were made available by the National Geodetic Survey in 1999. Within our database, some of the GPS/L data sources provide geoid undulations (N), while others provide height anomalies (ζ). We account for this in our comparisons, using *Rapp's* [1997] formulation and the spherical harmonic coefficients from the analysis of the DTM2006.0 database that we discussed in section 3.2, in order to compute the height anomaly to geoid undulation conversion terms, to a spherical harmonic degree commensurate to the maximum degree of the model being tested. Table 5 summarizes the results from our GPS/L comparisons over CONUS. In these comparisons, we segment the data by State within the USA (see also [*Smith and Roman*, 2001]), and we apply a ±2 meter editing criterion to the differences between GPS/L undulations and model-derived values. We compute statistics of GPS/L undulations minus model-derived values after removing a bias, as well as after removing a linear trend from the differences within each State. A bias, i.e., a non-zero mean value of the differences between the GPS/L and the gravimetric geoid undulations, represents primarily the combined effect of any offset that may be present in the realization of a leveling datum compared to the gravimetric geoid, and the difference between the semi-major axis of the ellipsoid used to derive ellipsoidal (geodetic) heights from the GPS-derived Cartesian coordinates and that of the "ideal" mean-Earth ellipsoid with respect to which our gravimetric geoid undulations were computed. This "ideal" mean-Earth is defined such that the geoid undulations defined with respect to its surface average to zero globally. In Table 5, the tabulated standard deviations represent values weighted by the number of points within each State.

Table 5 includes the statistics of the differences between the GPS/L undulations and those from EGM96 [*Lemoine et al.*, 1998] to degree 360, GGM02C [*Tapley et al.*, 2005] up to degree 200 augmented with EGM96 from degree 201 to 360, EIGEN-GL04C [*Förste et al.*, 2008] to degree 360, as well as with the detailed 1 arc-minute gravimetric geoid USGG2003, which is discussed in *Wang and Roman* [2004]. Progressing from EGM96 to the newer GRACE-based models that are expected to be more accurate, the number of points passing the ±2 meter editing criterion is generally increasing, while the standard deviation of the differences is generally decreasing, as they should. When truncated to degree 360, the EGM2008 combination solution performs noticeably better than its contemporary solutions GGM02C and EIGEN-GL04C. When extended to spherical harmonic degree 2190, EGM2008 results in no edited points using the ±2 meter criterion, and performs better than even the detailed gravimetric geoid USGG2003, whose 1 arc-minute resolution corresponds to spherical harmonic degree 10800.

In Table 6 we summarize the results from similar GPS/Leveling data comparisons, using the entire compliment of our globally distributed sets of points. Again, the truncated to degree 360 version of EGM2008 outperforms its contemporary solutions GGM02C and EIGEN-GL04C. Using EGM2008 to its maximum degree 2190, results in ±13.0 cm weighted standard deviation after removing a bias per data set, and ±10.3 cm after removing a linear trend per data set. These results indicate that EGM2008 may have reached or even surpassed the ±15 cm global RMS geoid undulation commission error goal set by NGA at the beginning of this project. Of course, we should note here that the distribution of GPS/Leveling data is confined to land areas only, and over these areas is certainly not uniform. In most cases, high quality GPS/Leveling data exist over the same areas covered with high quality gravity data.

The increased accuracy and resolution of EGM2008 as compared to other models, mandates special attention to the fact that the geoid undulations or height anomalies obtained from the GPS/Leveling data are not error free. Table 7 shows the results from GPS/L comparisons using 534 points distributed over mainland Australia. These comparisons were performed using the same ±2 meter editing criterion and the same computational procedure for the height anomaly to geoid undulation conversion terms as in all our GPS/L comparisons. Compared to CONUS, the results here are systematically poorer for all models. For EGM2008 to degree 2190 they are poorer by approximately a factor of two, compared to the corresponding results shown in Table 5. Interestingly, EGM2008 to degree 2190 outperforms the AUSGeoid98 geoid model [*Featherstone et al.*, 2001], whose 2 arc-minute resolution corresponds to spherical harmonic degree 5400. The poorer performance by the various geoid models may reflect errors in the GPS/L geoid undulations, rather than errors in the geoid models. This hypothesis is also supported by comparison results that we obtained using 48 high accuracy GPS/L points distributed over Australia's South West Seismic Zone [*Featherstone et al.*, 2004, Figure 1]. These results are shown in Table 8. Although our sample here is small, the various geoid models demonstrate a performance that is similar to their performance over CONUS.

Comparisons with Astrogeodetic Deflections of the Vertical

Astrogeodetic deflections of the vertical are particularly useful for the evaluation of the high degree part of a gravitational model. Two sets of such independent data were available to us. One consists of 3561 pairs of meridional and prime-vertical deflections (ξ, η) distributed over CONUS. This set is also discussed in [*Jekeli*, 1999]. The other set consists of 1080 (ξ, η) pairs, scattered over Australia [*Featherstone, personal communication*, 2006]. Using the specific procedures discussed in detail in [*Jekeli*, 1999], we compared the independent astrogeodetic (ξ, η) data in these two sets to the corresponding gravimetric values computed by various models. The RMS differences ($\Delta\xi, \Delta\eta$) are shown in Table 9.

As expected, the results are practically equivalent for all models extending to degree 360. A significant reduction of the RMS differences by approximately a factor of three occurs when EGM2008 is extended to degree 2190. Up to that degree, the performance of EGM2008 is marginally inferior to that of the detailed DEFLEC99 model that has a 1 arc-minute resolution, and marginally superior to the performance of the 2 arc-minute resolution AUSGeoid98 model. It should be emphasized here that despite the high resolution of EGM2008, there is a significant contribution to the deflections of the vertical arising from harmonics beyond the maximum degree of EGM2008. This omission error of EGM2008 may be reduced considerably, using the

Residual Terrain Modeling approach, in a fashion similar to our augmentation of the gravitational information beyond degree 720 in the fill-in anomalies, as [*Hirt et al.*, 2010] demonstrated recently.

Comparisons with TOPEX Altimeter Data

Over a set of reference locations on the 10-day repeat ground-track of the TOPEX/Poseidon altimeter satellite, we have formed temporally averaged values of the Sea Surface Heights (SSH), sampled at the rate of one-per-second, by "stacking" altimeter data over the 6-year period from 1993 to 1998. This mean track contains 517835 1 Hz SSH values. Over these locations we compute residual SSH as:

$$rSSH = SSH - N_{Mod} - \zeta_{Mod} \ , \tag{41}$$

where N_{Mod} is the geoid undulation implied by a gravitational model and ζ_{Mod} is the Dynamic Ocean Topography implied by a DOT model. The DOT2007A model discussed in section 4.1, complete to degree and order 50, was used in all the comparisons whose results are summarized in Table 10. We have also formed 494350 along-track residual SSH slope values, by differencing consecutive residual SSH values and dividing these differences by the distance of the sub-satellite points. To avoid data gaps, no slopes were formed if the consecutive sub-satellite points were further than 8 km apart from each other. We have also applied a 200 m depth threshold, in order to avoid shallow water areas where tidal corrections may be less accurate. Inland and enclosed seas, such as the Caspian, Mediterranean, Black and Red Seas, and the Hudson Bay were excluded from this comparison.

In Table 10 we show the maximum absolute residual SSH and residual SSH slope, as well as the standard deviation of these quantities, for the same global models as those compared in our GPS/L tests. Up to degree 360, the EGM2008 solution clearly outperforms its contemporary models GGM02C and EIGEN-GL04C, both in terms of the residual SSH and the residual SSH slope comparisons. Expanding the EGM2008 solution to its maximum degree yields an improvement by a factor of approximately three in the standard deviation of $rSSH$, and a factor of 6.3 in the standard deviation of $rSSH$ slopes, compared to its truncated to degree 360 version.

Dynamic Ocean Topography Comparisons

In late January of 2008, the DNSC provided to us the twelfth version of their MSS, which was designated DNSC08B [*Andersen, personal communication*, 2008]. This MSS model is identical to the DNSC08 MSS model discussed by *Andersen and Knudsen* [2009]. DNSC08B was delivered to us in 1, 2, and 5 arc-minute versions. We used this MSS to compare the DOT implied by EGM2008 and by its contemporary GRACE-based gravitational models GGM02C [*Tapley et al.*, 2005] and EIGEN-GL04C [*Förste et al.*, 2008]. For this test, we created three sets of residual SSH by subtracting area-mean values of height anomalies computed from the three gravitational models over 2 arc-minute cells, from the 2 arc-minute version of the DNSC08B MSS. As before, we augmented GGM02C with the EGM96 coefficients from degree 201 to 360, and we used EIGEN-GL04C to degree 360. We computed the height anomalies from all three models in the "Mean Tide" system, to be consistent with the permanent tide system in which the DNSC08B MSS is expressed. We averaged the 2 arc-minute residual SSH values over 6 arc-minute and over 1° equiangular cells, without applying any other smoothing or filtering. These residual SSH represent also "direct" estimates of the DOT. Apart from the DOT signal, they are composed of errors present in the MSS, as well as errors of commission and of omission associated with each gravitational model used to define the geoid. The accuracy and resolution of the GRACE-based geoid information is the primary factor affecting the accuracy and resolution of the DOT that can be extracted from these residual SSH. The three DOT estimates obtained in this fashion are shown in Figure 9.

The omission error associated with the two models that extend to degree 360 is visible in Figure 9, especially over trenches and sea mount chains, where these models fail to capture the large variations of the geoid, which are also present in the 6 arc-minute averages of the MSS. The DOT estimate based on GGM02C shows significant stripe artifacts. These are less pronounced in the estimate that is based on EIGEN-GL04C, which produces though some "ringing" artifacts that are most evident around the coast of New Zealand. The EGM2008 estimate of the DOT is largely free of the artifacts and shortcomings of the estimates based on the other two models. Over the 4247328 6 arc-minute cells displayed in Figure 9, the standard deviation of the residual SSH is ±66.60 cm based on GGM02C, ±66.58 cm based on EIGEN-GL04C, and ±63.97 cm based on EGM2008.

We also compared the residual SSH computed from these three gravitational models and averaged over 1°×1° cells, to the DOT output for the 12-year period [1993, 2004] of the MIT version of the ECCO general circulation model [*cf. Wunsch and Heimbach*, 2007], as we have also discussed before (see Table 3). The results are displayed in Figure 10. Again, the use of GGM02C results in significant stripe artifacts. EIGEN-GL04C shows reduced stripe artifacts compared to GGM02C, but creates the "ringing" artifacts that are absent from GGM02C. The differences based on the EGM2008-implied residual SSH are largely free of the artifacts and the shortcomings associated with the other two models, and, as expected, are highly correlated with areas of significant sea surface height variability over the Southern Ocean, and the Gulf Stream, Kuroshio, and Agulhas currents. Over the 33016 1° equiangular ocean cells displayed in Figure 10, the standard deviation of the differences is ±9.7 cm based on GGM02C, ±10.7 cm based on EIGEN-GL04C, and ±7.8 cm based on EGM2008.

4.5 The Dynamic Ocean Topography Model DOT2008A

The DOT estimate displayed in Figure 9c was the basis for the development of a spherical harmonic representation of the DOT model implied by the DNSC08B MSS and the EGM2008 geoid model. To develop this model, we first edited the EGM2008-implied residual SSH by smoothing over suspiciously high residual SSH spikes, particularly near the coastlines where the DNSC08B MSS is less accurate, and by tapering the edited field inland in order to eliminate discontinuities at the coastal boundaries. Specifically, since the DOT is defined only over ocean areas, we first extrapolated the edited DOT values inland, in an iterative fashion that estimates fictitious DOT values based on either valid DOT values or on previously extrapolated fictitious values, moving progressively inland. In a second step, we linearly taper the fictitious inland DOT values to zero with increasing distance from the coastline, so that all inland cells further than 10 degrees from the coastline contain a zero DOT value. We thus created a global set of "DOT" area-mean values, over a global 5 arc-minute equiangular grid. We then analyzed harmonically these gridded values, and determined a set of spherical harmonic coefficients of the DOT, complete to degree and order 180. We note here that the DNSC08B MSS was developed using the "standard" inverted barometer correction [*cf. Gill*, 1982] for the satellite altimeter data, with a 1013.3 mbar reference pressure. The use of alternative formulations accounting for the response of the ocean to the variable atmospheric pressure loading will produce, in general, a different MSS and therefore a different DOT model.

After plotting the DOT surface produced from our spherical harmonic coefficients to degree and order 180, we recognized that some small residual stripe artifacts were still visible in that surface. In order to reduce those, we applied to our spherical harmonic DOT model a Gaussian smoothing function $w(\psi)$, defined by:

$$w(\psi) = \exp\left[-a(1-\cos\psi)\right], \quad a = \ln 2 \big/ (1-\cos\psi_0), \quad w(\psi_0) = 1\big/2 \quad , \tag{42}$$

with the spherical angle ψ_0 set to $\psi_0 = 0.8°$, a value which we determined empirically. We applied this smoothing in the spectral domain, operating on the spherical harmonic coefficients of our DOT representation, using the eigenvalues of the Gaussian smoothing operator [*cf. Jekeli*, 1981]. We designated the resulting spherical harmonic DOT model as DOT2008A. Of course this smoothing that we applied to the DOT field also dampens its spectral power, progressively with increasing degree. Although the DOT2008A model is defined to degree and order 180, its signal spectrum dips below the EGM2008 geoid error spectrum at degree 69, which corresponds approximately to a 2.6° half wavelength angular resolution at the equator.

An independent set of data, which offered us the possibility to test the EGM2008 and DOT2008A models consisted of airborne LiDAR SSH data collected over 13 flight lines in the area of the Aegean Sea [*Papafitsorou et al.*, 2003]. A total of 106726 LiDAR-derived SSH were available to us. We generated model-derived values for these SSH and analyzed the differences between LiDAR-derived and model-derived SSH by flight line. The mean standard deviations of these differences, weighted by number of points per flight line, are shown in Table 11 for three model-derived sets of SSH generated from: (a) the EGM96 geoid to degree and order 360 and its associated TOPEX-specific DOT96 model to degree and order 20 [*Lemoine et al.*, 1998, section 7.3.3.2], (b) the EGM2008 geoid to degree 2190 and DOT2008A to degree and order 180, and (c) the DNSC08B MSS. The modeling gain achieved with EGM2008 and DOT2008A is obvious and amounts to a reduction of the mean standard deviation by about a factor of five. Interestingly, EGM2008 plus its associated DOT2008A model slightly outperform even the DNSC08B MSS, which has a 1 arc-minute resolution. Being a nearly enclosed sea, dotted with numerous islands, the Aegean Sea offers the setting for very challenging tests to MSS, DOT, and geoid models. Therefore, over open ocean areas, we expect the error in the model-derived SSH from the EGM2008 geoid plus the DOT2008A model to be less than 6 or 7 cm (see also Table 10), considering also that the independent LiDAR-derived SSH are not error free.

Concluding this section, in Figure 11 we present a result indicative of the resolving power supported by the use of EGM2008 to degree 2190, as compared to the resolving power obtained from expansions extending only to degree and order 360. In that figure we have plotted the gravity anomalies over the Yucatán Peninsula, computed from: (a) EIGEN-GL04C to degree and order 360, and (b) from EGM2008 to degree 2190. The “ring” of the Chicxulub impact crater is clearly visible in the EGM2008 gravity anomalies. This feature is indistinguishable in the plot showing the EIGEN-GL04C gravity anomalies. To our knowledge, EGM2008 is the first global gravitational model *ever*, that possesses sufficient resolving power to permit the clear identification of this geophysical feature.

5. ERROR ASSESSMENT

The use of any model for the computation of functionals of the gravitational field such as gravity anomalies, height anomalies, geoid undulations, deflections of the vertical, etc., implies a commission and an omission error. The commission (or propagated) error is due to the fact that any model based on actual observations can never be error-free since the data supporting its development can never be error-free. The omission (or truncation) error is due to the fact that a model can only have finite resolution; therefore it will always omit a portion of the Earth's true gravity field spectrum, which extends to infinity. Model users require geographically specific estimates of the commission error associated with the model that they are using. The rigorous computation of the commission error requires the complete error covariance matrix of the model's defining parameters. Given this matrix, one can compute the commission error of various model-derived functionals, using error propagation. The error covariance matrix of an ellipsoidal harmonic model complete to degree and order 2159 has dimension ~4.7 million, and as we discussed before, the computation of such a matrix is beyond our existing computational resources. Even for expansions to degree and order 360, like EGM96, which involve approximately 130000 parameters, the formation of the normal equation matrix, its inversion, and the subsequent error propagation using the resulting error covariance matrix is a formidable computational task. For EGM96 [*Lemoine et al.*, 1998], such error propagation was only possible for the portion of the model extending to degree and order 70. For EGM2008, which extends to degree 2159 in ellipsoidal harmonics, the alternative error propagation technique that was developed and implemented by *Pavlis and Saleh* [2005] was used. This technique is capable of producing geographically specific estimates of a model's commission error, *without* the need to form, invert, and propagate large matrices. Instead, this technique uses integral formulas with band-limited kernels and requires as input the error variances of the gravity anomaly data that are used in the development of the gravitational model.

5.1 Methodology

The main idea behind the technique of *Pavlis and Saleh* [2005] is the realization that in combination solutions like EGM96 and EGM2008, the satellite-only information influences the combined model only up to a relatively low degree, which is the maximum degree of the satellite-only solution. Up to this maximum degree, the combined solution is the outcome of a least-squares adjustment. Beyond this degree, the solution is determined *solely* from the complete, global grid of area-mean gravity anomaly data. Therefore, beyond the maximum degree and order of the available satellite-only solution, there is little need to form complete normal matrices, since no "adjustment" takes place within this degree range. The merged (terrestrial plus altimetry-derived) area-mean gravity anomalies are the only data whose signal and error content determine the model's signal and error properties over this degree range. This fact enables high degree error propagation, *with* geographic specificity, through the use of integral formulas with band-limited kernels, *without* the need to form, invert, and propagate extremely large matrices. We will use the geoid undulation as an example of a model-derived quantity to present the essential elements of the technique, whose details can be found in [*Pavlis* *and Saleh*, 2005]. The adjusted gravity anomaly computed from a composite model can be written as (*L* and *H* stand for *Low*- and *High-degree*):

$$\widehat{\Delta g} = \widehat{\Delta g}_L + \widehat{\Delta g}_H = \sum_{n=2}^{L} \widehat{\Delta g}_n + \sum_{n=L+1}^{H} \widehat{\Delta g}_n \; . \tag{43}$$

The corresponding geoid undulations is:

$$\widehat{N} = \widehat{N}_L + \widehat{N}_H = \sum_{n=2}^{L} \widehat{N}_n + \sum_{n=L+1}^{H} \widehat{N}_n \; , \tag{44}$$

and can also be written as [*Heiskanen and Moritz*, 1967]:

$$\widehat{N} = \frac{R}{4\pi\gamma} \iint_{\sigma} \widehat{\Delta g}\, S(\psi)\, d\sigma \; , \tag{45}$$

where $S(\psi)$ is Stokes' function [*ibid.*, section 2-16]. With $t = \cos(\psi)$, and $P_n(t)$ denoting the Legendre polynomial of degree n, one has [*ibid.*, equation 2-169]:

$$S(\psi) = \sum_{n=2}^{\infty} \frac{2n+1}{n-1} P_n(t)$$

$$= \sum_{n=2}^{L} \frac{2n+1}{n-1} P_n(t) + \sum_{n=L+1}^{H} \frac{2n+1}{n-1} P_n(t) + \sum_{n=H+1}^{\infty} \frac{2n+1}{n-1} P_n(t)$$

$$= S_L(\psi) + S_H(\psi) + S_\infty(\psi) \ . \tag{46}$$

Equations (43) through (46), due to the orthogonality of surface spherical harmonics imply that:

$$\check{N} = \frac{R}{4\pi\gamma} \iint_\sigma \left(\check{\Delta g}_L + \check{\Delta g}_H + 0 \right) \left[S_L(\psi) + S_H(\psi) + S_\infty(\psi) \right] d\sigma \quad \Rightarrow$$

$$\check{N} = \frac{R}{4\pi\gamma} \iint_\sigma \check{\Delta g}_L \, S_L(\psi) \, d\sigma + \frac{R}{4\pi\gamma} \iint_\sigma \check{\Delta g}_H \, S_H(\psi) \, d\sigma = \check{N}_L + \check{N}_H \ . \tag{47}$$

Therefore, a strict, degree-wise separation of spectral components can be achieved by restricting the spectral content of the kernel function accordingly, *as long as* the integration is performed globally. The band-limited version of Stokes' equation:

$$\check{N}_H = \frac{R}{4\pi\gamma} \iint_\sigma \check{\Delta g}_H \, S_H(\psi) \, d\sigma \ , \tag{48}$$

implies, for uncorrelated errors of $\check{\Delta g}_H$, the error propagation formula:

$$eVar(\check{N}_H) = \left(\frac{R}{4\pi\gamma} \right)^2 \iint_\sigma eVar(\check{\Delta g}_H) \, S_H^2(\psi) \, d\sigma \ , \tag{49}$$

for the computation of the high-degree component of the commission error variance in the geoid undulation. $eVar$ denotes the error variance of the quantity inside the parenthesis. This approach is applicable to any functional related to the gravity anomaly through a surface integral formula. Equation (49) employs the spherical approximation, which we consider adequate for error propagation work. Apart from this, (49) is rigorous, and its numerical implementation is only subject to discretization errors. Finally, the band limiting of integration kernels removes the singularity at the origin of kernels like Stokes' and Vening Meinesz's, therefore the innermost zone effects require no special treatment.

If we assume that the error correlation between $\check{\Delta g}_L$ and $\check{\Delta g}_H$ is negligible, then the total error variance of a field functional, f, at the geographic location (R, φ, λ), as computed from a specific gravitational model, can be written as:

$$
\begin{aligned}
eVar_f(R,\varphi,\lambda) \approx\; & eVar_f(R,\varphi,\lambda)\_commission\_L \\
& + eVar_f(R,\varphi,\lambda)\_commission\_H \\
& + eVar_f(R,\varphi,\lambda)\_omission
\end{aligned}
\quad , \tag{50}
$$

where $eVar_f(R,\varphi,\lambda)\_commission\_L$ may be computed through error propagation using the error covariance matrix from the combination solution corresponding to the maximum degree of the satellite-only model, $eVar_f(R,\varphi,\lambda)\_commission\_H$ is computed using global convolution based on a surface integral formula, and $eVar_f(R,\varphi,\lambda)\_omission$ may be estimated statistically using local covariance models or may be deduced from gravimetric information implied, e.g., by the topography. This approach circumvents the need to form, invert, and propagate extremely large matrices.

In the present case, we estimated the degree L to be equal to 86, based on Figure 7b. Due to the diagonal dominance of the ITG-GRACE03S normal equations, which also dominates the EGM2008 combination solution up to that degree, the term $eVar_f(R,\varphi,\lambda)\_commission\_L$ was estimated considering only the error variances of the EGM2008 coefficients. The high-degree component $eVar_f(R,\varphi,\lambda)\_commission\_H$, from degree 87 to degree and order 2159 was computed via the 1D FFT algorithm of [*Haagmans et al.*, 1993], using the calibrated gravity anomaly standard deviations discussed in section 4.3. Within this computation we also introduced a set of spectral weights. These are a function of spherical harmonic degree and multiply the corresponding surface spherical harmonic component of the appropriate kernel function in error propagation formulas like equation (49). Their purpose was to ensure that the propagated error estimates would also be consistent with the error degree variances of Figure 7a, and would not produce an error spectrum that overwhelms the signal spectrum of the model "prematurely", as we also discussed in section 4.3.

Using this approach, we computed the commission error implied by EGM2008 from degree 2 to degree and order 2159, on point values of gravity anomalies, height anomalies, and the two components (ξ,η) of the deflection of the vertical, over global 5 arc-minute equiangular grids. We note here that in our error propagation work we made no distinction between errors of height anomalies and those of geoid undulations. Strictly speaking, the geoid undulation error should also account for the errors in the elevation data that were used to compute the height anomaly to geoid undulation conversion terms. This may be accomplished if the elevation database contains also reliable estimates for the elevation data errors.

As examples, in Figure 12 we display the results obtained from our error propagation for the height anomalies and the meridional component (ξ) of the deflection of the vertical. Our error estimation implies height anomaly errors that range from ±3 cm to ±102 cm, with a global RMS value for the propagated errors of about ±11 cm. The errors in the meridional component of the deflection of the vertical range from ±0.13 arc-seconds to ±11.4 arc-seconds, with a global RMS value for these propagated errors of approximately ±1 arc-second. Note that the color-bars in Figure 12 have a reduced range compared to the range of the plotted values. This choice permits the illustration of the geographic variability of the plotted values, which otherwise would have been practically invisible, given the distribution of these values, as it may be deduced also from their histograms.

The availability of geographically specific estimates of the commission error implied by EGM2008 over its entire spectral bandwidth, permits also the comparison of the actual performance of the model, as gauged from comparisons with independent data, to its estimated error properties. As we also discussed in section 4.3, this was the basis for our "calibration" of the gravity anomaly error estimates that were used in the combination solution. In Table 12 we present the RMS values of the commission error of EGM2008 for geoid undulations and the deflections of the vertical, over five regions of the Earth. Wherever available, we include in parentheses comparable results from our comparisons with independent data. When comparing the estimated commission error to the model's actual performance, one should keep in mind on one hand the existence of the omission error and on the other the fact that our test data are not error free. These two contributions are present in the comparison results, but absent from the model's strictly commission error estimates. Results involving functionals like the deflections of the vertical, which are rich in high frequency contributions beyond degree 2159, are particularly affected by the omission error component, especially over land areas with significant terrain variation. Therefore, the fact that our commission error estimates on the deflections of the vertical over CONUS amount to approximately half the RMS value obtained from comparisons with astrogeodetic deflections, can be explained if one considers the omission error component. The latter can be estimated using Residual Terrain Modeling (RTM) approaches and detailed elevation data, to compute the component of the deflection of the vertical beyond degree 2159. *Hirt et al.* [2010] used this approach and validated our error estimates over Europe.

Finally, we should also note here that our ±10.3 cm RMS geoid undulation discrepancy representing "Land" reflects the geographic distribution of the GPS/Leveling data available to us. For the most part these data are available over areas that are also covered with gravity data of good quality. The ±18.3 cm RMS geoid undulation commission error for "Land" represents *all* land areas, including, e.g., Antarctica, and should therefore be expected to be significantly higher than the RMS GPS/L discrepancy.

6. EGM2008 MODEL PRODUCTS

The primary product of the EGM2008 model development is the set of estimated spherical harmonic coefficients, to degree 2190 and order 2159. From these coefficients the user may compute the values of various functionals of the gravitational potential such as gravity anomalies, height anomalies, deflections of the vertical, etc., on or above the physical surface of the Earth, using harmonic synthesis. *Holmes and Pavlis* [2006] made available a FORTRAN computer program called HARMONIC_SYNTH, which may be used to perform such harmonic synthesis tasks in various modes, e.g., for randomly scattered geographic locations, or for grids of point and/or area-mean values. This program, accompanied by test input and output files, and associated documentation is freely available from:

http://earth-info.nga.mil/GandG/wgs84/gravitymod/new_egm/new_egm.html

On Table 13 we list the estimated values and their standard deviations of the EGM2008 zonal spherical harmonic coefficients of the gravitational potential to degree 10.

For height anomaly and geoid computations, the user should also pay attention to some important issues related to the Permanent Tide, and the Geodetic Reference System (GRS) to which the computed values refer. For example, in applications involving ellipsoidal heights obtained from space techniques like the Global Positioning System, the user should be aware of the fact that the International Earth Rotation and Reference Systems Service (IERS) reports positions with respect to a conventional "Tide-Free" crust (also known as "Non-Tidal"). Therefore, in order to maintain consistency, geoid undulations and/or height anomalies involved in computations that use positions derived from space techniques, should be computed in the same Tide-Free system. In contrast, in applications involving satellite altimetry, the "Mean Tide" system is commonly used. Therefore, geoid undulations that are to be subtracted from altimetry-derived sea surface heights, in order to estimate the dynamic ocean topography, should also be computed in the Mean Tide system. The definition of the three systems in use with regards to the Permanent Tide (Tide-Free, Mean Tide, and Zero Tide), and the relationships between the geoid undulations expressed in different systems is also discussed in [*Lemoine et al.*, 1998, chapter 11]. This chapter is available on-line from:

http://cddis.nasa.gov/926/egm96/doc/S11.HTML

In the same chapter, the issue of expressing the geoid undulations and/or height anomalies with respect to a specific GRS is discussed. In the case of EGM2008, the conversion from an "ideal" mean-Earth ellipsoid, whose semi-major axis remains numerically unspecified, to the WGS 84 GRS in the Tide-Free system, involves the application of a zero-degree height anomaly, denoted by ζ_z in equation 11.2-1 of the above chapter, equal to -41 cm. The zero-degree height anomaly, ζ_z, that was computed when the WGS 84 EGM96 geoid was released was equal to -53 cm [*Lemoine et al*., 1998, chapter 11]. The main reason for the change in the numerical value of ζ_z from the EGM96 days to the current best estimate, is the discovery by Ouan-Zan Zanife (CLS, France) of an error in the Oscillator Drift correction applied to TOPEX altimeter data [*Fu and Cazenave*, 2001, p. 34]. The erroneous correction was producing TOPEX sea surface heights, biased by approximately 12 to 13 centimeters. Due to the fact that the height anomaly to geoid undulation conversion terms do not average to zero globally, the -41 cm ζ_z value results in a -46.3 cm zero-degree geoid undulation value (N_0). N_0 depends not only on ζ_z, but also on the formulation and the data used to compute the height anomaly to geoid undulation conversion terms.

Under:

http://earth-info.nga.mil/GandG/wgs84/gravitymod/egm2008/egm08_wgs84.html the user can find a modified version of the HARMONIC_SYNTH program, specifically designed to compute geoid undulations at arbitrarily scattered locations, in the Tide-Free system, with respect to the WGS 84 GRS. In the same web site, the user can also find pre-computed global grids of these geoid undulations, at both 1 and 2.5 arc-minute grid-spacing, as well as a FORTRAN program to interpolate from these grids. The interpolation error, i.e., the difference of interpolated values from those obtained via harmonic synthesis, associated with the use of the 1 arc-minute grid and of the interpolation program provided does not exceed ±1 millimeter. This error rises to ±1 centimeter with the use of the 2.5 arc-minute grid.

Several other products of the EGM2008 model development can be found under:

http://earth-info.nga.mil/GandG/wgs84/gravitymod/egm2008/index.html

These include the spherical harmonic coefficients of the Dynamic Ocean Topography model DOT2008A, complete to degree and order 180, as well as grids of height anomalies and of the DOT, computed with oceanographic applications in mind. In addition, grids of pre-computed gravity anomalies and deflections of the vertical, as well as grids of the propagated errors implied by EGM2008 in gravity anomalies, geoid undulations and deflections of the vertical (ξ, η) are available from the same web site.

7. CONCLUSIONS

This paper describes the development and evaluation of the Earth Gravitational Model 2008 (EGM2008), the first model of its kind ever to be developed to ellipsoidal harmonic degree 2159. EGM2008 was developed in a least squares adjustment that combined the ITG-GRACE03S model, which was available to degree and order 180 along with its complete error covariance matrix, with the gravitational information extracted from a global 5 arc-minute equiangular grid of area-mean gravity anomalies. This global set of gravity anomalies was formed by merging terrestrial and airborne data with altimetry-derived values. Over certain areas where the available gravity anomaly data could only be used at a lower resolution, their spectral content was supplemented with the gravitational information obtained from a detailed global topographic database.

The least squares adjustment combination was performed in terms of ellipsoidal harmonics. The combined solution and its error estimates were then converted to spherical harmonics. This conversion preserves the order but not the degree, thus giving rise to model coefficients extending to degree 2190 and order 2159. The fact that the normal equations of ITG-GRACE03S are predominately block-diagonal permitted the combination solution to be performed in a very efficient way, without compromising the accuracy of the results.

The analytical and numerical work that supported the development of the final EGM2008 solution was accomplished over a time span of approximately eight years. During this period, steady progress was achieved both in modeling refinements and in the quality of the data supporting the final model. Three sets of Preliminary Gravitational Models were developed during this period. The precursor of the final solution was also provided for evaluation to an independent Special Working Group (SWG), with international participation, functioning under the auspices of the International Association of Geodesy (IAG) and the International Gravity Field Service (IGFS). Feedback from this group was considered towards the development of the final EGM2008 solution, which was also evaluated by the same independent group.

The evaluation of EGM2008 performed by its developers as well as by the IAG/IGFS SWG both indicate that the model's performance in orbit fits is comparable to any other GRACE-based solution. Over areas covered with high quality gravity data (e.g., USA, Europe, Australia), the discrepancies between geoid undulations computed from EGM2008 and those computed from independent GPS/Leveling data are on the order of ±5 to ±10 cm. These results are comparable to, and in several cases better than, corresponding results obtained using regional detailed geoid models. Deflections of the vertical, derived from EGM2008 over USA and Australia are within ±1.1 to ±1.3 arc-seconds from corresponding values obtained from independent astronomical and geodetic observations. Compared to its predecessor, EGM96, the EGM2008 solution represents an improvement in resolution by a factor of six, and yields improvements in gravity modeling accuracy ranging from a factor of three to a factor of six, depending on the gravitational functional and the geographic area in question.

The EGM2008 solution is accompanied by a set of global 5 arc-minute grids that provide geographically specific estimates of its commission error, over the entire bandwidth of the model, in commonly used gravimetric quantities, such as gravity anomalies, height anomalies, and deflections of the vertical, being mindful, especially in the case of the deflections, of the significant but unaccounted omission error, as mentioned already in section 5. This allows the model user to assign appropriate error estimates to the model's quantities, without the need to propagate error covariance matrices of prohibitively large dimension. This represents a significant improvement over EGM96, whose geographically specific error estimates could be computed only up to degree and order 70.

The EGM2008 solution is accompanied by a Dynamic Ocean Topography model designated DOT2008A. This was developed based on the DNSC08B Mean Sea Surface and the EGM2008 geoid. DOT2008A is available in the form of spherical harmonic coefficients complete to degree and order 180, as well as in grid form.

Although the development of EGM2008 was the catalyst for the systematic re-evaluation of the gravity anomaly data involved in its development, significant margin for improvement remains in this area. This involves data that may be misidentified, e.g., with respect to terrain corrections, as we found the case to be in Turkey. Also, considerable effort is still required towards the acquisition of accurate gravity information over several areas of the Earth. Of these, Antarctica's land mass and surrounding coastal areas remain the least surveyed, and therefore most poorly modeled areas of the Earth's gravity field.

Appendix A

The processing and the analysis of the gravity anomaly data requires the adoption of a Geodetic Reference System (GRS), in the form of a reference ellipsoid of revolution, whose surface is an equipotential surface of its gravity field [*Heiskanen and Moritz,* 1967, section 2-7]. To maintain consistency with the gravity anomaly processing work that was performed in support of EGM96 [*Lemoine et al.,* 1998, section 3.3.1], we adopted early on this project the exact same GRS that was adopted at that time, which is defined by the following four parameters:

$$\left.\begin{array}{ll} \text{Gravitational constant: } GM = 3986004.415 \times 10^{8}\ m^{3}s^{-2} & \text{(a)} \\ \text{Semimajor axis: } a = 6378136.3\, m & \text{(b)} \\ 2nd \text{ degree zonal coeff. (tide-free): } \bar{C}^{s}_{2,\,0} = -484.1654767 \times 10^{-6} & \text{(c)} \\ \text{Mean Earth rotation rate: } \omega = 7292115 \times 10^{-11}\ rad\ s^{-1} & \text{(d)} \end{array}\right\} . \qquad \text{(A1)}$$

From these defining parameters, all derived parameters associated with the GRS, including those appearing in the normal gravity formula, can be computed using closed expressions [*Heiskanen and Moritz*, 1967, section 2-7]. For the speed of light, we used the value $c = 299792458\ m\, s^{-1}$ [*Mohr and Taylor*, 1999], which is consistent with the 2003 and 2006 Conventions of the International Earth Rotation and Reference Systems Service (IERS). The values of GM and a given in equations (A1a) and (A1b) respectively, are also the scaling parameters of the EGM2008 spherical harmonic coefficients of the gravitational potential $\bar{C}^{s}_{nm}$. We should note here that within the least squares adjustment combination of "terrestrial" coefficient estimates with estimates derived from GRACE, the different estimates are rigorously "shifted" to common *a priori* values, as described by *Rapp, et al.* [1991, pp. 20-21]. Furthermore, the EGM2008 coefficients $\bar{C}^{s}_{nm}$ can be rigorously re-scaled to any desired scaling parameters GM and a as described in [*Lemoine et al.*, 1998, section 7.3.5.3], before using them in the computation of functionals of the gravity field (e.g., gravity anomalies, height anomalies). This formulation is actually implemented in the HARMONIC_SYNTH program [*Holmes and Pavlis*, 2006], to ensure that the scaling parameters of the potential coefficients used in the harmonic synthesis are consistent with those of the selected GRS. Finally, we re-emphasize here that the computation of height anomalies and geoid undulations reckoned from the surface of a specific GRS requires the estimation of a zero-degree value, as we discussed in section 6 (see also [*Lemoine et al*., 1998, chapter 11] for details).

Acknowledgements

We thank Carl Wunsch, Patrick Heimbach, and Charmaine King of the Massachusetts Institute of Technology, for providing the MIT ECCO Dynamic Ocean Topography output and its associated documentation. We thank Michael Watkins and Dah-Ning Yuan of NASA's Jet Propulsion Laboratory, for providing the JEM01-RL03B gravitational model and its error covariance matrix that were used in the development of PGM2007A/B, and for bringing to our attention the poor fits of PGM2007B on the GRACE K-band range-rate data. We thank Torsten Mayer-Gürr, of the Technical University of Graz in Austria, for providing the ITG-GRACE03S gravitational model and its error covariance matrix that were used in the development of EGM2008. We thank Minkang Cheng of the Univ. of Texas at Austin, Center for Space Research, for performing the satellite laser ranging data orbit fit comparisons, and Scott Luthcke of NASA's Goddard Space Flight Center, for performing the GRACE K-band range-rate data orbit fit comparisons. We thank all of the members of the Joint Working Group of the International Gravity Field Service (IGFS) and Commission 2 of the International Association of Geodesy (IAG), who participated in the evaluation of PGM2007A and EGM2008, for their valuable feedback. We thank Jianliang Huang and Christopher Kotsakis who co-chaired that Joint Working group, and edited *Newton's Bulletin Issue n°4*. During the course of this project,

and for the preparation of this manuscript, numerous figures were produced using the Generic Mapping Tools (GMT) [*Wessel and Smith*, 1998].

REFERENCES

Andersen, O.B. (2006), personal communication.

Andersen, O.B. (2007), personal communication.

Andersen, O.B. (2008), personal communication.

Andersen, O.B., and P. Knudsen (2009), DNSC08 mean sea surface and mean dynamic topography models, *J. Geophys. Res.*, ***114***, *C11001*, doi: 10.1029/2008JC005179.

Andersen, O.B., P. Knudsen, P.A.M. Berry (2010), The DNSC08GRA global marine gravity field from double retracked satellite altimetry, *J. Geod.*, ***84***, 191–199, doi: 10.1007/s00190-009-0355-9.

Chambers, D.P. and V. Zlotnicki (2004), GRACE products for oceanography, paper presented at the *Ocean Surface Topography Science Team Meeting*, St. Petersburg, Florida, November 4-6, 2004.

Cheng, M., J.C. Ries, and D.P. Chambers (2009), Evaluation of the EGM2008 Gravity Model, in: Newton's Bulletin (2009), External Quality Evaluation Reports of EGM08, *Newton's Bulletin Issue n°4, April 2009*, ISSN 1810-8555, Publication of the International Association of Geodesy and the International Gravity Field Service.

Colombo, O.L. (1981), Numerical methods for harmonic analysis on the sphere, *Rep. 310,* Dep. of Geod. Sci. and Surv., Ohio State Univ., Columbus.

Colombo, O.L. (1986), Ephemeris errors of GPS satellites, *Bull. Geod.*, ***60***, 64-84.

Colombo, O.L. (1989), The dynamics of Global Positioning System orbits and the determination of precise ephemerides, *J. Geophys. Res.*, ***94****(B7)*, 9167-9182.

DiMarzio, J.P. (2005), personal communication.

Ekholm, S. (2005), personal communication.

ESA SP-1233 (1) (1999), – The Four Candidate Earth Explorer Core Missions – Gravity Field and Steady-State Ocean Circulation Mission, ESA Publications Division, *ESTEC*, Noordwijk, The Netherlands.

Factor, J.K. (2006), Modeling Surface Anomalies, unpublished presentation, January 24, 2006.

Featherstone, W.E. (2006), personal communication.

Featherstone, W.E., J.F. Kirby, A.H.W. Kearsley, J.R. Gilliland, G.M. Johnston, J. Steed, R. Forsberg and M.G. Sideris (2001), The AUSGeoid98 geoid model of Australia: data treatment, computations and comparisons with GPS-levelling data, *J. Geod.*, ***75***(5-6), 313–330, doi: 10.1007/s001900100177.

Featherstone, W.E., N.T. Penna, M. Leonard, D. Clark, J. Dawson, M.C. Dentith, D. Darby and R. McCarthy (2004), GPS-geodetic deformation monitoring of the south-west seismic zone of Western Australia: review, description of methodology and results from epoch-one, *J. Roy. Soc. West. Aust.*, ***87****(1)*, 1-8.

Forsberg, R. (1984), A study of terrain reductions, density anomalies and geophysical inversion methods in gravity field modeling, *Rep. 355*, Dep. of Geod. Sci. and Surv., Ohio State University, Columbus.

Förste, C., R. Schmidt, R. Stubenvoll, F. Flechtner, U. Meyer, R. König, H. Neumayer, R. Biancale, J.-M. Lemoine, S. Bruinsma, S. Loyer, F. Barthelmes, and S. Esselborn (2008), The GeoForschungsZentrum Potsdam/Groupe de Recherche de Gèodésie Spatiale satellite-only and combined gravity field models: EIGEN-GL04S1 and EIGEN-GL04C, *J. Geod.*, ***82****(6)*, 331-346, doi: 10.1007/s00190-007-0183-8.

Fu, L.-L. and A. Cazenave (Eds.), (2001), *Satellite Altimetry and Earth Sciences A Handbook of Techniques and Applications*, International Geophysics Series, *Vol.* ***69***, Academic Press.

Fu, L.-L., E. Christensen, C. Yamarone Jr., M. Lefebvre, Y. Ménard, M. Dorrer, and P. Escudier (1994), TOPEX/POSEIDON: mission overview, *J. Geophys. Res.*, ***99****(C12)*, 24369-24381.

Ganachaud, A., C. Wunsch, M.-C. Kim, and B. Tapley (1997), Combination of TOPEX/POSEIDON data with a hydrographic inversion for determination of the oceanic general circulation and its relation to geoid accuracy, *Geophys. J. Int.*, ***128***, 708-722.

Gill, A.E. (1982), *Atmosphere-Ocean Dynamics*, Academic Press, Inc..

Gleason, D.M. (1988), Comparing ellipsoidal corrections to the transformation between the geopotential's spherical and ellipsoidal spectrums, *Manusc. Geod.*, ***13***, 114-129.

GRACE (1998), – Gravity Recovery and Climate Experiment: Science and Mission Requirements Document, revision A, *JPLD-15928*, NASA's Earth System Science Pathfinder Program.

Haagmans, R., E. de Min, M. Van Gelderen (1993), Fast evaluation of convolution integrals on the sphere using 1D FFT, and a comparison with existing methods for Stokes' integral, *Manusc. Geod.*, ***18***, 227-241.

Heiskanen, W.A. and H. Moritz (1967), *Physical Geodesy*, W.H. Freeman and Co., San Francisco.

Hirt, C., U. Marti, B. Bürki, and W. E. Featherstone (2010), Assessment of EGM2008 in Europe using accurate astrogeodetic vertical deflections and omission error estimates from SRTM/DTM2006.0 residual terrain model data, *J. Geophys. Res.*, ***115***, *B10404*, doi: 10.1029/2009JB007057.

Holmes, S.A. and N.K. Pavlis (2006), A FORTRAN program for very-high-degree harmonic synthesis, HARMONIC_SYNTH, http://earth-info.nga.mil/GandG/wgs84/gravitymod/new_egm/new_egm.html

Holmes, S.A. and N.K. Pavlis (2007), Some aspects of harmonic analysis of data gridded on the ellipsoid, in: *Gravity Field of the Earth*, A. Kiliçoglu and R. Forsberg (Eds.), Proceedings of the 1st International Symposium of the International Gravity Field Service (IGFS), *Harita Dergisi*, Special Issue No. 18, General Command of Mapping, Ankara, Turkey.

Jekeli, C. (1981), Alternative Methods to Smooth the Earth's Gravity Field, *Rep. 327*, Dep. of Geod. Sci. and Surv., Ohio State Univ., Columbus.

Jekeli, C. (1988), The exact transformation between ellipsoidal and spherical harmonic expansions, *Manusc. Geod.*, ***13***, 106-113.

Jekeli, C. (1996), Spherical harmonic analysis, aliasing, and filtering, *J. Geod.*, ***70****(4)*, 214-223, doi: 10.1007/BF00873702.

Jekeli, C. (1999), An analysis of vertical deflections derived from high-degree spherical harmonic models, *J. Geod.*, ***73**(1)*, 10-22, doi: 10.1007/s001900050213.

Kenyon, S.C. and R. Forsberg (2008), New Gravity Field for the Arctic, *Eos Trans.*, ***89**(32)*, 289-290, American Geophysical Union.

Lemoine, F.G., S.C. Kenyon, J.K. Factor, R.G. Trimmer, N.K. Pavlis, D.S. Chinn, C.M. Cox, S.M. Klosko, S.B. Luthcke, M.H. Torrence, Y.M. Wang, R.G. Williamson, E.C. Pavlis, R.H. Rapp, and T.R. Olson (1998), The development of the joint NASA GSFC and the National Imagery and Mapping Agency (NIMA) geopotential model EGM96, *NASA Tech. Publ. TP-1998-206861*.

Luthcke, S.B. (2008), personal communication.

Mayer-Gürr, T. (2007), ITG-Grace03s: The latest GRACE gravity field solution computed in Bonn, presented at the Joint International GSTM and SPP Symposium, Potsdam, Germany, October 15-17, 2007 (http://www.igg.uni-bonn.de/apmg/index.php?id=itg-grace03).

Mayer-Gürr, T., A. Eicker, K.H. Ilk (2007), ITG-Grace02s: a GRACE gravity field derived from range measurements of short arcs, in: *Gravity Field of the Earth*, A. Kiliçoglu and R. Forsberg (Eds.), Proceedings of the 1st International Symposium of the International Gravity Field Service (IGFS), *Harita Dergisi*, Special Issue No. 18, General Command of Mapping, Ankara, Turkey.

Milbert D.G. (1998), Documentation for the GPS Benchmark Data Set of 23-July-1998, International Geoid Service, IGeS, *Bulletin n°8*, pp. 29-42.

Mohr, P.J. and B.N. Taylor (1999), *J. Phys. Chem. Ref. Data*, ***28**(6)*, p. 1713.

Moritz, H. (1978), Least-squares collocation, *Rev. Geophys.*, ***16****(3),* 421-430, doi:10.1029/RG016i003p00421.

Moritz, H. (1980), *Advanced Physical Geodesy*, Herbert Wichmann Verlag, Karlsruhe, Federal Republic of Germany.

Newton's Bulletin (2009), External Quality Evaluation Reports of EGM08, *Newton's Bulletin Issue n°4, April 2009*, ISSN 1810-8555, Publication of the International Association of Geodesy and the International Gravity Field Service.

Papafitsorou, A., H.-G. Kahle, M. Cocard and G. Veis (2003), Sea Surface Determination from Airborne Laser Altimetry over the Aegean Sea, *Gravity and Geoid 2002*, Proc. of the 3rd Meeting of the IGGC, Tziavos, I.N. (Ed.), Ziti, Thessaloniki, Greece.

Pavlis, N.K. (1988), Modeling and estimation of a low degree geopotential model from terrestrial gravity data, *Rep. 386*, Dep. of Geod. Sci. and Surv., Ohio State Univ., Columbus.

Pavlis, N.K. (1998), Observed inconsistencies between satellite-only and surface gravity-only geopotential models, in: *Geodesy on the Move*, R. Forsberg, M. Feissel and R. Dietrich (Eds.), IAG Symposia, *Vol. **119***, Springer-Verlag, Berlin, Germany.

Pavlis, N.K. and R.H. Rapp (1990), The development of an isostatic gravitational model to degree 360 and its use in global gravity modelling, *Geophys. J. Int.*, ***100***, 369-378.

Pavlis, N.K., C.M. Cox, E.C. Pavlis and F.G. Lemoine (1999), Intercomparison and evaluation of some contemporary global geopotential models, *Boll. di Geofisica teorica ed applicata*, *Vol. **40***, No. 3-4, pp. 245-254.

Pavlis, N.K. and J. Saleh (2005), Error Propagation with Geographic Specificity for Very High Degree Geopotential Models, in: *Gravity, Geoid and Space Missions*, C. Jekeli, L. Bastos, J. Fernandes (Eds.), IAG Symposia, *Vol.* ***129***, Springer-Verlag, Berlin, Germany.

Pavlis, N.K., S.A. Holmes, S.C. Kenyon, D. Schmidt, and R. Trimmer (2005), A preliminary gravitational model to degree 2160, in: *Gravity, Geoid and Space Missions*, C. Jekeli, L. Bastos, J. Fernandes (Eds.), IAG Symposia, *Vol.* ***129***, Springer-Verlag, Berlin, Germany.

Pavlis, N.K., S.A. Holmes, S.C. Kenyon, and J.K. Factor (2006a), Towards the next EGM: Progress in Model Development and Evaluation, paper presented to the First International Symposium of the International Gravity Field Service, Istanbul, Turkey, August 28-September 1, 2007.

Pavlis, N.K., S.A. Holmes, O.B. Andersen (2006b), Dynamic Ocean Topography Solutions Based on a New Mean Sea Surface Model and a GRACE-Based Geoid Model, paper presented at the 2006 Fall Meeting of the American Geophysical Union, San Francisco, California, December 11-16, 2006.

Pavlis, N.K., S.A. Holmes, S.C. Kenyon, and J.K. Factor (2007a), Earth Gravitational Model to Degree 2160: Status and Progress, paper presented to the XXIV General Assembly of the International Union of Geodesy and Geophysics (IUGG), Perugia, Italy, July 2-13, 2007.

Pavlis, N.K., J.K. Factor, and S.A. Holmes (2007b), Terrain-Related Gravimetric Quantities Computed for the Next EGM, in: *Gravity Field of the Earth*, A. Kiliçoglu and R. Forsberg (Eds.), Proceedings of the 1st International Symposium of the International Gravity Field Service (IGFS), *Harita Dergisi*, Special Issue No. 18, General Command of Mapping, Ankara, Turkey.

Pavlis, N.K., S.A. Holmes, S.C. Kenyon, and J.K. Factor (2008), An Earth Gravitational Model to Degree 2160: EGM2008, paper presented to the 2008 General Assembly of the European Geosciences Union, Vienna, Austria, April 13-18, 2008.

Rapp, R.H. (1997), Use of potential coefficient models for geoid undulation determinations using a spherical harmonic representation of the height anomaly/geoid undulation difference, *J. Geod.*, ***71***, 282-289.

Rapp, R.H. (1998), Past and future developments in geopotential modeling, in: *Geodesy on the Move – Gravity, Geoid, Geodynamics and Antarctica*, R. Forsberg, M. Feissel, and R. Dietrich (Eds.), IAG Symposia, *Vol. **119***, Springer-Verlag, Berlin, Germany.

Rapp, R.H. and N.K. Pavlis (1990), The development and analysis of geopotential coefficient models to spherical harmonic degree 360, *J. Geophys. Res.*, ***95**(B13)*, 21885-21911.

Rapp, R.H., Y.M. Wang and N.K. Pavlis (1991), The Ohio State 1991 geopotential and sea surface topography harmonic coefficient models, *Rep. 410*, Dep. of Geod. Sci. and Surv., Ohio State Univ., Columbus.

Rapp, R.H. and N. Balasubramania (1992), A conceptual formulation of a world height system, *Rep. 421*, Dep. of Geod. Sci. and Surv., Ohio State Univ., Columbus.

Reigber, C., R. Bock, C. Förste, L. Grunwaldt, N. Jakowski, H. Lühr, P. Schwintzer, and C. Tilgner (1996), CHAMP Phase-B Executive Summary, *GFZ, STR96/13*.

Rodriguez, E., C.S. Morris, J.E. Belz, E.C. Chapin, J.M. Martin, W. Daffer, S. Hensley (2005), An assessment of the SRTM topographic products, *Tech. Rep.* D-31639, Jet Propulsion Laboratory, Pasadena, California.

Rowlands, D.D. (2012), personal communication.

Saleh, J. and N.K. Pavlis (2003), The development and evaluation of the global digital terrain model DTM2002, *Gravity and Geoid 2002*, Proc. of the 3rd Meeting of the IGGC, Tziavos, I.N. (Ed.), Ziti, Thessaloniki, Greece.

Sandwell, D.T. and W.H.F. Smith (1997), Marine gravity anomaly from Geosat and ERS-1 satellite altimetry, *J. Geophys. Res.*, ***102****(B5)*, 10039-10054.

Sandwell, D.T. and W.H.F. Smith (2009), Global marine gravity from retracked Geosat and ERS-1 altimetry: Ridge segmentation versus spreading rate, *J. Geophys. Res.*, ***114***, *B01411*, doi: 10.1029/2008JB006008.

Schwarz, K.-P., M.G. Sideris, and R. Forsberg (1987), Orthometric heights without leveling, *J. Surv. Eng.*, ***113****(1)*, 28-40.

Smith, D.A., D.R. Roman (2001), GEOID99 and G99SSS: 1-arc-minute geoid models for the United States, *J. Geod.*, ***75***, 469-490.

Smith, W.H.F. and D.T. Sandwell (1997), Global sea floor topography from satellite altimetry and ship depth soundings, *Science*, ***277***, pp. 1956-1962.

Swenson, S., and J. Wahr (2006), Post-processing removal of correlated errors in GRACE data, *Geophys. Res. Lett.*, ***33***, L08402, doi: 10.1029/2005GL025285.

Tapley, B.D., S. Bettadpur, M. Watkins, and C. Reigber (2004), The gravity recovery and climate experiment: Mission overview and early results, *Geophys. Res. Lett.*, ***31***, L09607, doi: 10.1029/2004GL019920.

Tapley, B., J. Ries, S. Bettadpur, D. Chambers, M. Cheng, F. Condi, B. Gunter, Z. Kang, P. Nagel, R. Pastor, T. Pekker, S. Poole and F. Wang (2005), GGM02 – An Improved Earth

Gravity Field Model from GRACE, *J. Geod.*, ***79**(8)*, 467-478, doi: 10.1007/s00190-005-0480-z.

Tscherning, C.C. (1983), The role of high-degree spherical harmonic expansions in solving geodetic problems, in: *Proceedings of the International Association of Geodesy Symposia, IUGG XVII General Assembly*, *Vol.* ***1***, 431-441, Dep. of Geod. Sci. and Surv., Ohio State Univ., Columbus.

Uotila, U.A. (1986), Notes on adjustment computations, *Part I*, Dep. of Geod. Sci. and Surv., Ohio State Univ., Columbus.

Wang, Y.M. and D. Roman (2004), Effect of High Resolution Altimetric Gravity Anomalies on the North America Geoid Computations, paper presented at the 2004 Spring Meeting of the American Geophysical Union, Montreal, Canada, May 17-21, 2004.

Watkins, M.M. (2007), personal communication.

Wenzel, G. (1998), Ultra high degree geopotential models GPM98A, B and C to degree 1800, in: Proceedings of Joint Meeting of the International Gravity Commission and the International Geoid Commission, 2nd Continental Workshop on the Geoid in Europe, M. Vermeer and J. Adám J (Eds.), *Rep. Finnish Geod. Inst.*, ***98**:4*, pp. 71-80, Helsinki, Finland.

Werner, M. (2001), Shuttle Radar Topography Mission (SRTM), Mission overview. *J. Telecom.*, ***55***, 75-79.

Wessel, P. and W.H.F. Smith (1998), New, improved version of the Generic Mapping Tools released, *Eos Trans.*, ***79***, 579, American Geophysical Union.

Wunsch, C. (2006), personal communication.

Wunsch, C. and P. Heimbach (2007), Practical global oceanic state estimation, *Physica D*, ***230***, 197-208.

Yuan, D.-N. (2007), personal communication.

Zhongolovich, I.D. (1952), The external gravitational field of the Earth and the fundamental constants related to it, *Acad. Sci. Publ. Inst. Teor. Astron.*, Leningrad.

Figure Captions

Figure 1. The form of the combined normal equation system for a hypothetical combination of satellite-only normal equations to degree 4 (black elements), with "terrestrial" normal equations to degree 6 (gray elements). See text for details.

Figure 2. Gravity anomaly degree variances computed from the ellipsoidal harmonic coefficients representing two versions of the global 5 arc-minute file: v050707a did not limit the bandwidth of the predicted gravity anomalies, while file v021408a did. Unit is $mGal^2$.

Figure 3. Geographic display of some of the characteristics of the 5 arc-minute area-mean gravity anomalies in the merged file used to develop the EGM2008 model: (a) Data availability, (b) Data source identification. See text for details.

Figure 4. 5 arc-minute residual gravity anomalies over southern Alaska and western USA and Canada from the least squares adjustments of two combination solutions: (a) PGM2006A without downward continued gravity anomalies, (b) PGM2006B with downward continued gravity anomalies. Unit is mGal.

Figure 5. Dynamic Ocean Topography (DOT) estimates averaged over 6 arc-minute equiangular cells, obtained by subtracting model-implied height anomalies, computed to spherical harmonic degree 2190, from the DNSC07C Mean Sea Surface (MSS) model. (a) Using the PGM2006C model, (b) Using the PGM2007A model. Unit is cm.

Figure 6. (a) 5 arc-minute gravity anomaly residuals from the least squares adjustment that yielded the EGM2008 combination solution. (b) 5 arc-minute gravity anomaly differences ITG-GRACE03S minus EGM2008. Maximum degree and order is 180. Unit is mGal.

Figure 7. Gravity anomaly degree variances computed from the ellipsoidal harmonic coefficients of the signal and error spectra associated with the EGM2008 solution: (a) to degree 2159, (b) to degree 180. Unit is $mGal^2$.

Figure 8. RMS fits to GRACE K-band range-rate data computed from 26 daily arcs spanning the month of November 2005, using three gravitational models to degree and order 200. Unit is µm/s.

Figure 9. Dynamic Ocean Topography (DOT) estimates averaged over 6 arc-minute equiangular cells, obtained by subtracting model-implied height anomalies from the DNSC08B Mean Sea Surface (MSS) model. (a) Using GGM02C to degree 200, augmented with EGM96 from degree 201 to 360, (b) Using EIGEN-GL04C to degree 360, (c) Using EGM2008 to degree 2190. Unit is cm.

Figure 10. Differences between the ECCO DOT model and the DOT estimates implied by subtracting from the DNSC08B MSS three different geoid models, over the 1°×1° ocean cells between latitudes 65°N and 65°S. (a) Using GGM02C to degree 200, augmented with EGM96 from degree 201 to 360, (b) Using EIGEN-GL04C to degree 360, (c) Using EGM2008 to degree 2190. Unit is cm.

Figure 11. Free-air gravity anomalies over the Yucatán Peninsula from: (a) EIGEN-GL04C to degree and order 360, (b) EGM2008 to degree 2190. Unit is mGal.

Figure 12. Commission error implied by EGM2008 from degree 2, to degree and order 2159 on: (a) height anomalies (cm), (b) the meridional component (ξ) of the deflection of the vertical (arc-second).

Table 1. Statistics from the inter-comparison of three sets of 5 arc-minute area-mean gravity anomalies, over oceanic areas between latitudes 80°N and 80°S. Above the diagonal: area-weighted mean and standard deviation difference (mGal); below the diagonal: extreme differences (mGal). 5646416 values compared, covering 70.025 percent of the Earth's surface area.

	PGM2007B		DNSC07		SS v18.1	
PGM2007B	-	-	0.000	1.351	-0.047	2.201
DNSC07	-45	42	-	-	-0.048	1.939
SS v18.1	-88	114	-74	114	-	-

Table 2. Statistics of the 5 arc-minute area-mean gravity anomalies after editing and downward continuation of the merged file used to develop the EGM2008 model. Unit is mGal. The latitudes and longitudes listed identify the location of the extreme values in the merged file.

Data Source	% Area	Minimum	Maximum	RMS	RMS σ
ArcGP	3.0	-192.0	281.8	30.2	3.0
Altimetry	63.2	-361.8	351.1	28.4	3.0
Terrestrial	17.6	-351.9	868.4	41.2	2.8
Fill-in	16.2	-333.0	593.5	46.8	7.6
Non Fill-in	83.8	-361.8	868.4	31.6	2.9
All	100.0	-361.8	868.4	34.5	4.1
(φ, λ)		19.4°, 293.5°	10.8°, 286.3°		

Table 3. Standard deviation of the differences between the ECCO DOT model and the DOT models implied by the DNSC06E MSS minus each geoid model over the 1°×1° ocean cells between latitudes 65°N and 65°S. Unit is cm.

Model	DOT Difference Std. Deviation
GGM02C_EGM96	9.8
EIGEN-GL04C	10.8
PGM2006C	8.8
GGM02C_EGM96 (†)	7.0

(†) After application of the iterative filtering approach of *Chambers and Zlotnicki* [2004].

Table 4. Average laser ranging residual RMS from one year (2003) of 3-day orbit fits without and with one cycle-per-revolution (1-cpr) empirical accelerations being adjusted. Unit is cm.

	EGM96		GGM02C		EGM2008	
Satellite	No 1-cpr	1-cpr	No 1-cpr	1-cpr	No 1-cpr	1-cpr
LAGEOS-1	1.5	1.05	1.5	0.96	1.5	0.97
LAGEOS-2	1.3	0.96	1.3	0.84	1.4	0.85
Ajisai	5.9	5.6	5.2	4.8	5.3	4.4
Starlette	5.1	3.7	3.5	1.8	4.8	1.8
Stella	9.0	6.5	3.1	2.2	3.0	1.6
BE-C	11.1	9.1	9.1	7.6	9.4	7.6

Table 5. GPS/Leveling comparisons over CONUS.

Model (Nmax)	Bias Removed		Linear Trend Removed	
	Number Passed Edit	Weighted Std. Deviation (cm)	Number Passed Edit	Weighted Std. Deviation (cm)
EGM96 (360)	4096	21.4	4092	18.2
GGM02C_EGM96 (360)	4169	18.9	4165	17.6
EIGEN-GL04C (360)	4167	19.5	4163	18.1
EGM2008 (360)	4185	17.6	4181	16.4
EGM2008 (2190)	4201	7.1	4197	4.8
USGG2003 ($1' \rightarrow 10800$)	4201	9.1	4197	5.8

Table 6. GPS/Leveling comparisons globally.

Model (Nmax)	Bias Removed		Linear Trend Removed	
	Number Passed Edit	Weighted Std. Deviation (cm)	Number Passed Edit	Weighted Std. Deviation (cm)
EGM96 (360)	12220	30.3	12173	27.0
GGM02C_EGM96 (360)	12305	25.6	12258	23.2
EIGEN-GL04C (360)	12299	26.2	12252	23.5
EGM2008 (360)	12329	23.0	12283	20.9
EGM2008 (2190)	12352	13.0	12305	10.3

Table 7. GPS/Leveling comparisons over mainland Australia.

Model (Nmax)	Bias Removed		Linear Trend Removed	
	Number Passed Edit	Weighted Std. Deviation (cm)	Number Passed Edit	Weighted Std. Deviation (cm)
EGM96 (360)	533	37.7	533	35.8
GGM02C_EGM96 (360)	534	32.2	534	29.6
EIGEN-GL04C (360)	534	32.7	534	30.1
EGM2008 (360)	534	29.2	534	26.0
EGM2008 (2190)	534	26.6	534	23.0
AUSGeoid98 (2′→5400)	534	31.0	534	26.1

Table 8. GPS/Leveling comparisons over Australia's South West Seismic Zone.

Model (Nmax)	Bias Removed		Linear Trend Removed	
	Number Passed Edit	Weighted Std. Deviation (cm)	Number Passed Edit	Weighted Std. Deviation (cm)
EGM96 (360)	48	27.8	48	24.3
GGM02C_EGM96 (360)	48	25.2	48	23.6
EIGEN-GL04C (360)	48	25.7	48	24.9
EGM2008 (360)	48	23.4	48	20.1
EGM2008 (2190)	48	10.6	48	4.6
AUSGeoid98 ($2' \rightarrow 5400$)	48	12.7	48	4.8

Table 9. RMS differences between astrogeodetic and gravimetric deflections of the vertical over CONUS and Australia. Unit is arc-second.

Model (Nmax)	CONUS 3561 Stations		Australia 1080 Stations	
	$\Delta\xi''$	$\Delta\eta''$	$\Delta\xi''$	$\Delta\eta''$
EGM96 (360)	2.80	3.22	1.91	2.23
GGM02C_EGM96 (360)	2.80	3.22	1.89	2.22
EIGEN-GL04C (360)	2.81	3.20	1.92	2.23
EGM2008 (2190)	1.12	1.16	1.19	1.29
DEFLEC99 ($1'\rightarrow 10800$)	0.91	0.92	-	-
AUSGeoid98 ($2'\rightarrow 5400$)	-	-	1.31	1.37

Table 10. Comparisons with TOPEX altimeter data from a 6-year mean track containing 517835 1 Hz SSH and 494350 along-track SSH slopes.

Model (Nmax)	Residual SSH (cm)		Residual Along-Track Slope (arc-second)	
	Max | • |	Std. Dev.	Max | • |	Std. Dev.
EGM96 (360)	334	20.0	30.0	1.96
GGM02C_EGM96 (360)	300	18.2	29.3	1.96
EIGEN-GL04C (360)	288	19.2	29.7	1.98
EGM2008 (360)	307	16.0	28.5	1.90
EGM2008 (2190)	121	5.2	7.6	0.30

Table 11. Weighted mean standard deviation of the differences between the SSH obtained from airborne LiDAR data and from model-derived values, over 13 flight lines in the Aegean Sea. Weights are proportional to the number of points per flight line. Unit is cm.

Model (Nmax)	Bias Removed	Linear Trend Removed
EGM96 (360)+DOT96 (20)	34.0	32.1
EGM2008 (2190)+DOT2008A (180)	7.2	6.0
DNSC08B (1′→10800)	7.4	6.2

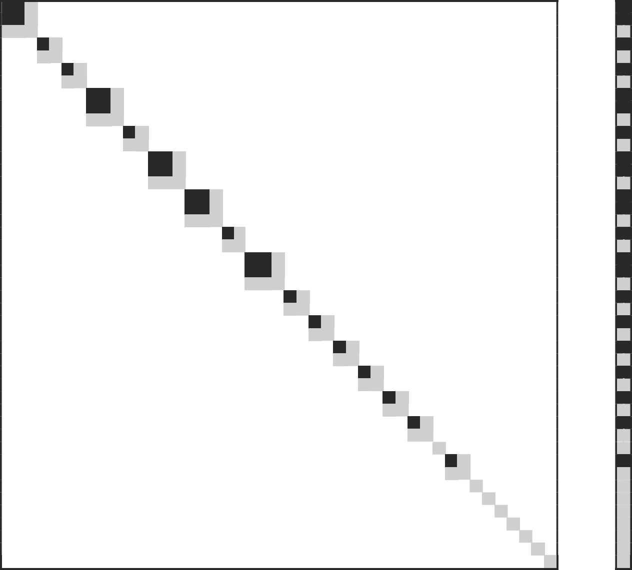
Table 12. RMS value of the commission error of EGM2008, from harmonic degree 2 to harmonic degree and order 2159, for geoid undulations (N) and deflections of the vertical components (ξ, η), computed over five regions of the Earth. Parenthetical values represent estimates based on comparisons with independent data.

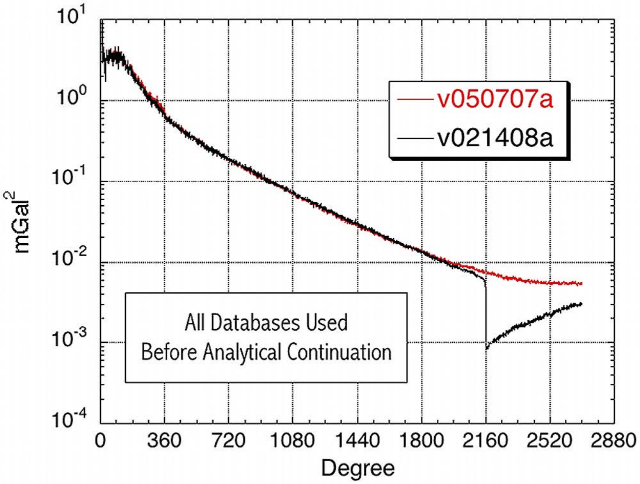
Region	RMS σ_N (cm)	RMS σ_ξ (arc-second)	RMS σ_η (arc-second)
Ocean areas with $\lvert\varphi\rvert < 66°$	5.8 (5.2 [1])	0.38 (~0.30 [1])	0.39 (~0.30 [1])
CONUS	5.9 (4.8 [2])	0.47 (1.12 [3])	0.47 (1.16 [3])
Land	18.3 (~10.3 [4])	1.69	1.69
Ocean	6.1	0.42	0.42
Globally	11.1	0.98	0.98

[1] See Table 10; [2] see Table 5; [3] see Table 9; [4] see Table 6.

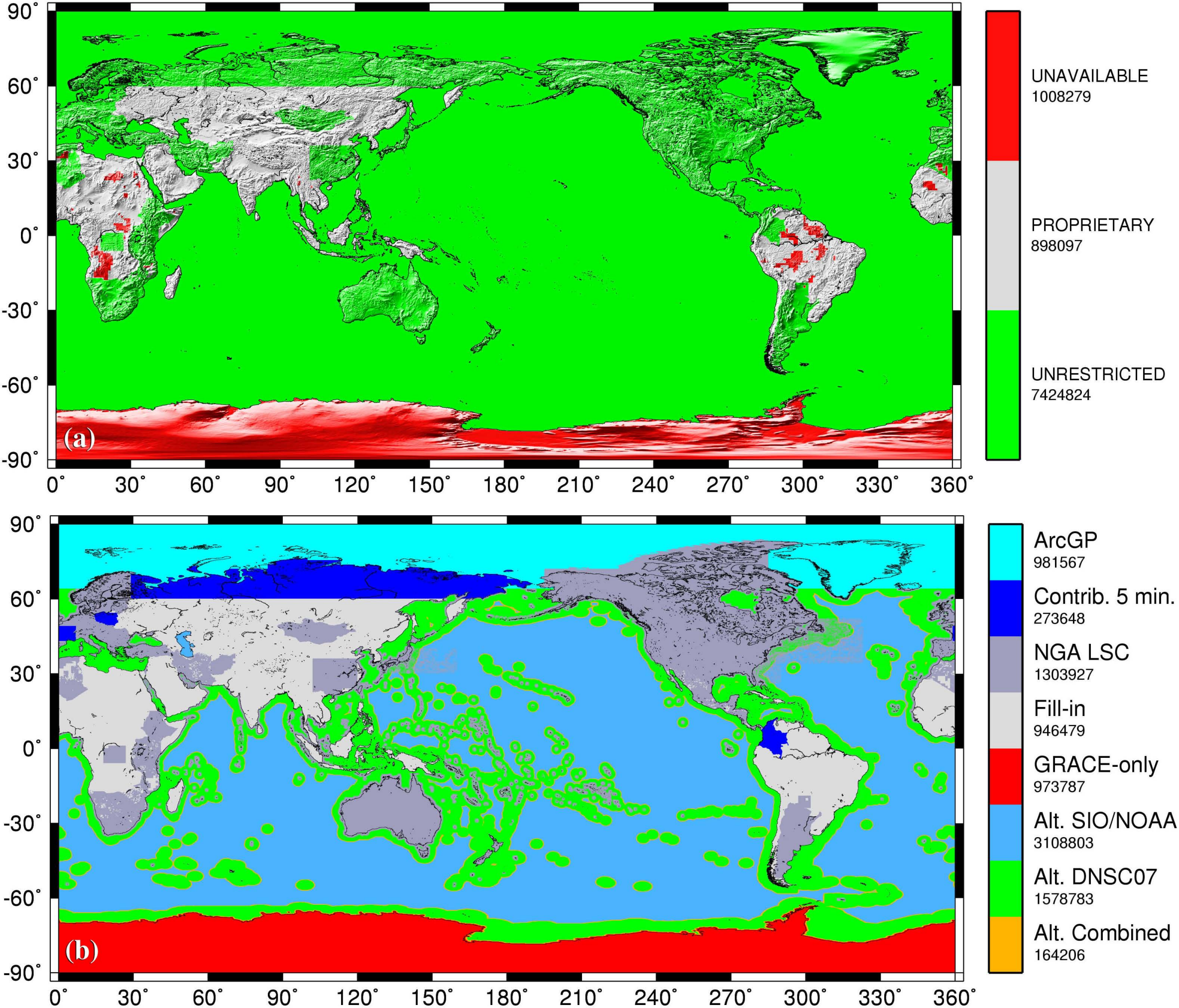
Table 13. Estimated values and their standard deviations of the EGM2008 zonal spherical harmonic coefficients of the gravitational potential to degree 10. The $\bar{C}^{s}_{2,0}$ value is in the tide-free system.

Degree (n)	$\bar{C}^{s}_{n,0}$	$\sigma(\bar{C}^{s}_{n,0})$
2	-0.484165143790815D-03	0.7481239490D-11
3	0.957161207093473D-06	0.5731430751D-11
4	0.539965866638991D-06	0.4431111968D-11
5	0.686702913736681D-07	0.2910198425D-11
6	-0.149953927978527D-06	0.2035490195D-11
7	0.905120844521618D-07	0.1542363963D-11
8	0.494756003005199D-07	0.1237051133D-11
9	0.280180753216300D-07	0.1023487582D-11
10	0.533304381729473D-07	0.8818400481D-12

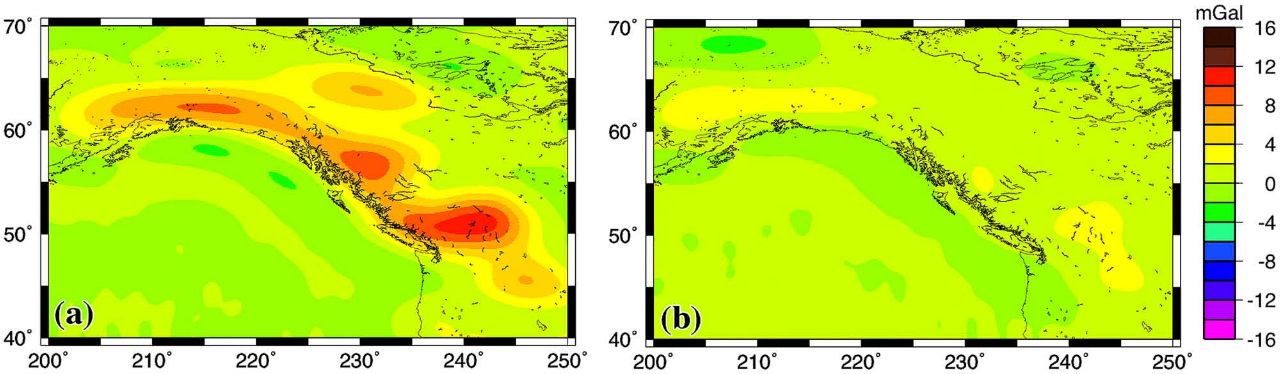




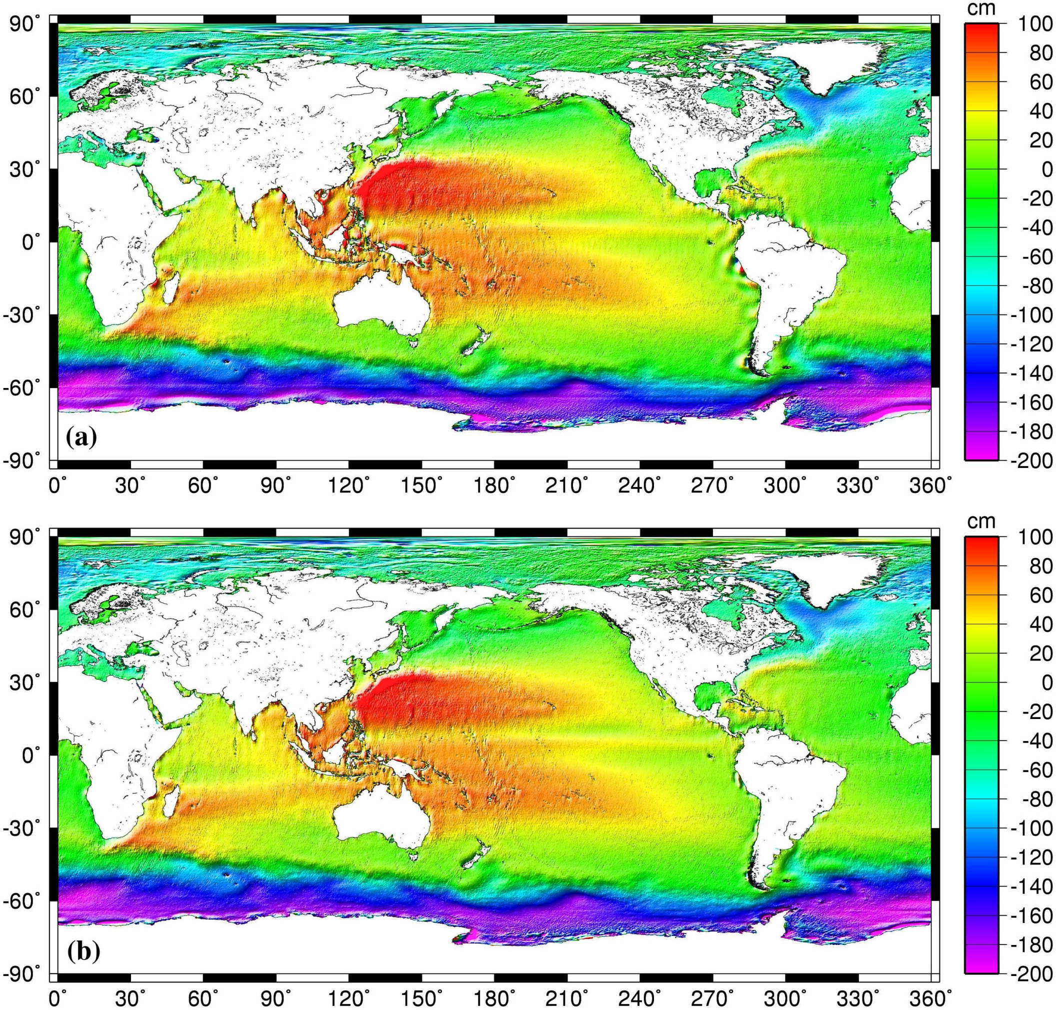
v050707a
v021408a
All Databases Used
Before Analytical Continuation
mGal^2
Degree
10^1
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
0
360
720
1080
1440
1800
2160
2520
2880



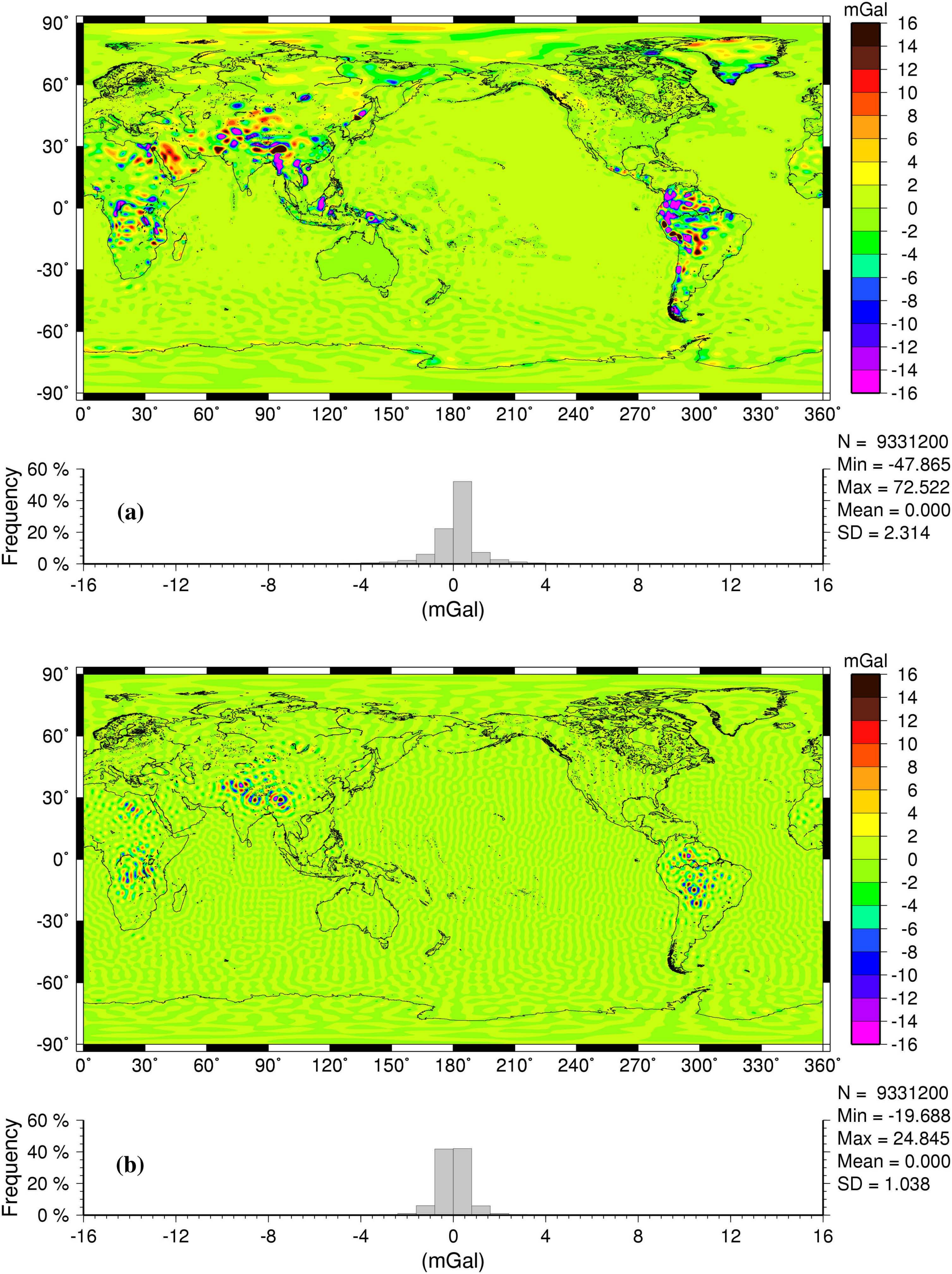
90°
60°
30°
0°
-30°
-60°
-90°
0°
30°
60°
90°
120°
150°
180°
210°
240°
270°
300°
330°
360°
(a)
UNAVAILABLE
1008279
PROPRIETARY
898097
UNRESTRICTED
7424824
(b)
ArcGP
981567
Contrib. 5 min.
273648
NGA LSC
1303927
Fill-in
946479
GRACE-only
973787
Alt. SIO/NOAA
3108803
Alt. DNSC07
1578783
Alt. Combined
164206

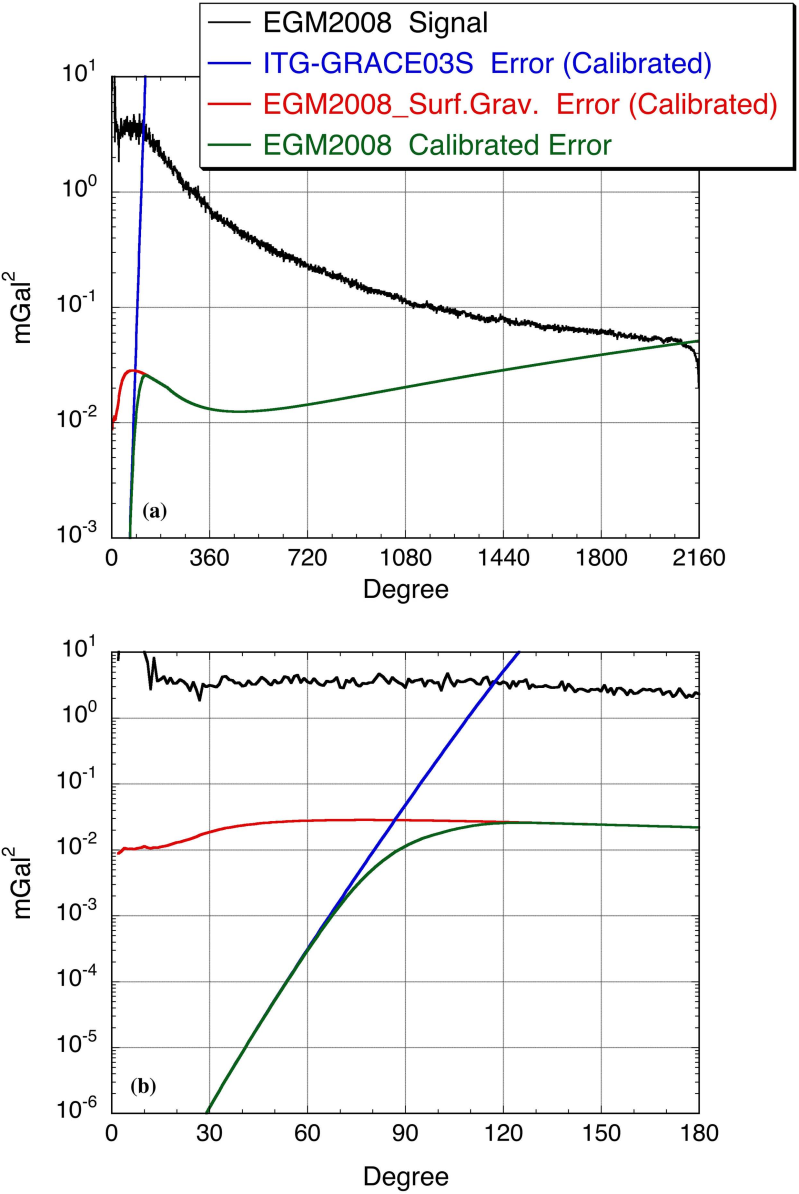


(a)
(b)
mGal
16
12
8
4
0
-4
-8
-12
-16
70°
60°
50°
40°
200°
210°
220°
230°
240°
250°

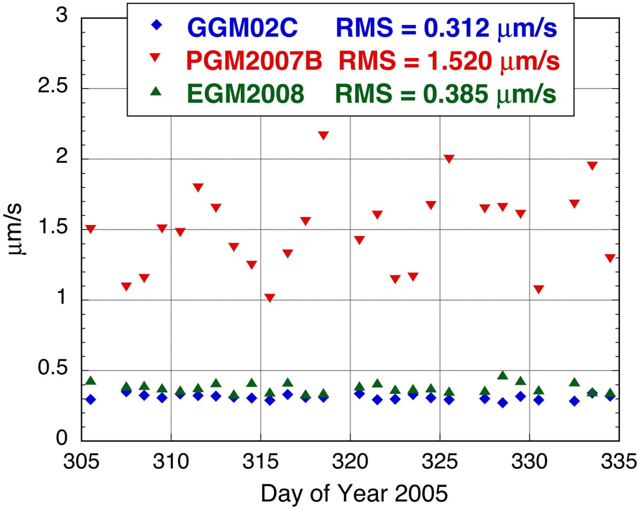


(a)
(b)
cm
100
80
60
40
20
0
-20
-40
-60
-80
-100
-120
-140
-160
-180
-200
90°
60°
30°
0°
-30°
-60°
-90°
0°
30°
60°
90°
120°
150°
180°
210°
240°
270°
300°
330°
360°

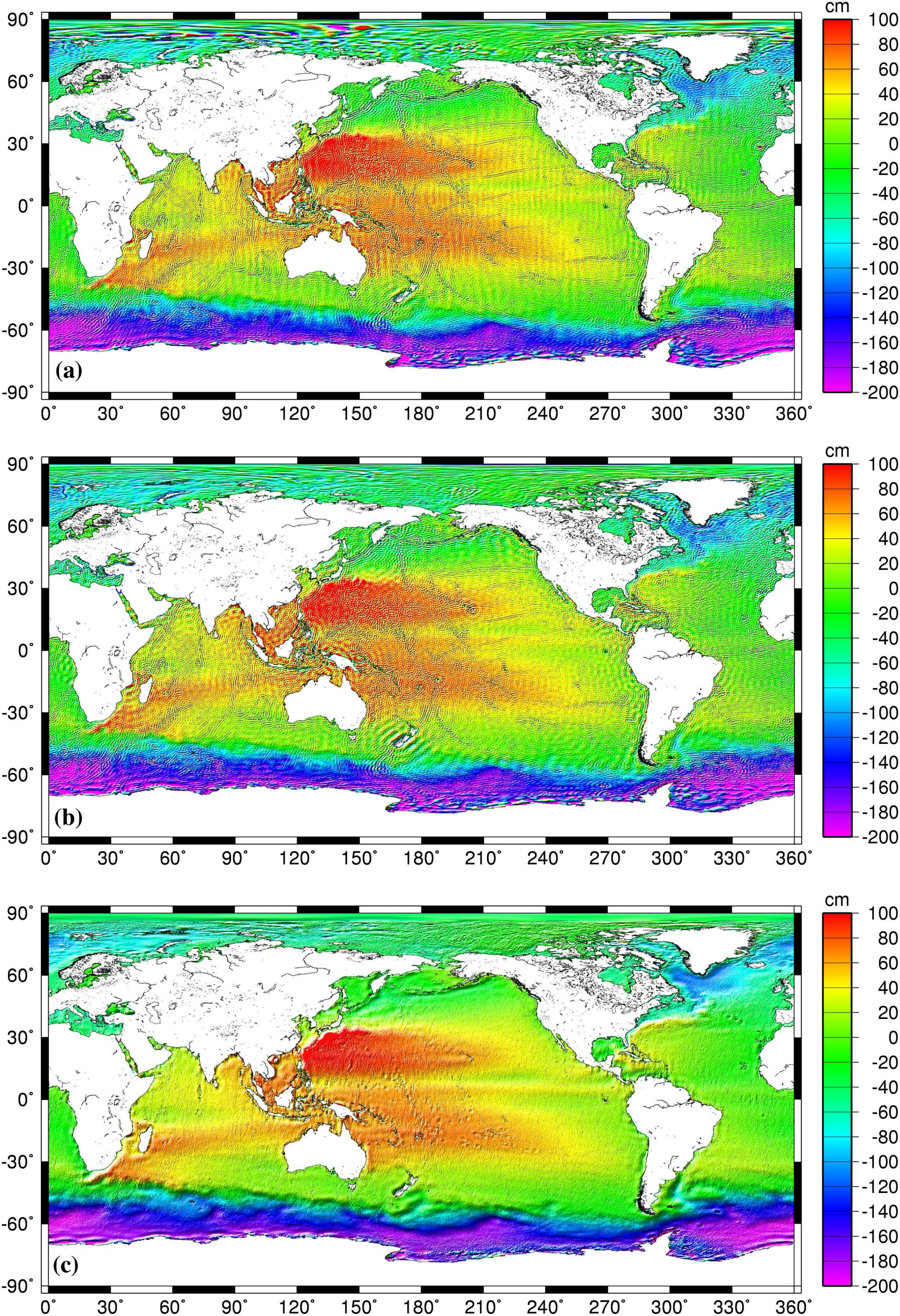

mGal
N = 9331200
Min = -47.865
Max = 72.522
Mean = 0.000
SD = 2.314
Frequency
(a)
(mGal)
mGal
N = 9331200
Min = -19.688
Max = 24.845
Mean = 0.000
SD = 1.038
Frequency
(b)
(mGal)



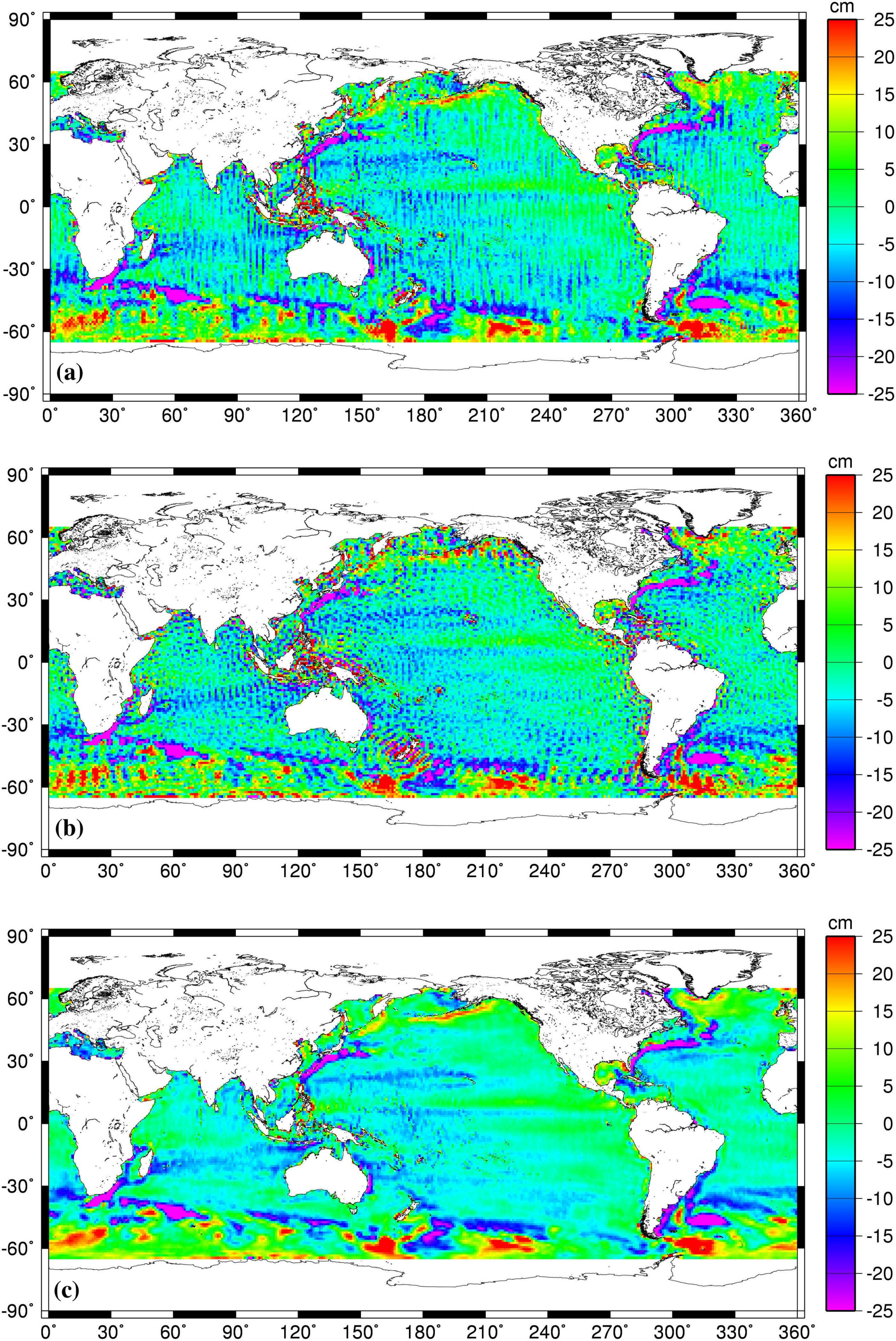
EGM2008 Signal
ITG-GRACE03S Error (Calibrated)
EGM2008_Surf.Grav. Error (Calibrated)
EGM2008 Calibrated Error
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
mGal^2
(a)
0
360
720
1080
1440
1800
2160
Degree
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
mGal^2
(b)
0
30
60
90
120
150
180
Degree



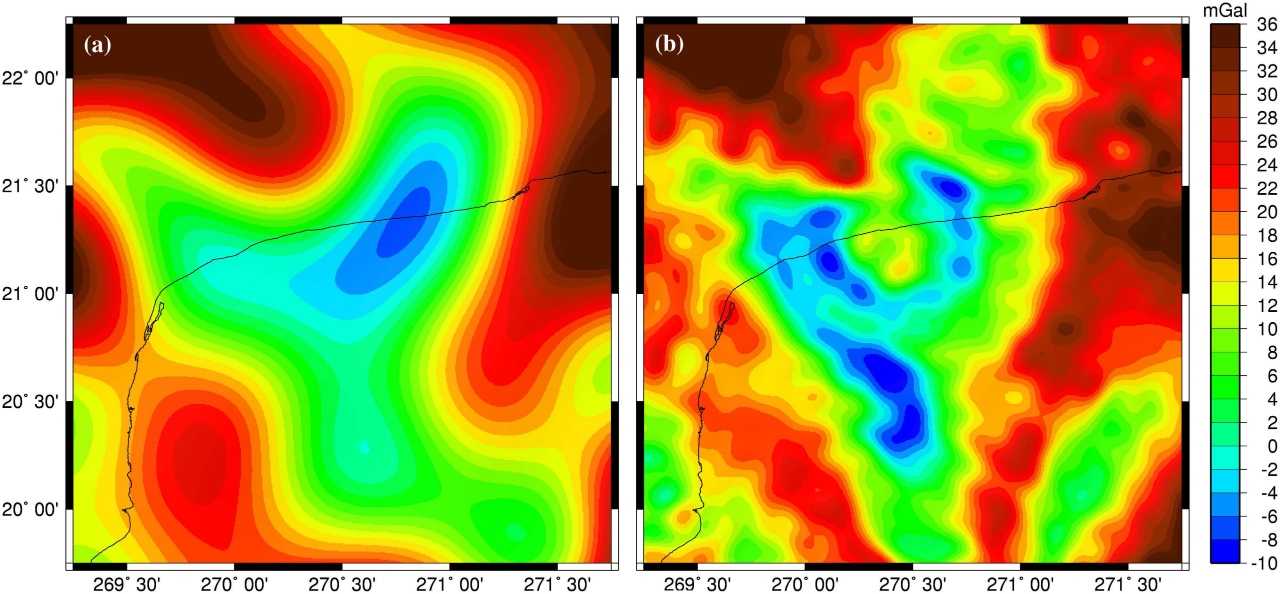
GGM02C RMS = 0.312 μm/s
PGM2007B RMS = 1.520 μm/s
EGM2008 RMS = 0.385 μm/s
μm/s
0
0.5
1
1.5
2
2.5
3
305
310
315
320
325
330
335
Day of Year 2005

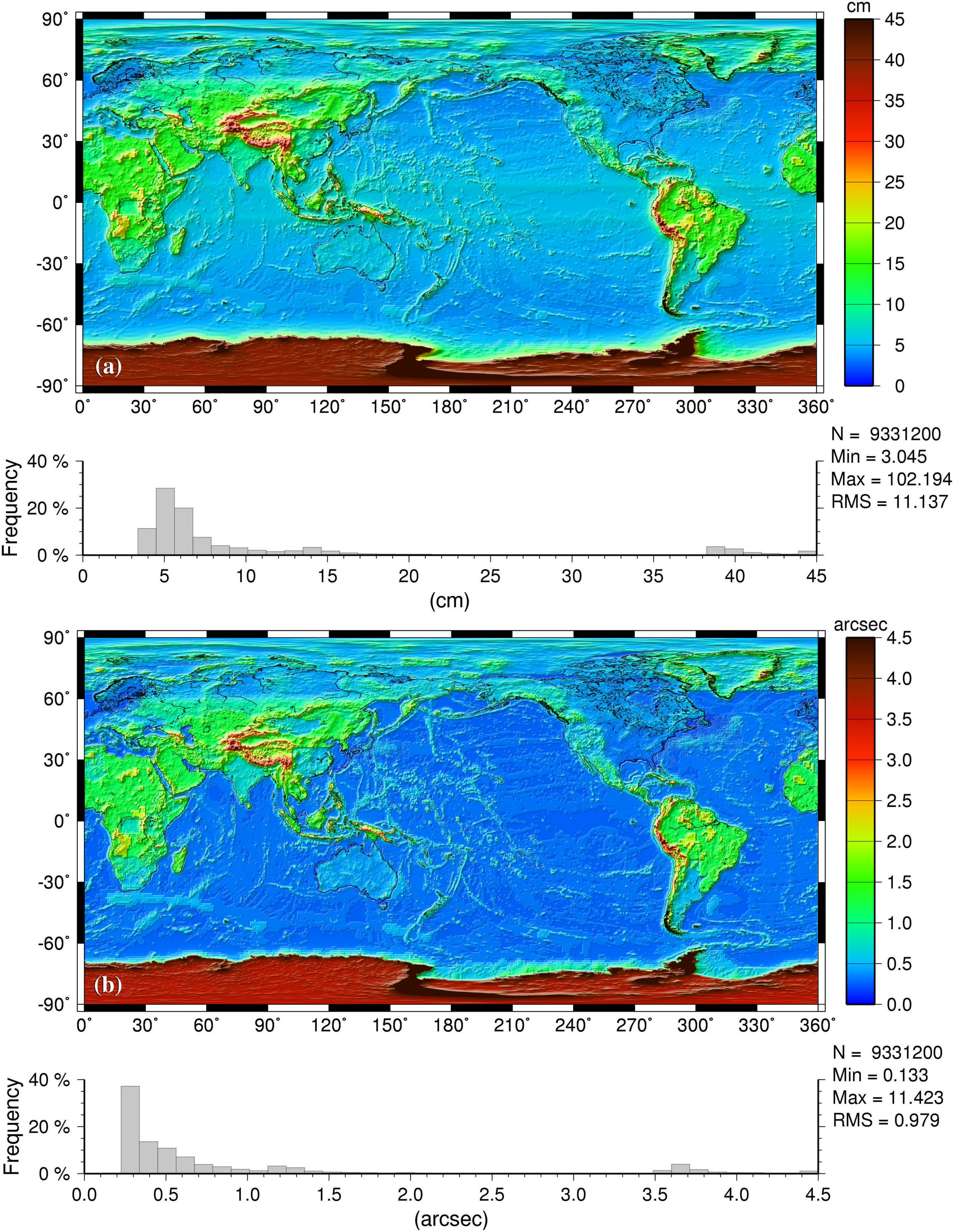


cm
100
80
60
40
20
0
-20
-40
-60
-80
-100
-120
-140
-160
-180
-200
90°
60°
30°
0°
-30°
-60°
-90°
0°
30°
60°
90°
120°
150°
180°
210°
240°
270°
300°
330°
360°
(a)
(b)
(c)



(a)
(b)
(c)
cm
25
20
15
10
5
0
-5
-10
-15
-20
-25
90˚
60˚
30˚
0˚
-30˚
-60˚
-90˚
0˚
30˚
60˚
90˚
120˚
150˚
180˚
210˚
240˚
270˚
300˚
330˚
360˚


(a)
(b)
mGal
36
34
32
30
28
26
24
22
20
18
16
14
12
10
8
6
4
2
0
-2
-4
-6
-8
-10
22° 00'
21° 30'
21° 00'
20° 30'
20° 00'
269° 30'
270° 00'
270° 30'
271° 00'
271° 30'



cm
(a)
N = 9331200
Min = 3.045
Max = 102.194
RMS = 11.137
Frequency
(cm)
arcsec
(b)
N = 9331200
Min = 0.133
Max = 11.423
RMS = 0.979
Frequency
(arcsec)