Meridional Circulation and the Maintenance of the Venus Atmospheric Rotation

PETER J. GIERASCH

Center for Radiophysics and Space Research, Cornell University, Ithaca, N. Y. 14850

(Manuscript received 24 July 1974, in revised form 16 January 1975)

ABSTRACT

A meridional cell, with rising motion near the equator and sinking near the poles, transports angular momentum upward in an atmosphere whenever equatorial regions of the atmosphere have an angular momentum surplus relative to polar regions. This process may contribute to the maintenance of the Venus atmospheric super-rotation.

Super-rotation by this process is exhibited in a simple analytical model. The super-rotation ratio in the model is derived to be $\exp(HD^2/\nu_v t_m)$, where H is depth in scale heights, D the mean scale height, ν_v the vertical eddy diffusivity, and t_m the meridional overturning time.

For the mechanism to work, some eddy process must maintain an angular momentum surplus in equatorial regions. Vorticity mixing is suggested. It is also demonstrated that if the Richardson number is large in a cyclostrophic atmosphere, the mean thermal structure is given by global radiative equilibrium, and local deviations from equilibrium are balanced by adiabatic cooling or warming associated with vertical motions.

1. Introduction

The rapid rotation of the Venus atmosphere between altitudes of about 20 to 70 km now seems to be a well-established fact. Although the ultraviolet images received from Mariner 10 (Murray *et al.*, 1974) demonstrate that a great deal of complex dynamical activity takes place, the rapid mean flow is the dominant component of the circulation. It is apparent in the Mariner 10 pictures, it is consistent with spectroscopic CO_2 absorption line Doppler shifts (Traub and Carleton, 1975), and it has been measured by analysis of horizontal drifts of descending Venera probes (Marov *et al.*, 1973).

The general character of the flow as indicated by Venera data seems to be an irregular but monotonic increase of wind speed with height. The Mariner 10 data (Murray *et al.*, 1974; Suomi, 1975) show an increase of angular velocity from equator toward mid-latitudes, up to about 40° or 50°. Poleward of this the angular velocity is approximately constant. The rate of increase from equator toward mid-latitudes is not quite as rapid as a profile with uniform angular momentum would display (Suomi, 1975). The wind speed near the cloud tops at the equator is about 100 m s^{-1}. The direction is in the same sense as the rotation of the solid planet, and about 60 times faster.

The puzzle presented by these observations is one of the most exciting facing the atmospheric sciences today. More detailed data will be needed to resolve the problem, but a number of theoretical suggestions have been made which serve to provide a framework for planning future observations and numerical experiments. The central problem is to explain how angular momentum is fed into the upper levels of the atmosphere to maintain the super-rotation. Gold and Soter (1971) have suggested an external tidal torque. Other theories have dealt with internal fluid dynamical momentum redistribution. The present paper extends the latter discussions.

Young and Schubert (1973) have reviewed the fluid dynamical theories. Briefly, there are three different suggestions for the upward flux of angular momentum. Thompson (1970) proposes Reynolds stresses due to tilted thermal convection cells. The cells are tilted in the direction of the mean flow, by the action of the mean flow itself. Schubert and Whitehead (1969) propose that tilted cells are produced by the response of the atmosphere to the motion of the subsolar point. In both theories the Reynolds stresses are due to two large convection cells whose cross section lies in the equatorial plane (although Thompson's mechanism could conceivably operate with small-scale convection near the subsolar point). Both theories are two-dimensional, and the mechanisms could operate in a cylindrical geometry.

Leovy (1973) qualitatively discusses the realistic three-dimensional case. He points out that the mean flow must be in approximate cyclostrophic balance (see below), satisfying a thermal wind equation with a latitudinal temperature gradient. He then suggests that small deviations from balance will exist, and the resulting adjustments will excite Kelvin waves (in equatorial regions), which can transport angular momentum upward.

In these three theories, vertical transport of angular momentum is accomplished by convection cells or waves with longitudinal variations. The purpose of the present paper is to point out that a meridional cell (rising near the equator and sinking near the poles) can also accomplish an upward angular momentum flux. All that is necessary is that the angular momentum per unit mass decrease toward the poles along a surface of constant height. The global balance would be an upward angular momentum transport by the meridional cell balancing a downward flux by whatever diffusive or eddy processes exist. Fig. 1 shows a sketch to illustrate the flow.

The difficulty would then be to explain how equatorial regions of the atmosphere are maintained with a surplus of angular momentum relative to polar regions. It is clear from Fig. 1 that the meridional cell transports angular momentum poleward, since the angular momentum per unit mass increases with height in the atmosphere. What processes might act to provide a compensating equatorward transport?

The purpose of this paper is merely to point out that the global balances may be of the three-dimensional character described above, rather than to discuss in detail the nature of the eddy processes that would occur in such a flow. We will eventually need detailed observations or realistic numerical calculations to definitively answer such questions. However, it is worthwhile to mention one or two possibilities for the compensating equatorward transport mechanisms, to help make the overall picture plausible. One of these is the mixing of vorticity by non-axisymmetric eddies. It is well known (see, e.g. Rossby, 1947) that axisymmetric (meridional) overturning tends to establish a zonal wind variation with latitude in which angular momentum per unit mass is constant, but that non-axisymmetric eddies tend to mix vorticity, and create a zonal wind distribution in which the vertical component of absolute vorticity is constant. Such a distribution will show an angular momentum increase away from the poles; in fact, near the poles it approximates constant angular velocity. To be precisely correct, it is potential vorticity that is conserved in the adiabatic, inviscid flow of a stratified atmosphere [see, e.g., Holton (1972) or Phillips (1963), for the generalization to the nongeostrophic case]. The consequences are approximately the same as vorticity conservation if the atmosphere is strongly stratified, and we shall see below that this is the case, at least for high levels, on Venus.

The zonal velocity profile on Venus, as described by Suomi (1975), suggests latitude variation corresponding to axisymmetric overturning between equatorial and mid-latitudes, and corresponding to constant vorticity at high latitudes. Whether or not this is due to vorticity mixing at high latitudes, the profile is consistent with the general picture we have presented, since high latitudes are deficient in angular momentum relative to low latitudes.

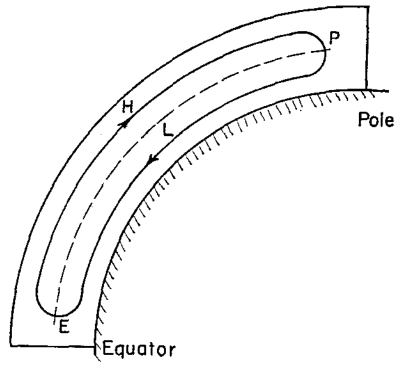

Fig. 1. Schematic of meridional cell. If the angular momentum per unit mass is greater at point E than at P, the meridional cell produces an upward flux of angular momentum. If it is greater at point H that at L, there is a poleward transport.

Another mechanism that may be important is equatorward momentum flux by Reynold's stresses due to tides induced by solar heating. Ingersoll and Orton (1974) show that the solar-related component of infrared emission maps displays a phase shift with latitude, suggesting such stresses.

In the next two sections of the paper a quantitative axisymmetric flow model is presented. The purpose is twofold. First, it is demonstrated that if the Richardson's number is sufficiently large in the Venus atmosphere, then the mean thermal structure is given by planetary average radiative equilibrium, and the meridional overturning rate is simply given by a balance between adiabatic cooling (or heating) and radiative heating (or cooling). This permits us to make an estimate of the overturning rate in the model. The second objective is to demonstrate the angular momentum balances in a simple model. The simplest possible parameterization of eddy processes is utilized. It is assumed that an anisotropic viscosity exists, and that in both horizontal and vertical directions the eddy viscosity acts analogously to a molecular viscosity.

It should be stressed that the purpose is only to illustrate that a strong atmospheric super-rotation can be driven in the general way outlined above. It is not necessary that the eddy process act precisely as parameterized in the model. The model merely provides a simple example, and helps make the idea more concrete by displaying numerical values for parameters and flow variables in an example.

2. Model equations and parameters

We are concerned with pressure levels between 100 atm and about 0.1 atm, or about 7 scale heights of depth. The aspect ratio is still small, however, and the hydrostatic approximation should be valid. The most convenient vertical coordinate is $\hat{z}=\ln(p_0/p)$, where p

is pressure, p_0 basal pressure, and $\hat{z}$ measures pressure scale heights. For the sake of familiarity, we take a coordinate system oriented on the planet so that rotation is positive. The equations for steady axisymmetric motion are

$$\frac{1}{a\cos\lambda}\frac{\partial}{\partial\lambda}[va\cos^2\lambda(u+\Omega a\cos\lambda)] + e^{\hat{z}}\frac{\partial}{\partial\hat{z}}[e^{-\hat{z}}\hat{w}a\cos\lambda(u+\Omega a\cos\lambda)] = a\cos\lambda\, e^{\hat{z}}\frac{\partial}{\partial\hat{z}}\left(e^{-\hat{z}}\frac{\nu_v}{D^2}\frac{\partial u}{\partial\hat{z}}\right)+\frac{a}{\cos\lambda}\frac{\partial}{\partial\lambda}\left(\frac{\nu_H}{a^2}\cos^3\lambda\frac{\partial}{\partial\lambda}\frac{u}{\cos\lambda}\right), \tag{1}$$

$$v\frac{\partial v}{\partial\lambda}+\hat{w}\frac{\partial v}{\partial\hat{z}}+u\sin\lambda\left(\frac{u}{a\cos\lambda}+2\Omega\right)+\frac{1}{a}\frac{\partial\Phi}{\partial\lambda} = e^{\hat{z}}\frac{\partial}{\partial\hat{z}}\left(e^{-\hat{z}}\frac{\nu_v}{D^2}\frac{\partial v}{\partial\hat{z}}\right)+\frac{2}{\cos\lambda}\frac{\partial}{\partial\lambda}\left(\frac{\nu_H}{a^2}\cos\lambda\frac{\partial v}{\partial\lambda}\right) - 2\frac{\nu_H}{a^2}v\tan^2\lambda, \tag{2}$$

$$\frac{\partial\Phi}{\partial\hat{z}}=RT, \tag{3}$$

$$c_p\frac{v}{a}\frac{\partial T}{\partial\lambda}+c_p\hat{w}\left(\frac{\partial T}{\partial\hat{z}}+\frac{RT}{c_p}\right)=Q, \tag{4}$$

$$\frac{1}{a\cos\lambda}\frac{\partial}{\partial\lambda}(v\cos\lambda)+e^{\hat{z}}\frac{\partial}{\partial\hat{z}}e^{-\hat{z}}\hat{w}=0. \tag{5}$$

The notation is as follows:

$\hat{w}$	vertical motion $[=D\hat{z}/Dt]$
λ	latitude
a	planetary radius
u, v	zonal, latitudinal velocities
Ω	planetary angular velocity
ν_H, ν_v	horizontal, vertical diffusion coefficients
R	gas constant
Φ	geopotential
T	temperature
c_p	specific heat at constant pressure
Q	radiative and diffusive heating
D	mean scale height ($\sim$11 km).

The frictional terms in the equations have been written approximately, for an atmosphere with small aspect ratio, based on the full expressions for spherical coordinates given by Durney and Roxburgh (1971). The important feature is that in the zonal momentum equation the momentum flux is proportional to the angular velocity gradient, as in molecular diffusion. As a consequence, angular momentum can be transferred against its gradient; for example, horizontal diffusion could act to speed up equatorial regions of the atmosphere if their angular velocity were deficient. We shall assume that ν_H and ν_v are constants. The scale height D, which appears in the vertical diffusion terms, will be approximated by a constant mean value.

We shall now nondimensionalize the equations to display parameter dependences. Let dimensionless (primed) variables be defined by

$$\left.\begin{aligned} &u=Uu', && \frac{\partial T_s}{\partial\hat{z}}+\frac{R}{C_p}T_s=\bar{S}S'(\hat{z})\\ &\hat{w}=W\hat{w}', && T=\frac{U^2}{RH}T'+T_s(\hat{z})\\ &v=aWv', && \hat{z}=Hh\\ &\Phi=U^2\Phi', && Q=Q_0Q' \end{aligned}\right\}, \tag{6}$$

where H is the total depth (scale heights), and h will run from zero to about unity. The mean static stability is given by $\bar{S}$. The equations become (with primes dropped)

$$\frac{1}{\cos\lambda}\frac{\partial}{\partial\lambda}[v\cos^2\lambda(u+\epsilon\cos\lambda)] + \frac{1}{H}e^{Hh}\frac{\partial}{\partial h}[e^{-Hh}\hat{w}\cos\lambda(u+\epsilon\cos\lambda)] = \frac{\nu_v}{W(DH)^2}\cos\lambda e^{Hh}\frac{\partial}{\partial h}\left(e^{-Hh}\frac{\partial u}{\partial h}\right) + \frac{\nu_H}{Wa^2}\frac{1}{\cos\lambda}\frac{\partial}{\partial\lambda}\left(\cos^3\lambda\frac{\partial}{\partial\lambda}\frac{u}{\cos\lambda}\right), \tag{7}$$

$$\frac{a^2W^2}{U^2}\left(v\frac{\partial v}{\partial\lambda}+\frac{\hat{w}}{H}\frac{\partial v}{\partial h}\right)+u\sin\lambda\left(2\epsilon+\frac{u}{\cos\lambda}\right)+\frac{\partial\Phi}{\partial\lambda} = \frac{a^2W^2}{U^2}\frac{\nu_v}{W(DH)^2}e^{Hh}\frac{\partial}{\partial h}\left(e^{-Hh}\frac{\partial v}{\partial h}\right) + \frac{a^2W^2}{U^2}\frac{\nu_H}{Wa^2}\left(\frac{1}{\cos\lambda}\frac{\partial}{\partial\lambda}\cos\lambda\frac{\partial v}{\partial\lambda}-v\tan^2\lambda\right), \tag{8}$$

$$\frac{\partial\Phi}{\partial h}=T, \tag{9}$$

$$\frac{1}{\mathrm{Ri}}\left[Hv\frac{\partial T}{\partial\lambda}+\hat{w}\left(\frac{\partial T}{\partial h}+H\frac{R}{c_p}T\right)\right]+S\hat{w}=\frac{Q_0}{c_pW\bar{S}}Q. \tag{10}$$

The continuity equation (5) remains unchanged. The parameter $\epsilon=\Omega a/U$. The parameter Ri (the Richardson number) is given by

$$\mathrm{Ri}=\frac{R\bar{S}}{(U/H)^2}. \tag{11}$$

This equation defines a mean Richardson number for the whole layer.

a. The energy equation.

The inverse Richardson number appears before the nonlinear advective terms in the energy equation because of the assumption that temperature perturbations are related to U by thermal wind balance. This means that we need to check the consistency of the "cyclostrophic balance" assumption, which is implicit in the scaling. It is most convenient, however, to perform the consistency check after examining the energy equation, since an estimate of W emerges.

Unfortunately, we do not have sufficiently detailed observations to evaluate Ri. We shall make the conjecture that $\mathrm{Ri}/H\gg 1$, and can offer only the following suggestive support for the conjecture. At upper levels in the atmosphere the static stability can be evaluated relatively accurately because the lapse rate is substantially subadiabatic. For example, between 40 and 70 km, the Mariner 10 radio occultation results reported by Howard *et al.* (1974) show a mean lapse rate of 5.8 K $\mathrm{km^{-1}}$, in contrast to an adiabatic lapse rate of about 10 K $\mathrm{km^{-1}}$. Between these same two levels, a zonal wind increment of about 50 m $\mathrm{s^{-1}}$ is indicated by the difference between the Venera 8 speed of about 50 m $\mathrm{s^{-1}}$ at 40 km and the cloud top speed of about 100 m $\mathrm{s^{-1}}$. If we evaluate the local Richardson's number

$$\mathrm{Ri}_{\mathrm{local}}=\frac{\dfrac{g}{T}\left(\dfrac{dT}{dz}+\dfrac{g}{c_p}\right)}{\left(\dfrac{du}{dz}\right)^2}, \tag{12}$$

with $dT/dz=-5.8$ K $\mathrm{km^{-1}}$, $g=860$ cm $\mathrm{s^{-2}}$, $c_p=0.86\times 10^7$ ergs $\mathrm{g^{-1}\,K^{-1}}$, $T=325$ K, $R=0.19\times 10^7$ ergs $\mathrm{g^{-1}\,K^{-1}}$, and $du/dz=50$ m $\mathrm{s^{-1}}$ (30 km)$^{-1}$, we obtain $\mathrm{Ri}_{\mathrm{local}}=37$. For this layer, therefore, the observations suggest that the Richardson number is large. As further support, we may offer the theoretical speculation that the observed rapid flow itself implies a large Richardson number, since its very existence suggests that the vertical turbulent diffusivity is small, and a large Richardson number is probably necessary to suppress the onset of instabilities.

If $\mathrm{Ri}/H\gg 1$, the approximate balance in (10) must be between adiabatic cooling (due to vertical motion) and radiative heating. The magnitude of W is determined by

$$\frac{Q_0}{c_pW\bar{S}}=1, \tag{13}$$

and the equation reduces to

$$S\hat{w}=Q. \tag{14}$$

We have made use of the observed fact that the Venus atmosphere is not in radiative equilibrium, i.e., $Q\neq 0$. The small latitudinal temperature gradient establishes this fact. Thus, Q must be balanced by one of the terms on the left-hand side of (10).

The order of magnitude of Q_0 is $(g/p_0)\sigma T_e^4$, where σ is the Stefan-Boltzmann constant and T_e the effective temperature of the planet. There is also a numerical factor which depends on the profile of Q with height. For the particular numerical example to be presented in Section 3, the value of W deduced from (13) is $10^{-7}\ \mathrm{s^{-1}}$. Details of the evaluation are below; for our purposes here we need only the order of magnitude for scaling.

The area-weighted horizontal average of the left-hand side of (14) is zero, since the net vertical mass flux vanishes. Therefore the equation for $T_s(h)$ is

$$\langle Q(\lambda,h)\rangle=0, \tag{15}$$

where the angle braces indicate the average. Eq. (15) states that $T_s(h)$ is determined by the requirement that the atmosphere be in radiative-convective equilibrium, horizontally averaged. This is a consequence of the fact that horizontal temperature variations are small, and that therefore the vertical energy flux by large-scale motions is negligible. Notice that the *poleward* energy flux by large-scale motions is not small. In fact, the meridional cell is adjusted precisely to transport sufficient heat poleward to compensate for lack of local radiative equilibrium.

The vertical motion field is given by (14). We shall assume that $Q\rightarrow 0$ toward the bottom and top of the atmosphere, so that $\hat{w}$ also does. If this were not the case, boundary conditions on $\hat{w}$ could not be met, and horizontal convection or diffusion would need to exist near top or bottom of the atmosphere.

b. The momentum equations

Examination of the meridional momentum equation (8) shows that cyclostrophic balance obtains if the three parameters

$$\left(\frac{aW}{U}\right)^2,\quad \frac{\nu_v}{W(DH)^2}\left(\frac{aW}{U}\right)^2,\quad \frac{\nu_H}{Wa^2}\left(\frac{aW}{U}\right)^2$$

are all small. With $a=6\times 10^8$ cm, $W=10^{-7}\ \mathrm{s^{-1}}$, and $U=10^4$ cm $\mathrm{s^{-1}}$, we find $(aW/U)^2=3.6\times 10^{-5}$. The other two factors in these parameters involve the ratios of diffusive to advective time constants. We show below that the values of interest for these ratios are in the range 10^{-2} to 10^{+2}; therefore (8) reduces to

$$u\sin\lambda\left(2\epsilon+\frac{u}{\cos\lambda}\right)+\frac{\partial\Phi}{\partial\lambda}=0 \tag{16}$$

for the parameter regime of interest.

In the zonal momentum equation (7) the parameter regime of interest to demonstrate a rapid super-rotation is

$$\frac{Wa^2}{\nu_H}\ll 1,\quad \frac{a^2\nu_v}{(DH)^2\nu_H}\ll 1, \tag{17}$$

so that advection by the meridional cell is slower than horizontal diffusion, and vertical diffusion is slower than horizontal. In this limit, the equation reduces to

$$\frac{1}{\cos\lambda}\frac{\partial}{\partial\lambda}\left(\cos^3\lambda\frac{\partial}{\partial\lambda}\frac{u}{\cos\lambda}\right)=0. \tag{18}$$

The quantity in parentheses is proportional to the pole-flux of angular momentum by diffusion, and therefore is zero at the pole. With this boundary condition, (18) can be integrated to give [to leading order in the parameters identified in (17)]

$$u(\lambda,h)=g(h)\cos\lambda, \tag{19}$$

where $g(h)$ is a function to be determined. This equation states that the zonal flow is rigid rotation of each spherical shell of atmosphere.

The function $g(h)$ is determined as follows. Multiply (7) by $\cos\lambda$ and integrate from equator to pole. The poleward flux of angular momentum vanishes at the pole and also at the equator (by symmetry). Only the vertical flux terms remain, and the equation is

$$\frac{1}{H}e^{Hh}\frac{\partial}{\partial h}\left[e^{-Hh}\int_0^{\pi/2}[g(h)+\epsilon]\hat{w}(h,\lambda)\cos^3\lambda d\lambda\right]$$
$$=\frac{\nu_v}{(DH)^2W}e^{Hh}\frac{\partial}{\partial h}\left[e^{-Hh}\int_0^{\pi/2}g(h)\cos^3\lambda d\lambda\right]. \tag{20}$$

The left-hand side of this equation is proportional to the divergence of the net vertical angular momentum flux by the large-scale flow, and the right-hand side, by diffusion. With the boundary conditions that the net angular momentum flux and also $g(h)$ vanish at $h=0$, (20) can be integrated to give the solution

$$\frac{1}{\epsilon}g(h)+1=\exp\left[\frac{3}{2}\frac{D^2HW}{\nu_v}\int_0^h dh'\right.$$
$$\left.\times\int_0^{\pi/2}d\lambda\hat{w}(h',\lambda)\cos^3\lambda\right]. \tag{21}$$

This represents the general solution for the particular parameter regime we have identified. The temperature, geopotential, and meridional flow fields can be calculated from (5), (9) and (16). A numerical example is presented in the next section.

For a strong flow ($\epsilon\ll 1$) to develop, two factors are necessary. The quantity in the exponential in (21) must be positive; this requires that the sense of the meridional cell be rising motion near the equator and sinking near the poles. Note that the reverse would lead to $g(h)<0$; in fact, to $\epsilon=1$ and approach by the upper atmospheric levels to zero absolute rotation. The second necessary factor is that D^2HW/ν_v must be large, but not exceedingly so because of the exponential dependence. To order of magnitude,

$$\frac{1}{\epsilon}=\frac{U}{\Omega a}\sim\exp(D^2HW/\nu_v). \tag{22}$$

For the observed flow, $U=10^4$ cm s^{-1}, $\Omega=2\pi/244$ (day)$^{-1}=3\times10^{-7}$ s^{-1}, $a=6\times10^8$ cm, and $U/\Omega a=56$. Thus $D^2HW/\nu_v\sim\ln 56=4.0$. With $H=7$, $W=10^{-7}$ s^{-1} and $D=1.1\times10^6$ cm, this implies that $\nu_v=2.1\times10^5$ cm^2 s^{-1}. We see that even with a fairly large vertical diffusivity, the strong flow could be maintained.

c. Summary of parameter conditions in the model

For a meridional cell to be driven in the manner we have described, two conditions are necessary. The one discussed above is that $\mathrm{Ri}/H\gg 1$. The other is that horizontal heat transport not be affected by diffusion. It can be shown that this criterion is

$$\frac{K_H}{Wa^2}\frac{H}{\mathrm{Ri}}\ll 1,$$

where K_H is a horizontal diffusivity for heat, analogous to ν_H. Since another criterion is that $\nu_H/Wa^2\gg 1$, we see that if $\nu_H\approx K_H$, it is necessary that

$$\frac{\mathrm{Ri}}{H}\gg\frac{\nu_H}{Wa^2}\gg 1.$$

An important reminder, however, is that the angular momentum arguments we have made do not depend on the precise way that the meridional cell is driven.

The criteria from the momentum balance equations can be summarized by

$$\frac{\nu_v}{WD^2H}\ll 1\ll\frac{\nu_H}{Wa^2}\ll\left(\frac{U}{aW}\right)^2. \tag{23}$$

Vertical diffusion must be slower than meridional overturning, horizontal diffusion must be faster than meridional overturning, and the zonal flow must be strong for cyclostrophic balance. When all these conditions are satisfied, the order of magnitude of U is given by (22).

3. A numerical example

We do not know the distribution of heating and cooling, $Q(h,\lambda)$, in the Venus atmosphere. For the sake of examining a particular numerical case, we shall calculate flow quantities for the arbitrary choice

$$Q(\lambda,h)=4h^2e^{-2h}\left(\frac{4}{\pi}\cos\lambda-1\right). \tag{24}$$

This function has a latitude variation simulating incident solar radiation and uniform cooling, and a smooth variation with height with heating and cooling (per unit mass) fairly uniformly distributed in height.

The dimensional constant Q_0 can be obtained as follows. At the pole ($\lambda=\pi/2$), the dimensional radiative-convective heat flux F must satisfy

$$\int_0^\infty \frac{\partial F}{\partial h}dh=\sigma T_e^4. \tag{25}$$

Using the definitions of h and $\hat{z}$, and the hydrostatic and ideal gas relations, we find

$$Q_{\mathrm{DIM}}=Q_0Q=-\frac{1}{\rho}\frac{\partial F}{\partial z}=-\frac{g}{p_0}e^{Hh}\frac{1}{H}\frac{\partial F}{\partial h}, \tag{26}$$

where z is geometrical height and ρ is density; $\partial F/\partial h$ can be evaluated from this expression, and the integration (25) can be carried out. The resulting value of Q_0 is given by

$$Q_0=\frac{g}{p_0}\frac{(H+2)^3}{8H}\sigma T_e^4=25 \text{ ergs g}^{-1}\text{ s}^{-1}, \tag{27}$$

for the numerical values $g=860$ cm s^{-2}, $p_0=10^8$ dyn cm^{-2}, $H=7$, $T_e=250$ K. Solving (13) for W then gives

$$W=\frac{Q_0}{c_p\bar{S}}=1.0\times10^{-7}\text{ s}^{-1}, \tag{28}$$

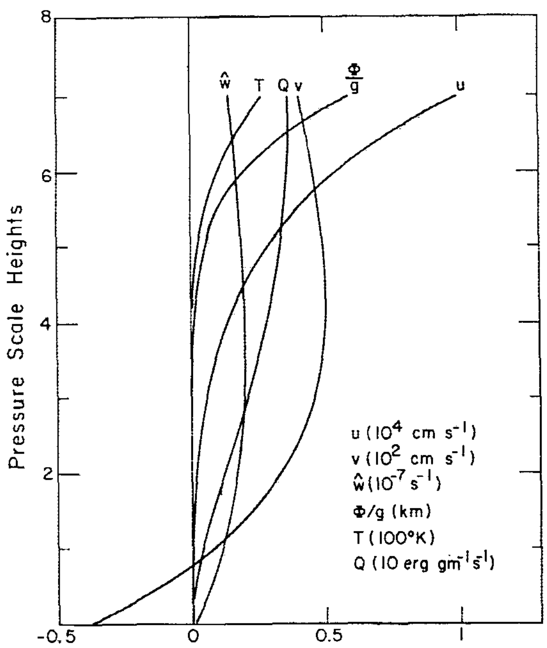

FIG. 2. Vertical profiles. All quantities are evaluated at the equator, except for meridional velocity v, which is at 45° latitude. Units for each variable are indicated.

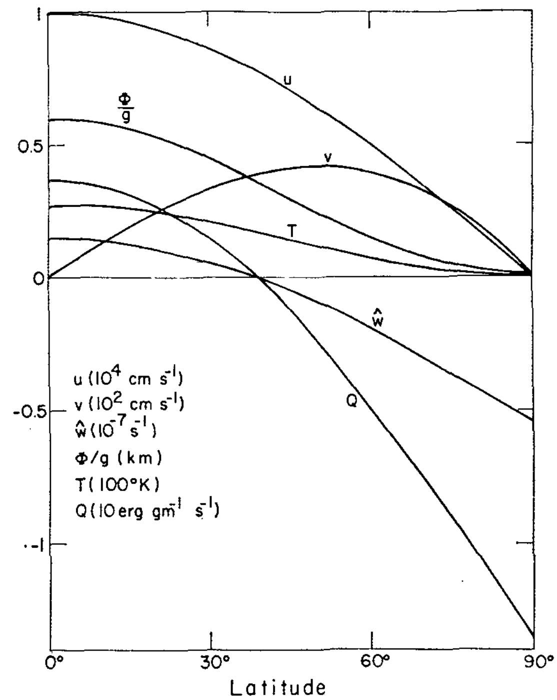

FIG. 3. Horizontal profiles. All quantities are evaluated at $h=1$, or a height of approximately 7 pressure scale heights.

for $c_p=0.86\times10^7$ ergs gm^{-1} K^{-1}, and $\bar{S}=28$ K [based on the numerical values for lapse rate and temperature quoted in the discussion following (12)]. This value of W implies a meridional overturning time of about 100 days.

From observation, we adopt $U=10^4$ cm s^{-1}. The following scaling factors are then determined.

Zonal flow:	$U=10^4$ cm s^{-1}
Meridional flow:	$aW=60$ cm s^{-1}
Vertical motion:	$W=10^{-7}$ s^{-1}
Temperature:	$U^2/RH=7.2$ K
Geopotential height (Φ/g):	$U^2/g=1.2\times10^5$ cm

Finally, we need to specify the variation of static stability $S(h)$ with height. We shall assume

$$S(h)=h. \tag{29}$$

The solution for $\hat{w}$ is then obtained from (14). The result is

$$\hat{w}=4he^{-2h}\left(\frac{4}{\pi}\cos\lambda-1\right). \tag{30}$$

Solving (21) for $g(h)$ and substituting into (19) gives

$$u=g(h)\cos\lambda$$

$$=\epsilon\left\{\exp\left[\frac{1}{8}\frac{D^2HW}{\nu_v}[1-(1+2h)e^{-2h}]\right]-1\right\}\cos\lambda. \tag{31}$$

To match the observed flow speed, we must have $u(h=1, \lambda=0)=1$. Since $\epsilon=\Omega a/U=0.018$, this condition leads to an equation for ν_v, whose solution is

$$\frac{D^2HW}{\nu_v}=54 \Rightarrow \nu_v=1.6\times10^4 \text{ cm}^2 \text{ s}^{-1}, \quad (32)$$

for $H=7$, $W=10^{-7}$ s^{-1} and $D=1.1\times10^6$ cm.

The other flow fields can be found as outlined above. The results are

$$v=4e^{-2h}[h(H+2)-1]\left[\frac{2}{\pi}\left(\sin\lambda-\frac{\lambda}{\cos\lambda}\right)-\tan\lambda\right], \quad (33)$$

$$T=\frac{1}{2}\frac{D^2HW}{\nu_v}(g+\epsilon)^2he^{-2h}\cos^2\lambda, \quad (34)$$

$$\Phi=\tfrac{1}{2}g(g+2\epsilon)\cos^2\lambda', \quad (35)$$

where constants of integration have been taken to arbitrarily set Φ and T equal to zero at $h=0$ and $\lambda=\pi/2$.

Height and latitude variations are displayed in Figs. 2 and 3. Notice that although the heating Q is fairly uniformly distributed in depth, the zonal velocity and, particularly, the temperature variation are concentrated near the top. The meridional return flow is confined to the lowest scale height.

4. Remarks

In the example, the heating and cooling are distributed in height. It seems likely that they are in reality concentrated at levels within the cloud deck, fairly high in the atmosphere. Examples have been worked to investigate this case. The vertical velocity W becomes larger, because less mass is involved in the flow. The effective depth H becomes smaller. The net effect is that ν_v can be larger than the value quoted in the preceding section.

A number of numerical circulation experiments have been performed for the Venus atmosphere (Turikov and Chalikov, 1971; Kálnay de Rivas, 1975), and these have not produced a strong equatorial current. If the mechanism discussed in this paper is correct, any of several deficiencies in the numerical models could be at fault. The vertical friction coefficients could be too large. The vertical resolution could be insufficient. The distribution of heating and cooling could be inaccurate, leading to inaccurate static stability. It would be particularly important to correctly simulate the stable layers near the cloud tops, since we have seen that the wind shear and the horizontal temperature variations can be concentrated at high levels. Finally, an extremely long time might be necessary to build up the flow. For example, the time constant for vertical diffusion is $t\sim D^2H^2/\nu_v$, and for $D=1.1\times10^6$ cm, $H=7$ and $\nu_v=1.6\times10^4$ cm^2 s^{-1}, this time is about 100 years.

The observational questions of particular importance to the theory we have discussed are obvious. Some are: What is the static stability and its distribution with height? What is the wind shear and the Richardson number; how do they vary with height? What is the distribution of heating and cooling? Is there a meridional cell? What is the nature of smaller scale motions; in particular, what is the nature of heat and angular momentum transport in the horizontal by non-axisymmetric flow components?

Acknowledgments. This research was supported in part by the Atmospheric Sciences Section, National Science Foundation, by Grant GA-37012 to Cornell University. Thoughtful and helpful comments by anonymous referees are much appreciated.

REFERENCES

Durney, B. R., and I. W. Roxburgh, 1971: Inhomogeneous convection and the equatorial acceleration of the sun. *Solar Phys.*, **16** 3–20.

Gold, T., and S. Soter, 1971: Atmospheric tides and the 4-day circulation on Venus. *Icarus*, **14**, 16–20.

Holton, J. R., 1972: *An Introduction to Dynamic Meteorology*, Academic Press, 319 pp.

Howard, H. T., G. L. Tyler, G. Fjeldbo, A. J. Kliore, G. S. Levy, D. L. Brunn, R. Dickinson, R. E. Edelson, W. L. Martin, R. B. Postal, B. Seidel, T. T. Sesplaukis, D. L. Shirley, C. T. Stelzreid, D. N. Sweetnam, A. I. Zygeilbaum, P. B. Esposito, J. D. Anderson, I. I. Shapiro and R. D. Reasenberg, 1974: Venus: mass, gravity field, atmsophere, and ionosphere as measured by the Mariner 10 dual-frequency radio system. *Science*, **183**, 1298–1302.

Ingersoll, A. P., and G. S. Orton, 1974: Lateral inhomogeneities in the Venus atmosphere: Analysis of thermal infrared maps. *Icarus*, **21**, 121–128.

Kálnay de Rivas, E., 1975: Further numerical calculations of the circulation of the atmosphere of Venus. *J. Atmos. Sci.*, **32**, 1017–1024.

Leovy, C. B., 1973: Rotation of the upper atmosphere of Venus. *J. Atmos. Sci.*, **30**, 1218–1220.

Marov, M. Ya., V. S. Avdnevsky, V. V. Kerzhanovich, M. K. Rozhdestvensky, N. F. Borodin and O. L. Ryabov, 1973: Venera 8: Measurements of temperature, pressure, and wind velocity on the illuminated side of Venus. *J. Atmos. Sci.*, **30**, 1210–1214.

Murray, B. C., M. J. S. Belton, G. E. Danielson, M. E. Davies, D. Gault, B. Hapke, B. O'Leary, R. G. Strom, V. Suomi and N. Trask, 1974: Venus: Atmospheric motion and structure from Mariner 10 pictures. *Science*, **183**, 1307–1315.

Phillips, N. A., 1963: Geostrophic motion. *Rev. Geophys.*, **1**, 123–176.

Rossby, C.-G., 1947: On the distribution of angular velocity in gaseous envelopes under the influence of large-scale horizontal mixing processes. *Bull. Amer. Meteor. Soc.*, **28**, 53–68.

Schubert, G., and J. A. Whitehead, 1969: Moving flame experiment with liquid mercury: Possible implications for the Venus atmosphere. *Science*, **163**, 71–72.

Suomi, V., 1975: Mariner 10 observations of cloud motions. Presented at the Conference on the Atmosphere of Venus.

Thompson, R., 1970: Venus' general circulation is a merry-go-round. *J. Atmos. Sci.*, **27**, 1107–1116.

Traub, W. A., and N. P. Carleton, 1975: Spectroscopic observations of winds on Venus. *J. Atmos. Sci.*, **32**, 1045–1059.

Turikov, V. J., and D. V. Chalikov, 1971: Computation of the general circulation of the Venusian atmosphere. *Izv. Atmos. Ocean Phys.*, **7**, 705–722.

Young, R. E., and G. Schubert, 1973: Dynamical aspects of the Venus 4-day circulation. *Planet. Space Sci.*, **21**, 1563 1580.