PLATE KINEMATICS

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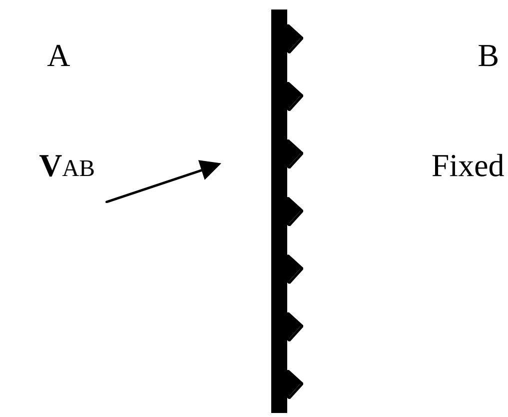
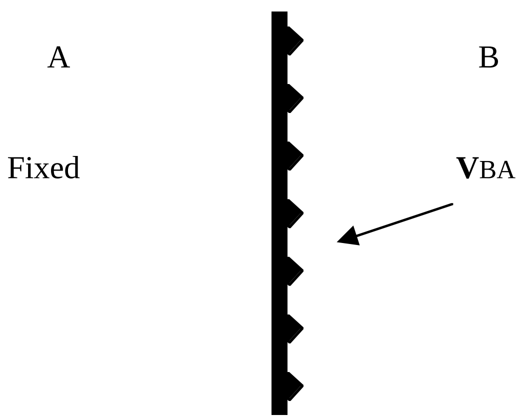
(Reference - The Solid Earth, C.M.R. Fowler, Cambridge University Press, 1990, Chapter 2)

Plate Motions on a Flat Earth

For the next few minutes we'll discuss the relative motions among plates on a flat earth. Consider 2 plates A and B which have a subduction zone boundary between them.





$\mathbf{V}_{AB}$ – velocity of plate A relative to plate B.

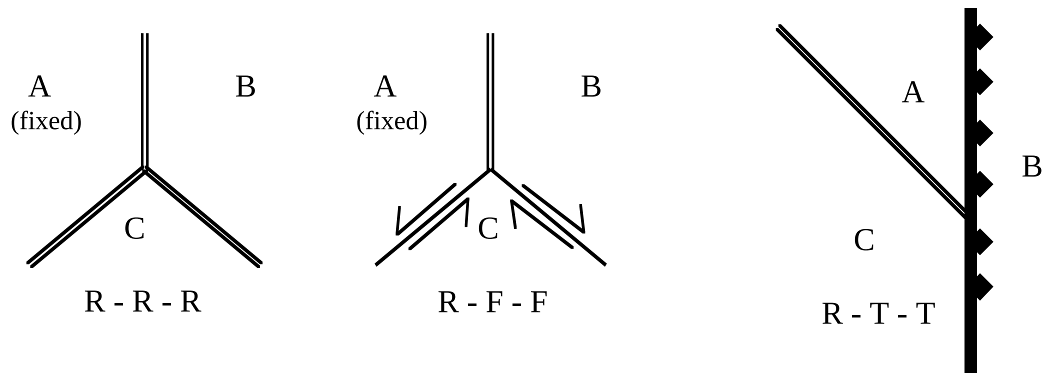
$\mathbf{V}_{BA}$ – velocity of plate B relative to plate A.

$$\mathbf{V}_{AB} = -\mathbf{V}_{BA}$$

$$\mathbf{V}_{AB} = V_x i + V_y j$$

Triple Junction

A triple junction is the intersection of three plate boundaries. The most common types of triple junctions are ridge-ridge-ridge (R-R-R), ridge-fault-fault (R-F-F), and ridge-trench-trench (R-T-T).



Each type of plate boundary has rules about relative velocities:

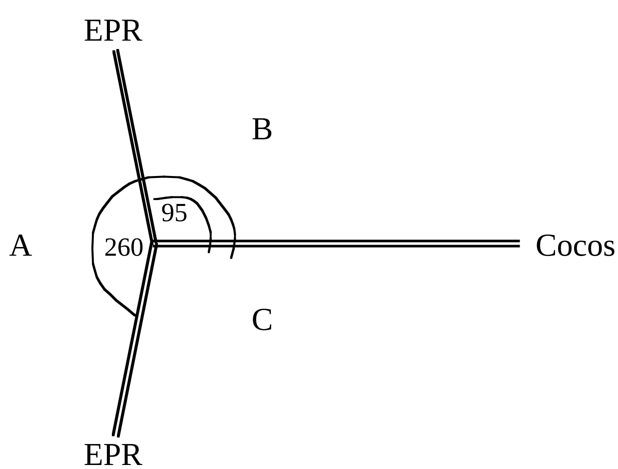
i) ridge - relative velocity must be divergent and is usually perpendicular to the ridge.
ii) transform fault - relative velocity must be parallel to the fault
iii) trench - relative velocity must be convergent but no direction is preferred

All triple junctions must satisfy a velocity condition such that the vector sum around the plate circuit is zero.

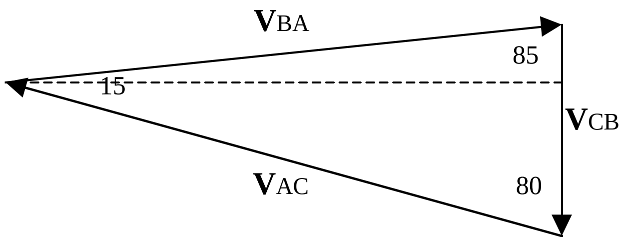
$$\text{Fix plate A}: \quad \mathbf{V}_{BA} + \mathbf{V}_{CB} + \mathbf{V}_{AC} = 0$$

In the real world we usually can map the geometry of the spreading ridges, transform faults and trenches but cannot always measure the relative velocities.

Example: Galapagos Triple Junction - RRR



Given the above geometry and one spreading velocity $|\mathbf{V}_{AC}|$ = 120 mm/yr, what are the other two spreading rates?



sum of interior angles = 180˚

$$\cos 10° = \frac{V_h}{V_{AC}}$$

V_h = 118 mm/yr

V_{b1} = 20.8 mm/yr

$$|\mathbf{V}_{BA}| = 118.45\text{mm/yr}$$

V_{b2} = 10.32

$$|\mathbf{V}_{CB}| = 20.8 + 10.32 = 31.12 mm/yr$$

One of these flat-earth triple junction closure problems will be on the quiz. The map on the next page shows the other triple junctions. As an exercise, use a bathymetric map to determine the geometry of the triple junction and use Table 2.1 (below) to calculate the spreading rate at one of the ridges. The next section develops the mathematics for calculation of plate motions on a spherical earth.

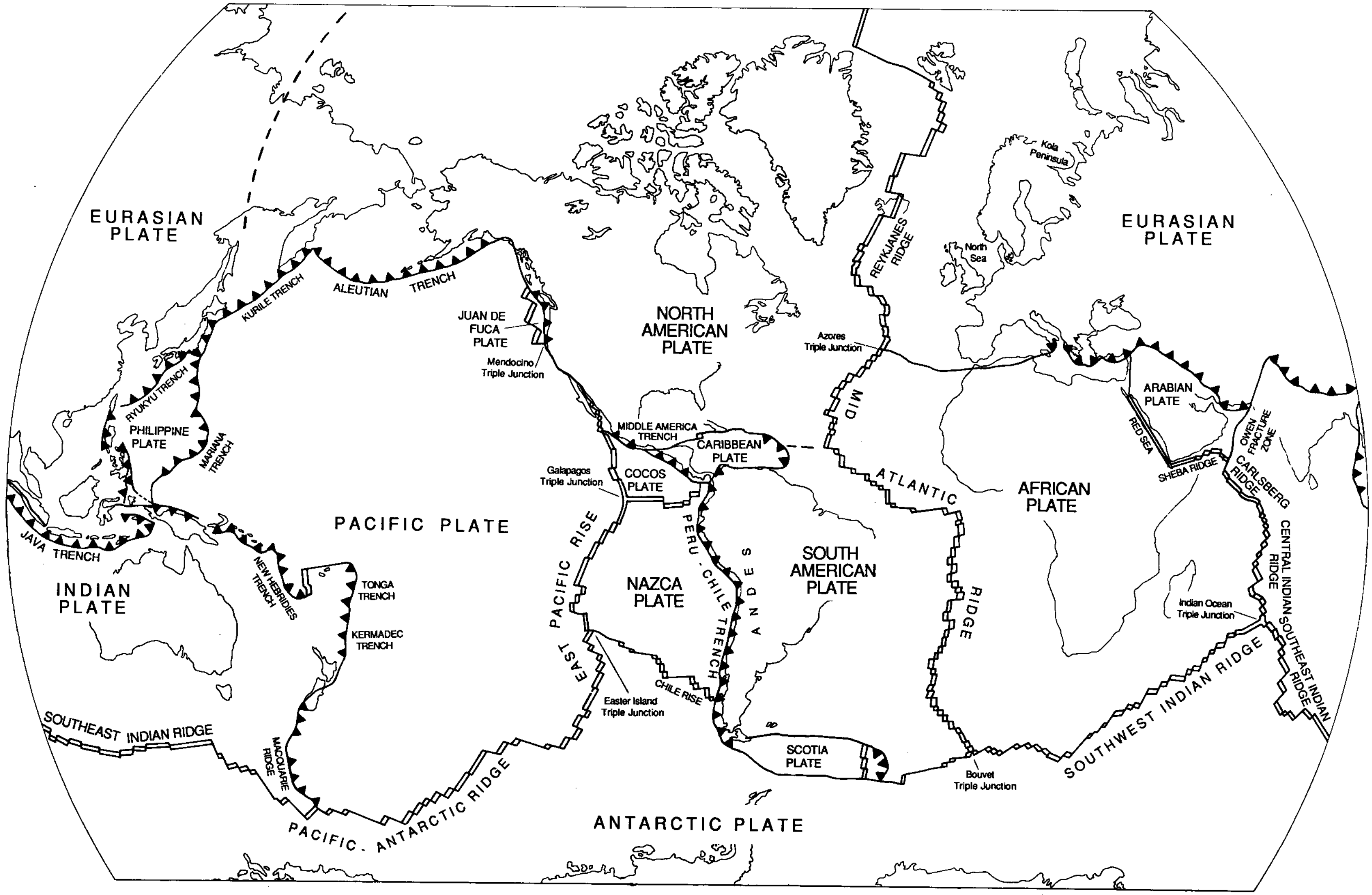
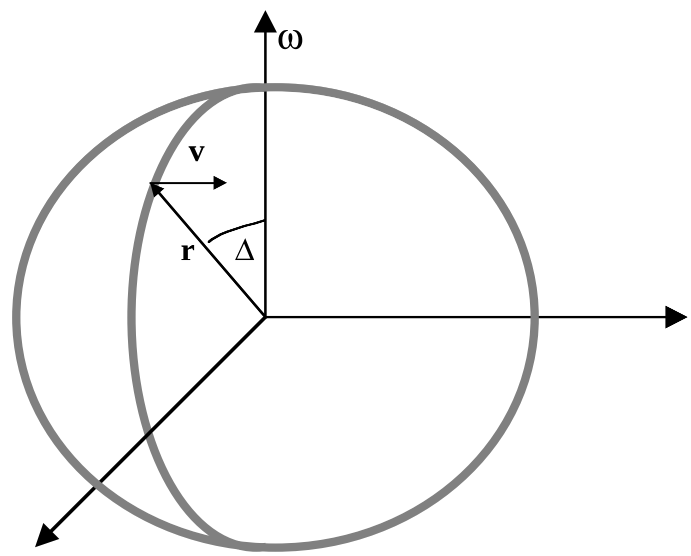


Figure 2.2. The major tectonic plates, midocean ridges, trenches and transform faults.

Plate Motions on a Sphere

(Minster, J-B, T.H. Jordan, Present-day plate motions, J. Geophys. Res., v. 85, p. 5331-5354, 1978. DeMets et al., Current plate motions, Geophys. J. R. Astr. Soc., v. 101, p. 425-478, 1990.)



Given:

ω – angular velocity vector $\left(\frac{\text{rad}}{\text{s}}\right)$

$\mathbf{r}$ – position on earth (m)

Calculate:

$\mathbf{v}$ – velocity vector at $\mathbf{r}$ $\left(\frac{\text{m}}{\text{s}}\right)$

Of course, the velocity of the plate must be tangent to the surface of the earth so the velocity is the cross product of the position vector and the angular velocity vector.

$$\mathbf{v} = \omega \times \mathbf{r} \qquad (1)$$

or

$$\mathbf{v} = \bar{i}\left(\omega_y z - \omega_z y\right) - \bar{j}\left(\omega_x z - \omega_z x\right) + \bar{k}\left(\omega_x y - \omega_y x\right) \qquad (2)$$

where *i*, *j*, and *k* are unit vectors. The magnitude of the velocity is given by

$$|\mathbf{v}| = |\omega||\mathbf{r}|\sin(\Delta) \qquad (3)$$

where Δ is the angle between the position vector and the angular velocity vector. It is given by the following formula.

$$\cos(\Delta) = \frac{\omega \bullet \mathbf{r}}{|\omega||\mathbf{r}|} \qquad (4)$$

The formulas above assume that the angular velocity vector and the position vector are provided in Cartesian coordinates. However, usually they are specified in terms of latitude and longitude. Thus one must transform both vectors. The usual case is to calculate the relative velocity between two plates somewhere along their common boundary. Table 2.1 (next page) lists the pole position and rates of rotation for relative motion between plate pairs. The Cartesian position of a point along the plate boundary is

$$x = a\cos\theta\cos\phi \qquad (5)$$
$$y = a\cos\theta\sin\phi$$
$$z = a\sin\theta$$

You should memorize the conversion from latitude longitude to the Cartesian co-ordinate system where the x-axis runs from the center of the earth, to a point at 0˚ latitude and 0˚ longitude (i.e. the Greenwich meridian), the y-axis runs through a point at 0˚ latitude and 90˚ east longitude and the z-axis runs along the spin-axis to the north pole.

Similarly the pole positions must be converted from geographic co-ordinates (θ_p, ϕ_p) into the Cartesian system

$$\begin{aligned} \omega_x &= |\omega| \cos\theta_p \cos\phi_p \\ \omega_y &= |\omega| \cos\theta_p \sin\phi_p \\ \omega_z &= |\omega| \sin\theta_p \end{aligned} \qquad (6)$$

where $|\omega|$ is the magnitude of the rotation vector provided in Table 2.1. There are two ways to compute the magnitude of the velocity. One could compute the cross product of the rotation vector and the position vector (equation 1). Then the magnitude of the velocity is

$$|\mathbf{v}| = \left(v_x^2 + v_y^2 + v_z^2\right)^{1/2} \qquad (7)$$

A second approach is to calculate the angle Δ between the position vector and the angular velocity vector using equation 4 and then use that value in equation 3 to calculate the magnitude of the velocity. Indeed, both Fowler and Turcotte & Schubert use this second approach. However, they use the rather cumbersome spherical trigonometry to calculate the angle Δ. Since I can never remember the spherical trigonometry formulas, I prefer to use equation 3 above after converting everything to Cartesian coordinates.

Velocity Azimuth

We know that the velocity vector is tangent to the sphere. Given the Cartesian velocity components from equation 2, we would like to compute the latitude v_θ and longitude v_ϕ components of velocity. Begin by taking the time derivative of equation 5.

$$\begin{aligned} v_x &= a\left(-\cos\phi \sin\theta \; v_\theta - \cos\theta \sin\phi \; v_\phi\right) \\ v_y &= a\left(-\sin\phi \sin\theta \; v_\theta + \cos\theta \cos\phi \; v_\phi\right) \\ v_z &= a\left(\cos\theta \; v_\theta\right) \end{aligned} \qquad (8)$$

TABLE 2. NUVEL-1 Euler Vectors

Plate Pair	Latitude °N	Longitude °E	ω (deg–m.y.$^{-1}$)	Error Ellipse σ_{max}	Error Ellipse σ_{min}	Error Ellipse ζ_{max}	σ_ω (deg–m.y.$^{-1}$)
			Pacific Ocean				
na-pa	48.7	-78.2	0.78	1.3	1.2	-61	0.01
co-pa	36.8	-108.6	2.09	1.0	0.6	-33	0.05
co-na	27.9	-120.7	1.42	1.8	0.7	-67	0.05
co-nz	4.8	-124.3	0.95	2.9	1.5	-88	0.05
nz-pa	55.6	-90.1	1.42	1.8	0.9	-1	0.02
nz-an	40.5	-95.9	0.54	4.5	1.9	-9	0.02
nz-sa	56.0	-94.0	0.76	3.6	1.5	-10	0.02
an-pa	64.3	-84.0	0.91	1.2	1.0	81	0.01
pa-au	-60.1	-178.3	1.12	1.0	0.9	-58	0.02
eu-pa	61.1	-85.8	0.90	1.3	1.1	90	0.02
co-ca	24.1	-119.4	1.37	2.5	1.2	-60	0.06
nz-ca	56.2	-104.6	0.58	6.5	2.2	-31	0.04
			Atlantic Ocean				
eu-na	62.4	135.8	0.22	4.1	1.3	-11	0.01
af-na	78.8	38.3	0.25	3.7	1.0	77	0.01
af-eu	21.0	-20.6	0.13	6.0	0.7	-4	0.02
na-sa	16.3	-58.1	0.15	5.9	3.7	-9	0.01
af-sa	62.5	-39.4	0.32	2.6	0.8	-11	0.01
an-sa	86.4	-40.7	0.27	3.0	1.2	-24	0.01
na-ca	-74.3	-26.1	0.11	25.5	2.6	-52	0.03
ca-sa	50.0	-65.3	0.19	15.1	4.3	-2	0.03
			Indian Ocean				
au-an	13.2	38.2	0.68	1.3	1.0	-63	0.00
af-an	5.6	-39.2	0.13	4.4	1.3	-42	0.01
au-af	12.4	49.8	0.66	1.2	0.9	-39	0.01
au-in	-5.6	77.1	0.31	7.4	3.1	-47	0.07
in-af	23.6	28.5	0.43	8.8	1.5	-74	0.06
ar-af	24.1	24.0	0.42	4.9	1.3	-65	0.05
in-eu	24.4	17.7	0.53	8.8	1.8	-79	0.06
ar-eu	24.6	13.7	0.52	5.2	1.7	-72	0.05
au-eu	15.1	40.5	0.72	2.1	1.1	-45	0.01
in-ar	3.0	91.5	0.03	26.1	2.4	-58	0.04

The first plate moves counterclockwise relative to the second plate. Plate abbreviations: pa, Pacific; na, North America; sa, South America; af, Africa; co, Cocos; nz, Nazca; eu, Eurasia; an, Antarctica; ar, Arabia; in, India; au, Australia; ca, Caribbean. See Figure 3 for plate geometries. Azimuths are in degrees clockwise of north. One sigma-error ellipses are specified by the angular lengths of the principal axes and by the azimuths (ζ_{max}) of the major axis. The rotation rate uncertainty is determined from a one-dimensional marginal distribution, whereas the lengths of the principal axes are determined from a two-dimensional marginal distribution.

From the last equation in (8), we can solve for the latitude velocity component.

$$v_\theta = \frac{v_z}{a\cos\theta} \qquad (9)$$

Now plug v_θ into either the v_x or v_y equation and solve for v_ϕ.

$$v_\phi = \frac{v_y - v_z \sin\phi \tan\theta}{a\cos\theta\cos\phi} \qquad (10)$$

If this equation turns out to be singular, then use the v_x equation.

Recipe for Computing Velocity Magnitude

In summary, to calculate the magnitude of the velocity:

i) Transform lat, lon into $\mathbf{x}$ = (x, y, z) unit vector using equation 5.
ii) Transform pole lat and lon into $\mathbf{x}_p$ = (x_p, y_p, z_p) unit vector
iii) $\cos\Delta = \mathbf{x} \bullet \mathbf{x}_p$
iv) $v = \omega\, a \sin\Delta$

Example: Given the rotation pole between the Pacific and Nazca plates, calculate the spreading rate at –20˚ 113.5˚W. ***One of these calculations will be on the quiz***

POLE	POINT
55.6˚N 90.1˚W 1.42x10^{-6} deg/yr	20.0˚S 113.5˚W
55.6 269.90 2.478 x 10^{-8} rad/yr	-20.0 246.5
x_p = -0.000986	x = -0.375
y_p = -0.565	y = -0.862
z_p = 0.825	z = -0.342

$\cos\Delta = \mathbf{x}\bullet\mathbf{x}_p$ = (0.000369 + .487 - .282) = .205

Δ = 78˚ $\qquad$ $v = \omega\, a \sin\Delta$ = 154.5 mm/yr

Triple Junctions on a Sphere

Triple junction closure on a sphere is similar to triple junction closure on a flat earth except that the sum of the rotation vectors must be zero.

$$\omega_{BA} + \omega_{CB} + \omega_{AC} = 0$$

Example: Galapagos Triple Junction

Given the rotation vectors of the Cosos plate relative to the Pacific plate and the Pacific plate relative to the Nazca plate, calculate the spreading rate at 2˚N, 260˚E.

$$\omega_{CP} + \omega_{NC} + \omega_{PN} = 0$$
$$\omega_{NC} = -\omega_{CP} - \omega_{PN}$$
$$\mathbf{v}_{NC} = \omega_{NC} \times \mathbf{r}\ (\theta, \phi)$$

$|\mathbf{v}|$ - magnitude of spreading rate

Hot Spots and Absolute Plate Motions

So far we have only considered relative plate motions because there was no way to tie the positions of the plates to the mantle. One method of making this connection and thus determining absolute plate motions is to assume that “hot spots” remain fixed with respect to the lower mantle.

A. A hot spot is an area of concentrated volcanic activity. There is a subset of hot spots that have the following characteristics:
B. They produce linear volcanic chains in the interiors of the plates.
C. The youngest volcanoes occur at one end of the volcanic chain and there is a linear increase in age away from that end.
D. The chemistry of the erupted lavas is significantly different from lava erupted at mid-ocean ridges or island arcs.
E. Some hotspots are surrounded by a broad topographic swell about 1000 m above the surrounding ocean basin.

These features are consistent with a model where the plates are moving over a relatively fixed mantle plume. After identifying the linear volcanic chains associated with the mantle plumes, it has been shown that the relative motions among hot spots is about 10 times less than the relative plate motions.