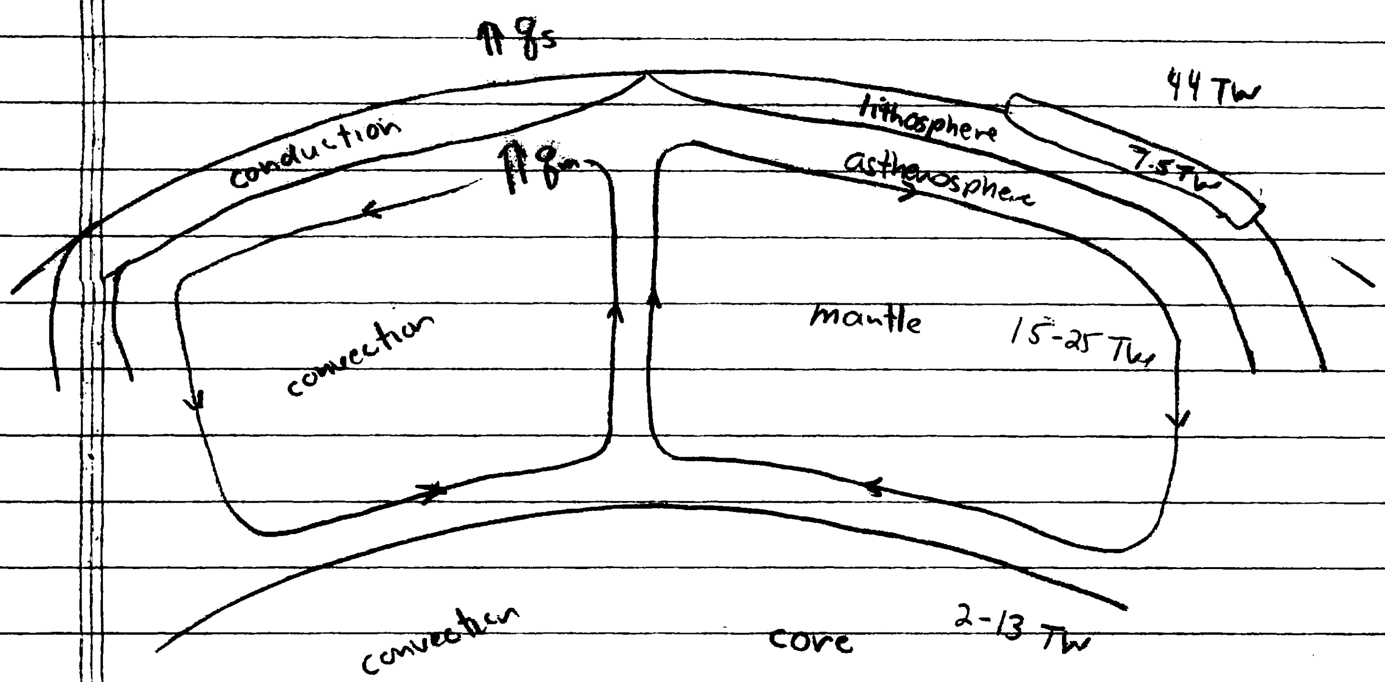
HEAT CONDUCTION (Geodynamics, Chapter 4)



- decay of radioactive elements produces heat which must escape to the surface of the earth
- secular cooling also contributes to surface heat flux
- modes of heat transport:

radiation	$q \propto T^4$
conduction	$q \propto \nabla T$
convection	$q \propto V \cdot \nabla T$

- today's lecture

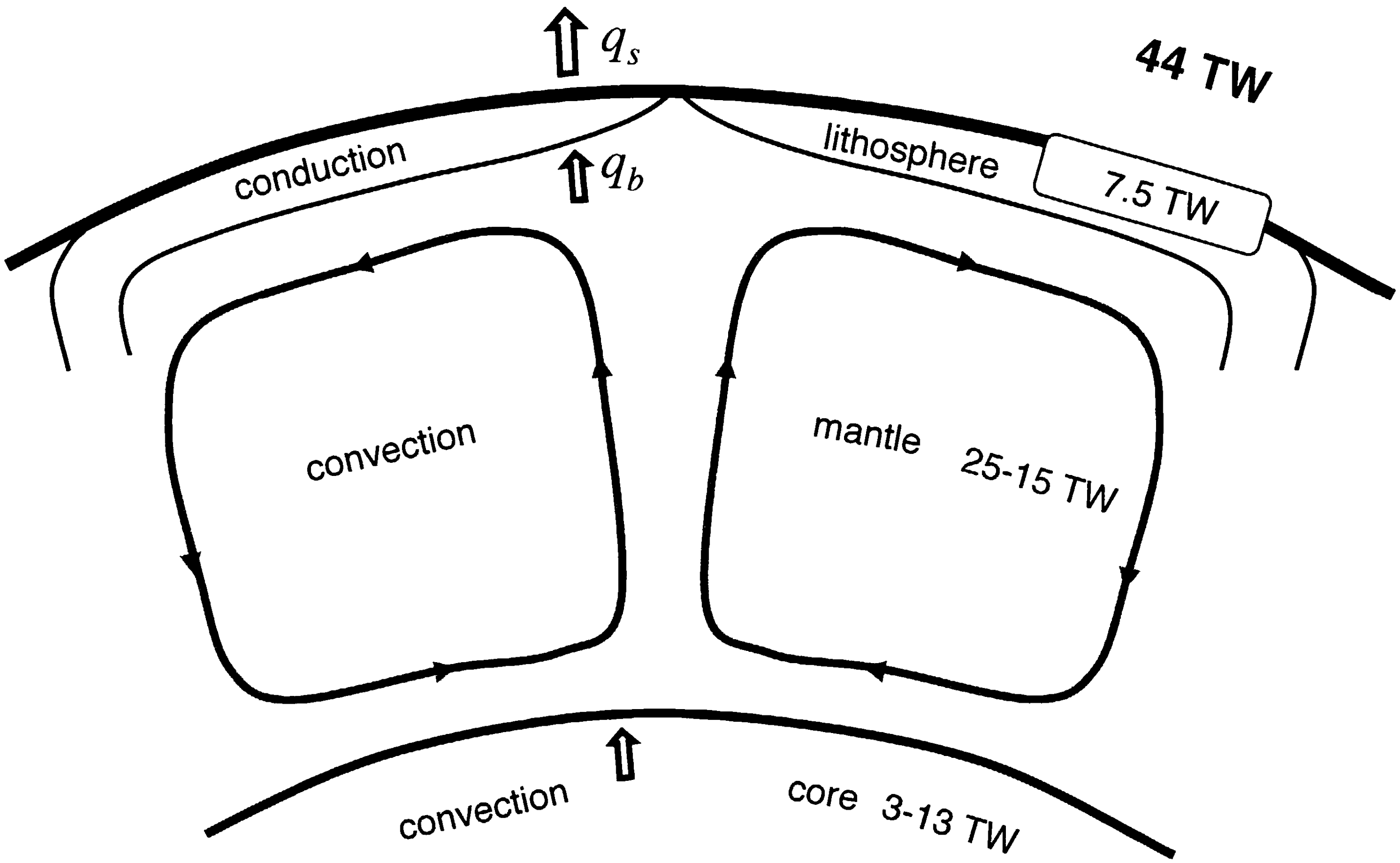
  What is the average surface heat flow q_s?

  What is the average heat flow into the base of the lithosphere q_m?

  How does temperature increase with depth in continents and oceans?

  How thick is the lithosphere and how does lithosphere thickness vary?

global heat budget



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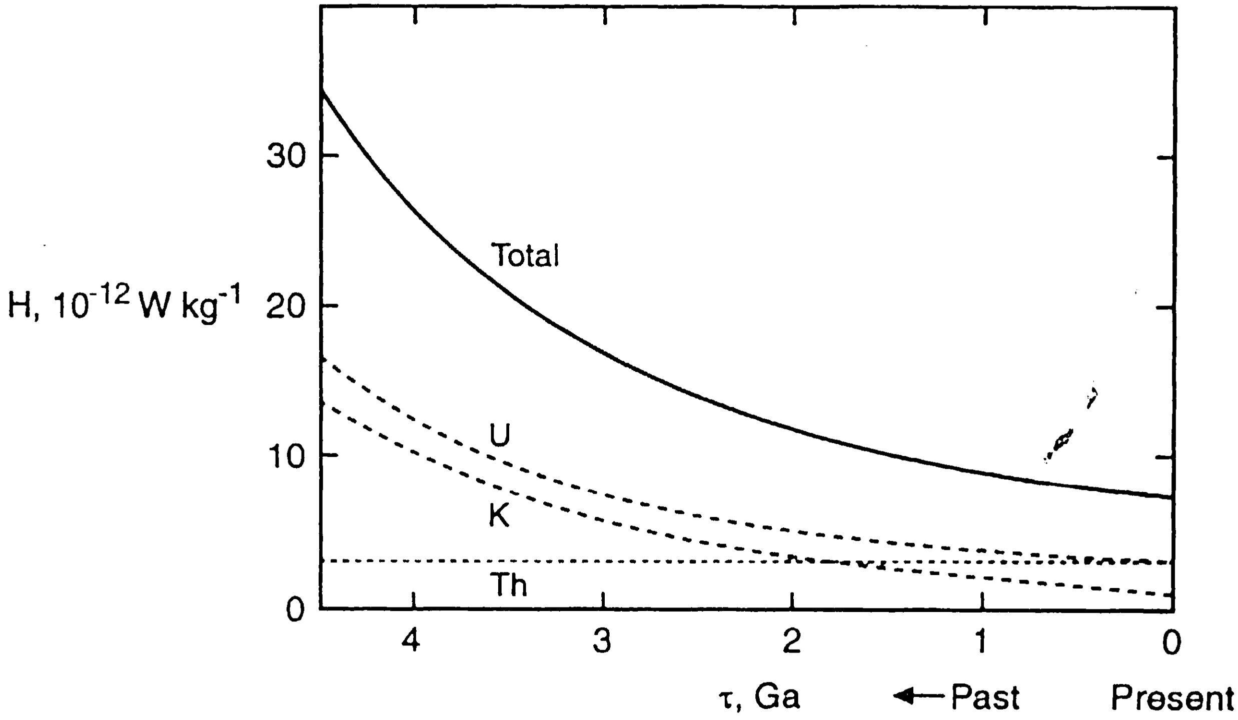


4–4 Mean mantle heat production rates due to the decay of the radioactive isotopes of U, Th, and K as functions of time measured back from the present.

plotted as a function of time before the present in Figure 4–4. The past contributions of the individual radioactive elements are also shown. We see that the rate of heat production 3×10^9 yr ago was about twice the

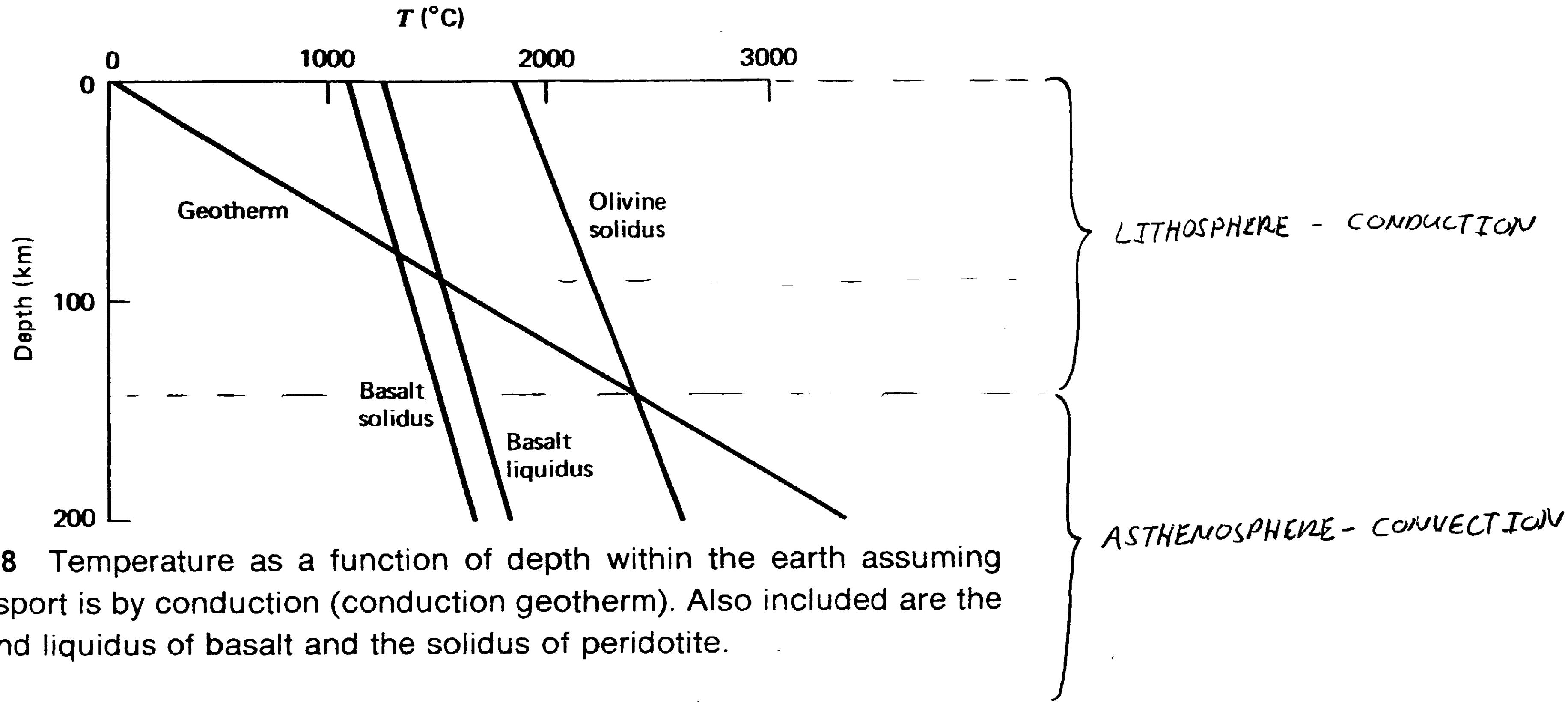


Figure 4-8 Temperature as a function of depth within the earth assuming heat transport is by conduction (conduction geotherm). Also included are the solidus and liquidus of basalt and the solidus of peridotite.

Conduction - Transfer of heat by molecular collisions. It is a diffusive process where heat flows from areas of higher T to lower T.

Convection - Transfer of heat by the motion of the rocks in the mantle.

Fourier's law of heat conduction:

(see also: Carslaw, H.S. and J.C. Jaeger, Conduction of Heat in Solids, Oxford, 1959)

$$q = -k \frac{\partial T}{\partial z}$$

q - heat flow per area W/m^2

k - thermal conductivity $W/m\,°K$ (material property)

T - temperature $°K$ or $°C$

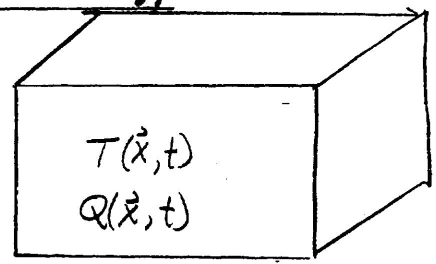
For most solids, the thermal conductivity is relatively insensitive to temperature and it is nearly isotropic.

$$\vec{q}(\vec{x}) = -k \nabla T$$

heat flows in the direction of decreasing temperature.

Using the law of energy conservation one can derive the equation for diffusion of heat in an isotropic solid.

Heat Equation - Conservation of Energy



$$-\nabla \cdot k \nabla T \quad + \quad \rho C_p \vec{V} \cdot \nabla T \quad + \quad \rho C_p \frac{\partial T(\vec{x},t)}{\partial t} \quad = \quad Q(\vec{x},t)$$

divergence heat flux	advection of heat	change in temp. with time	internal heat generation

k	-	thermal conductivity	$W/m\,^\circ K$
$Q(\vec{x},t)$	-	heat generation per volume	W/m^3
ρ	-	density	kg/m^3
C_p	-	heat capacity at constant P	$J/kg\,^\circ K$

$\kappa = \dfrac{k}{\rho C_p}$ - thermal diffusivity $\qquad \dfrac{m^2}{s}$

$H \equiv Q/\rho$ - heat generation per kg $\qquad W\,kg^{-1}$

divide through by ρC_p

$$\vec{V} \cdot \nabla T + \frac{\partial T}{\partial t} = \kappa \nabla^2 T + \frac{Q}{\rho C_p}$$

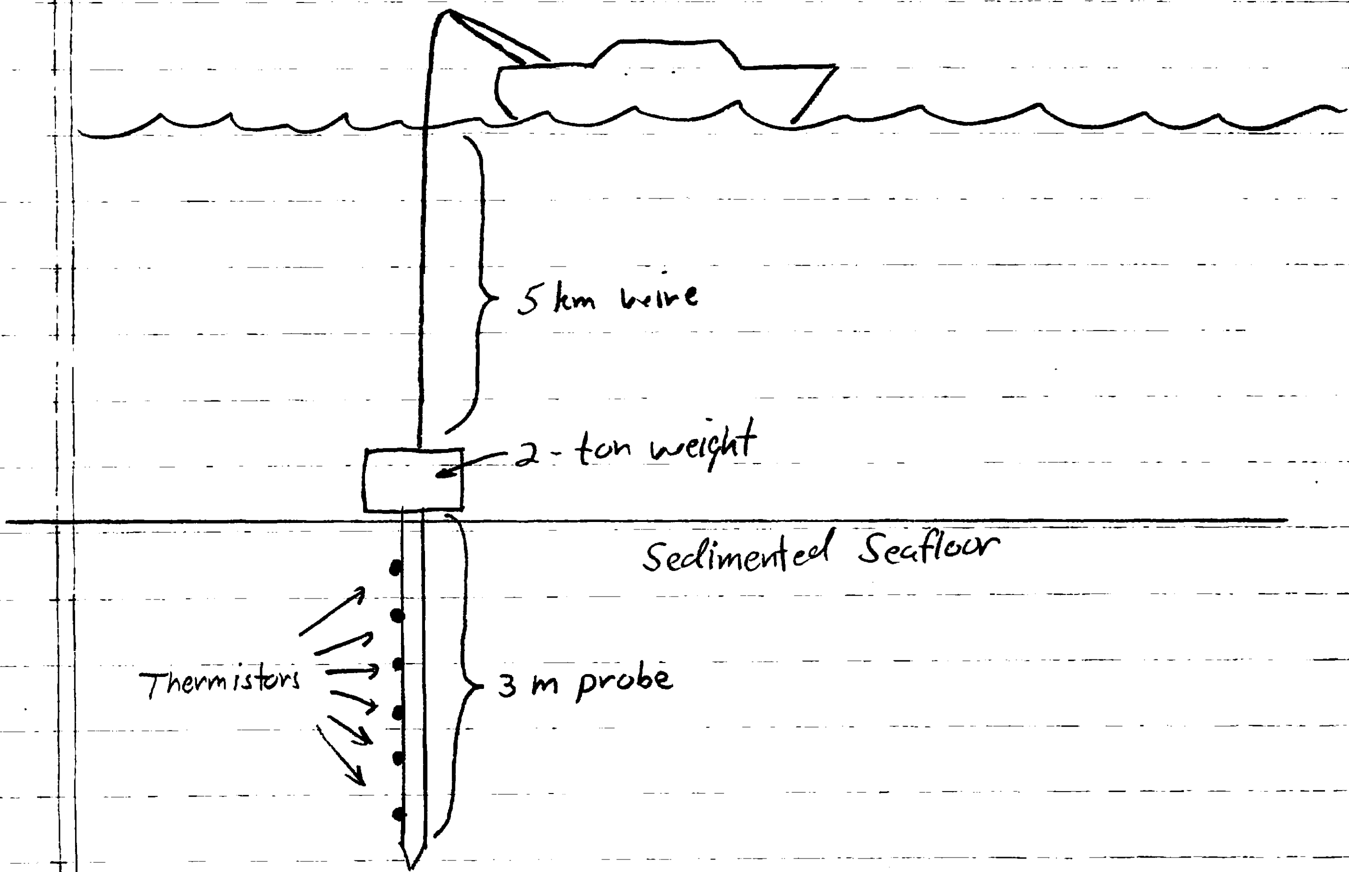
The thermal diffusivity of a solid is a measure of the amount of time it takes for a heat pulse to travel a distance l. $\quad \tau \equiv \dfrac{l^2}{\kappa}$.

Measurement of Heat Flow from the Earth:

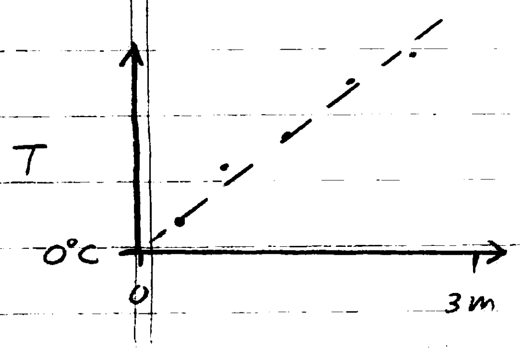
$$q_s = -k \frac{\partial T}{\partial z}$$

Need to measure surface temperature gradient and the surface thermal conductivity.

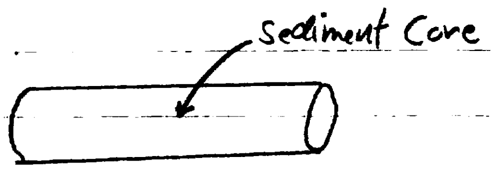
Ocean Measurements:



Temperature Gradient



Conductivity



measure conductivity in the lab.

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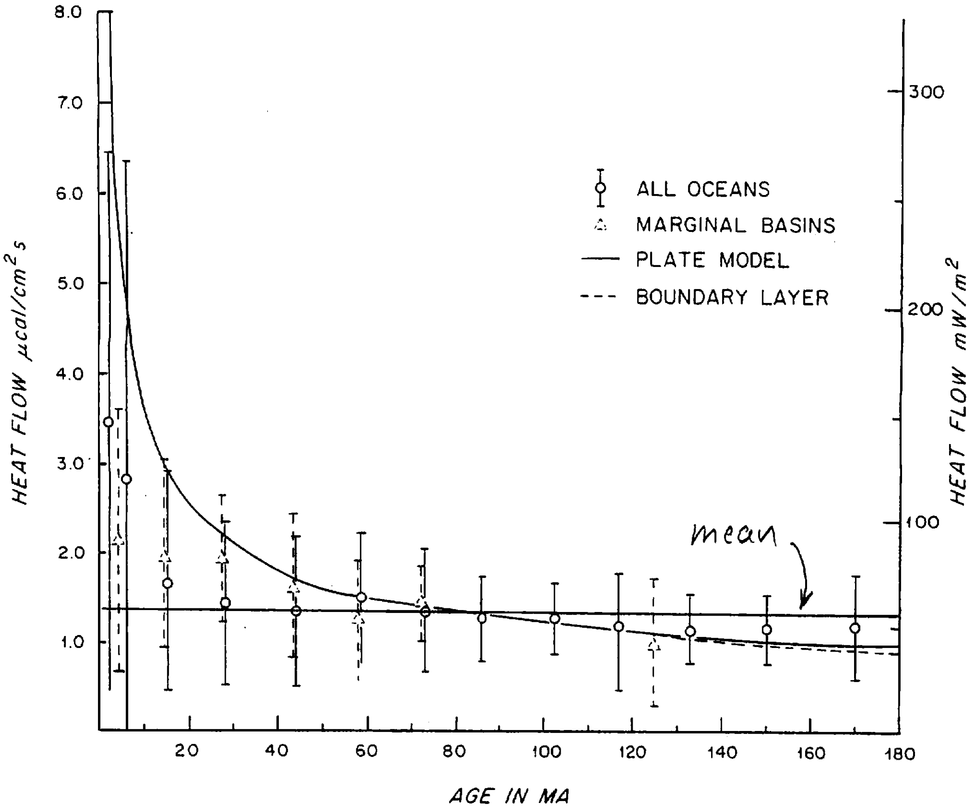


Fig. 4*a*. Mean heat flow and standard deviation for all the oceans and for the marginal basins as a function of age. Also shown are the expected heat flows from the plate and boundary layer models.

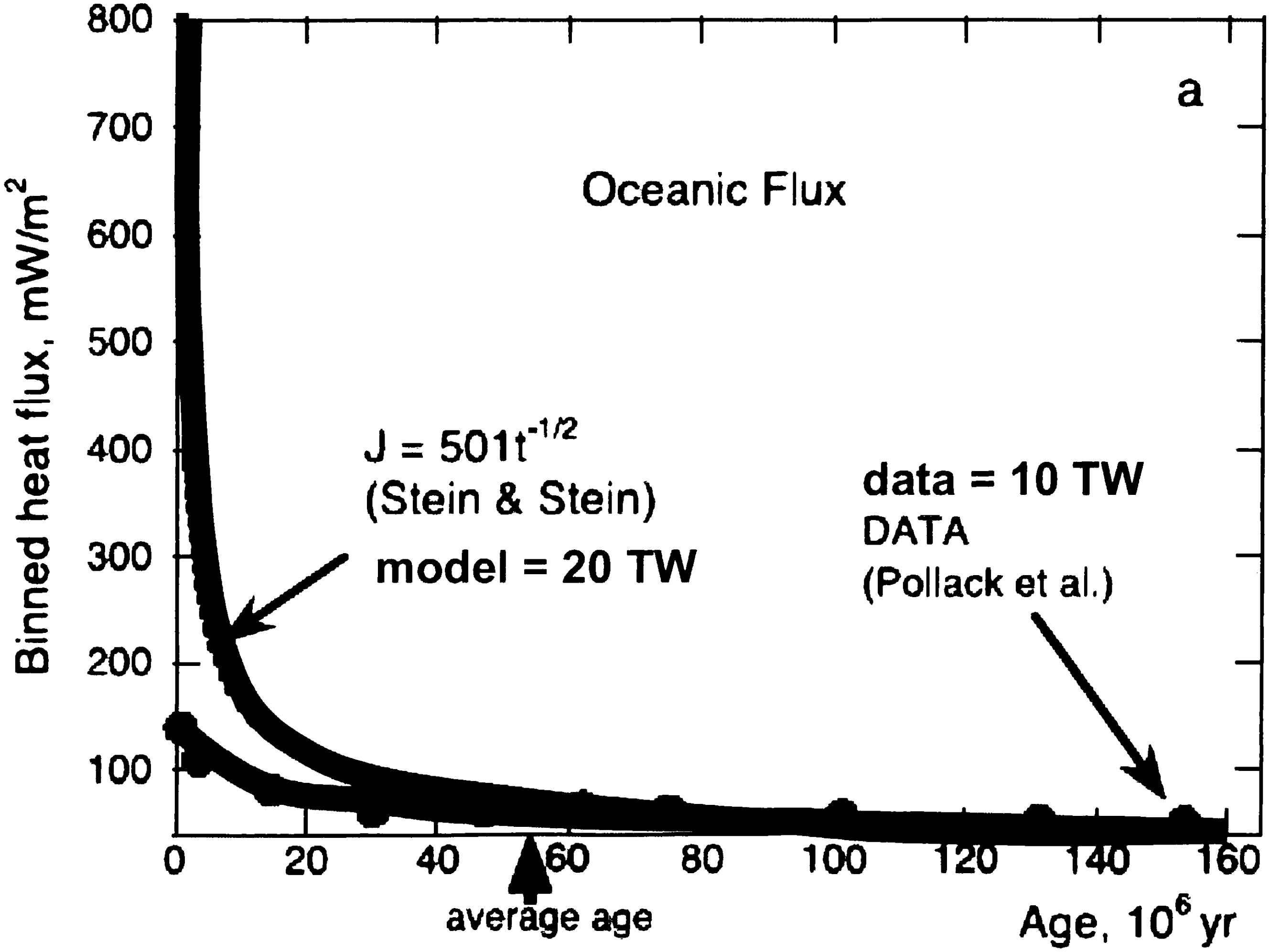
- Mean oceanic heat flow is ~ 66 mW/m^2.

- Heat flow decreases with increasing seafloor age.

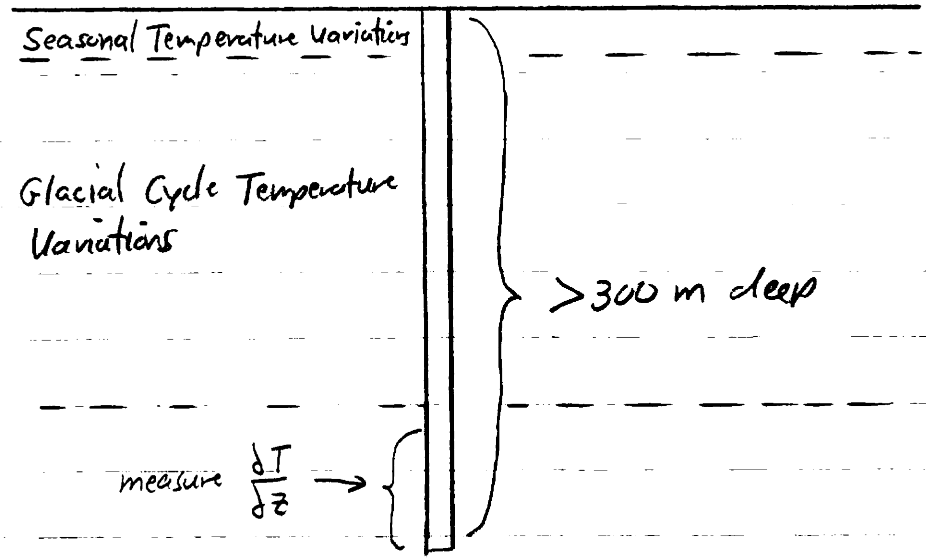
- For ages < 20 Ma the oceanic <u>conductive</u> heat flow is than expected!

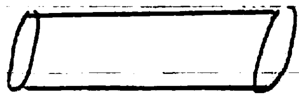
heat flow vs age ➡ $q(t) = k\frac{\partial T}{\partial z}$ ➡ $q(t) \cong 480t^{-1/2}$

A.M. Hofmeister, R.E. Criss / Tectonophysics 395 (2005) 159–177



Continent Measurements:

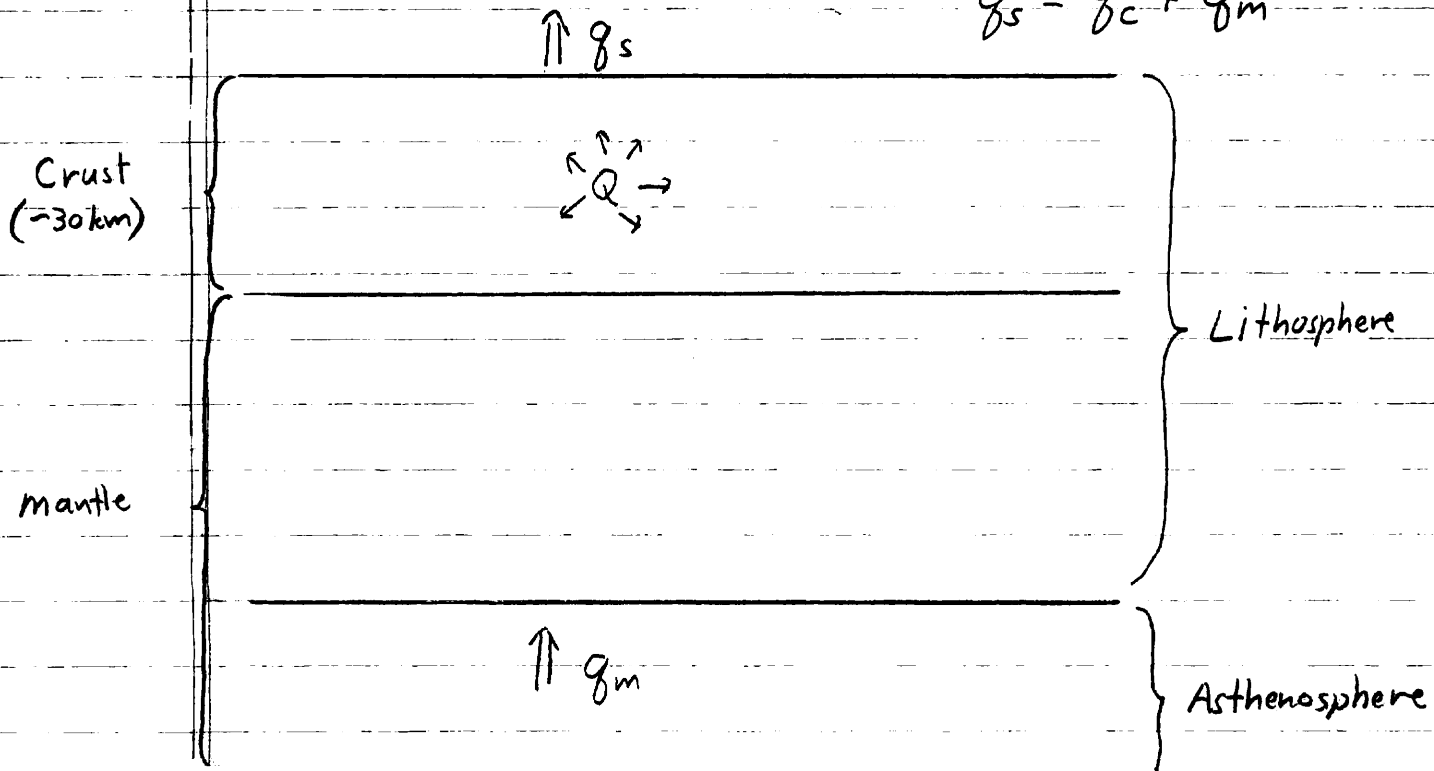


Measure conductivity of core in the lab: 

☆ on board

Continental crustal rocks contain higher concentrations of radioactive elements.

$$q_s = q_c + q_m$$



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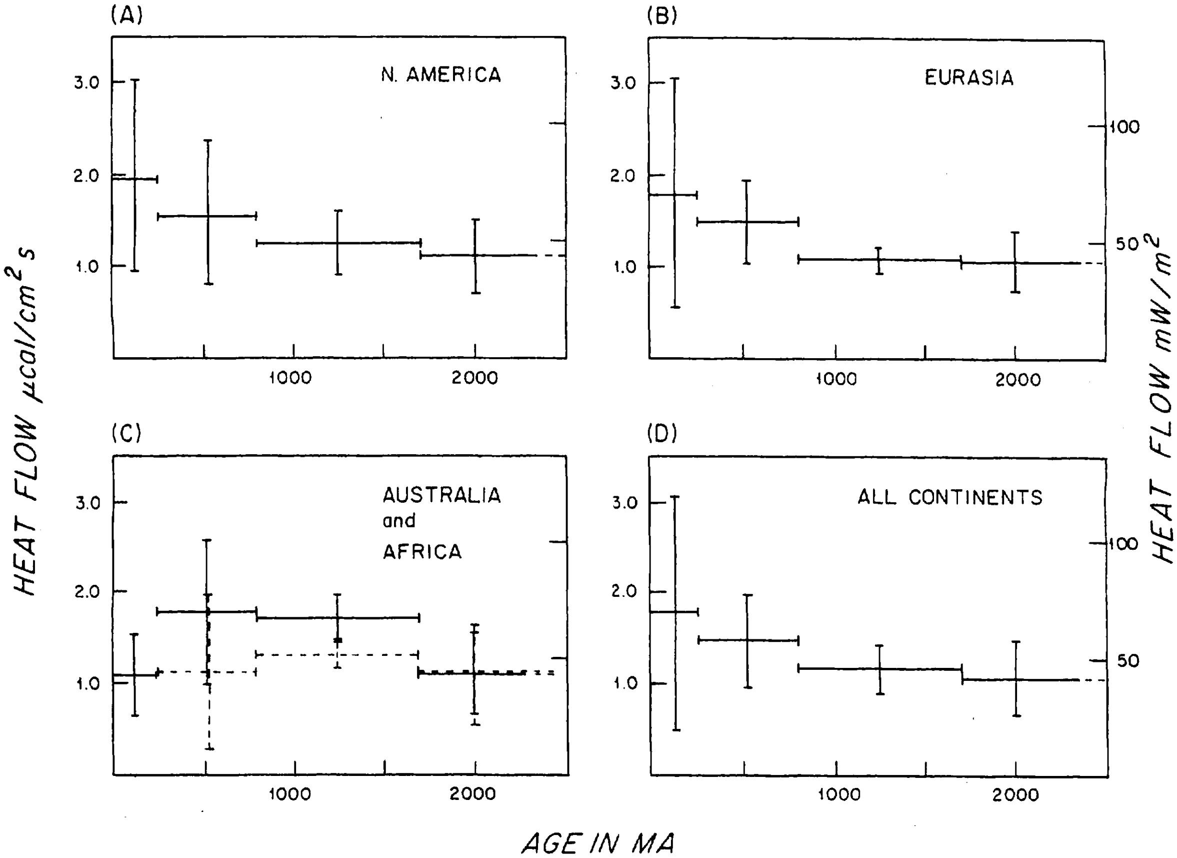


Fig. 16. Heat flow as a function of age for (*a*) North America, (*b*) Eurasia, (*c*) Australia and Africa (dashed lines), and (*d*) all continents.

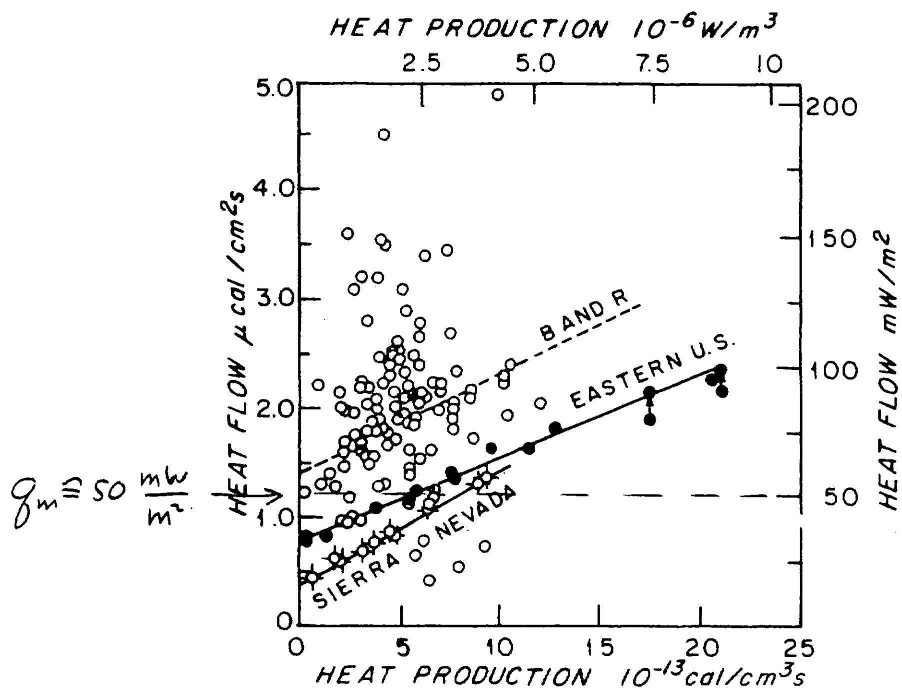


Fig. 17. Observations of heat flow and radioactive heat production from crystalline rocks in the United States; linear regression curves are from *Roy et al.* [1968] for the Basin and Range (dashed curve), eastern United States, and Sierra Nevada provinces. Solid circles represent points east of the Great Plains, and crossed circles represent points interior to the Sierra Nevada physiographic province.

Lets assume that the heat production of crustal rocks decreases exponentially with depth.

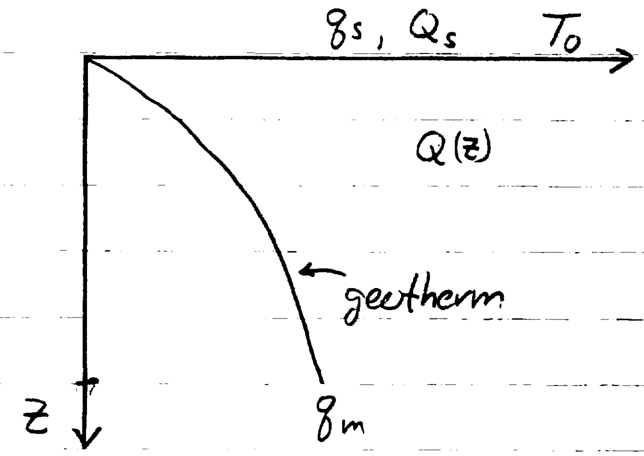
$$Q(z) = Q_s \, e^{-z/h_r}$$

- $Q(z)$ ↑ heat generation at depth
- Q_s ↑ measured surface heat generation
- e^{-z/h_r} ← assumed decay with depth

$$\boxed{\nabla^2 T = -\frac{Q}{k}}$$

What is the crustal contribution to the continental heat flow?
What does a typical continental geotherm look like?

Assume steady heat conduction.



$$\frac{\partial^2 T}{\partial z^2} = -\frac{Q_s}{k} e^{-z/h_r}$$

$$\left.\frac{\partial T}{\partial z}\right|_0 = \frac{q_s}{k}$$

$$\left.\frac{\partial T}{\partial z}\right|_\infty = \frac{q_m}{k}$$

$$\int_0^z e^{-\frac{\xi}{h_r}} d\xi = -h_r\left[e^{-\frac{z}{h_r}} - 1\right] = h_r\left[1 - e^{-\frac{z}{h_r}}\right]$$

Integrate Once: $-k\frac{\partial T}{\partial z} = q(z) = -Q_s h_r\left[1 - e^{-z/h_r}\right] + C_1$

Use boundary condition $q(\infty) = q_m$

Heat Flow: $q(z) = Q_s h_r e^{-z/h_r} + q_m$

Surface Heat Flow: $q_s = Q_s h_r + q_m$

- q_s ↑ surface heat flow
- $Q_s h_r$ ↑ surface heat production × effective thickness
- q_m ↑ mantle heat flow

Show Heat flow vs Heat Production Again

Show Global Heat Flow

h_r = slope of q_s vs Q_s plot ~ 10 km

$q_m = q_s - Q_s h_r$ - reduced heat flow

Theoretical Continental Geotherm:

Integrate again and use $T(0) = T_0$ boundary condition

$$T(z) = T_0 + \frac{q_m z}{k} + \frac{Q_s h_r^2}{k}\left[1 - e^{-z/h_r}\right]$$

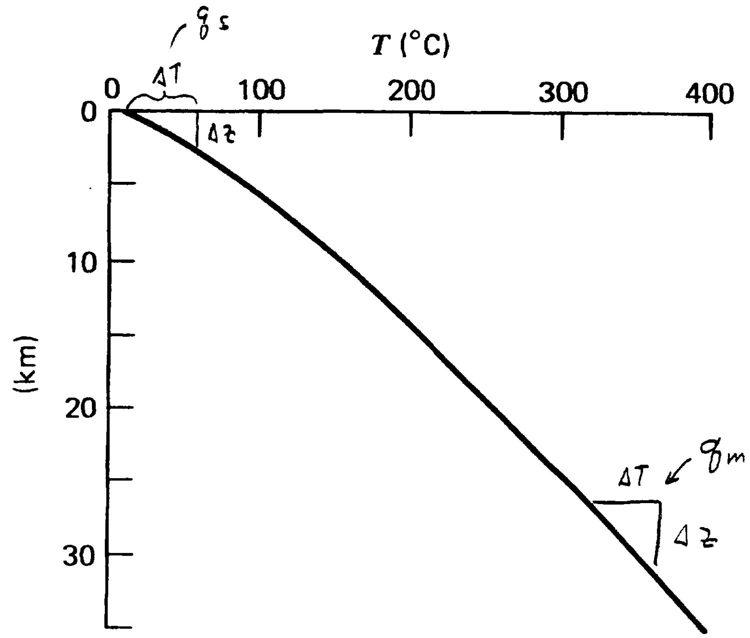


Figure 4-12 A typical geotherm in the continental crust.

TABLE 12. Summary of Area and Heat Loss Information

	Area, 10^6 km^2	Heat Loss, 10^{10} cal/s (10^{12} W)	
Continents (including effect of volcanoes)	149.3	208 (8.8)	
Continental shelves	52.2	67 (2.8)	
Total (continent plus shelves)	201.5	275 (11.5)	
Deep oceans	281.7	656 (27.4)	
Marginal basins	26.9	71 (3.0)	
Total (oceans plus marginal basins)	308.6	727 (30.4)	
Worldwide values	510.1	1002 (42.0)	← TOTAL LOSS
Heat loss by water flow and conduction through bare rock		241 (10.1)	
Heat loss by plate creation not including radiogenic heat loss		626 (26.2)	
Mean heat flow			
Continents and shelves		1.37* (57)	Average Heat Flow
All oceans (observed)		1.57* (66)	
All oceans (theoretical)		2.36* (99)	

*Heat flow in 10^{-6} cal/cm^2 s (mW/m^2).

Heat Flow Considerations

(from Jerry Schubert's lecture at IGPP, Fall, 2000)

- Heat flow from the Earth is 44 TW.

- Heat flow from radiogenic heat production in the continents is 7.5 TW.

  This is uncertain since we do not really know the amount of U, Th, and K in the continents, nor do we know the heat flow from the mantle into the base of the continental lithosphere.

- Heat flow from the mantle is 36.5 TW =

  Heat from the core +

  Heat from mantle cooling +

  Heat from radioactivity.

Heat Flow From the Mantle 36.5 TW

(from Jerry Schubert's lecture at IGPP, Fall, 2000)

Heat flow from core: 2.4 TW [Sleep, Davies]

13.4 TW [Malamud & Turcotte]

Guy masters says this much is needed to run the dynamo

Urey Number = (radiogenic heat)/(~~mantle cooling~~ total heat)

Heat flow from mantle cooling

if Ur = 0.75 9.1 TW

Heat produces by radiogenic sources in the mantle is 14 - 25 TW?

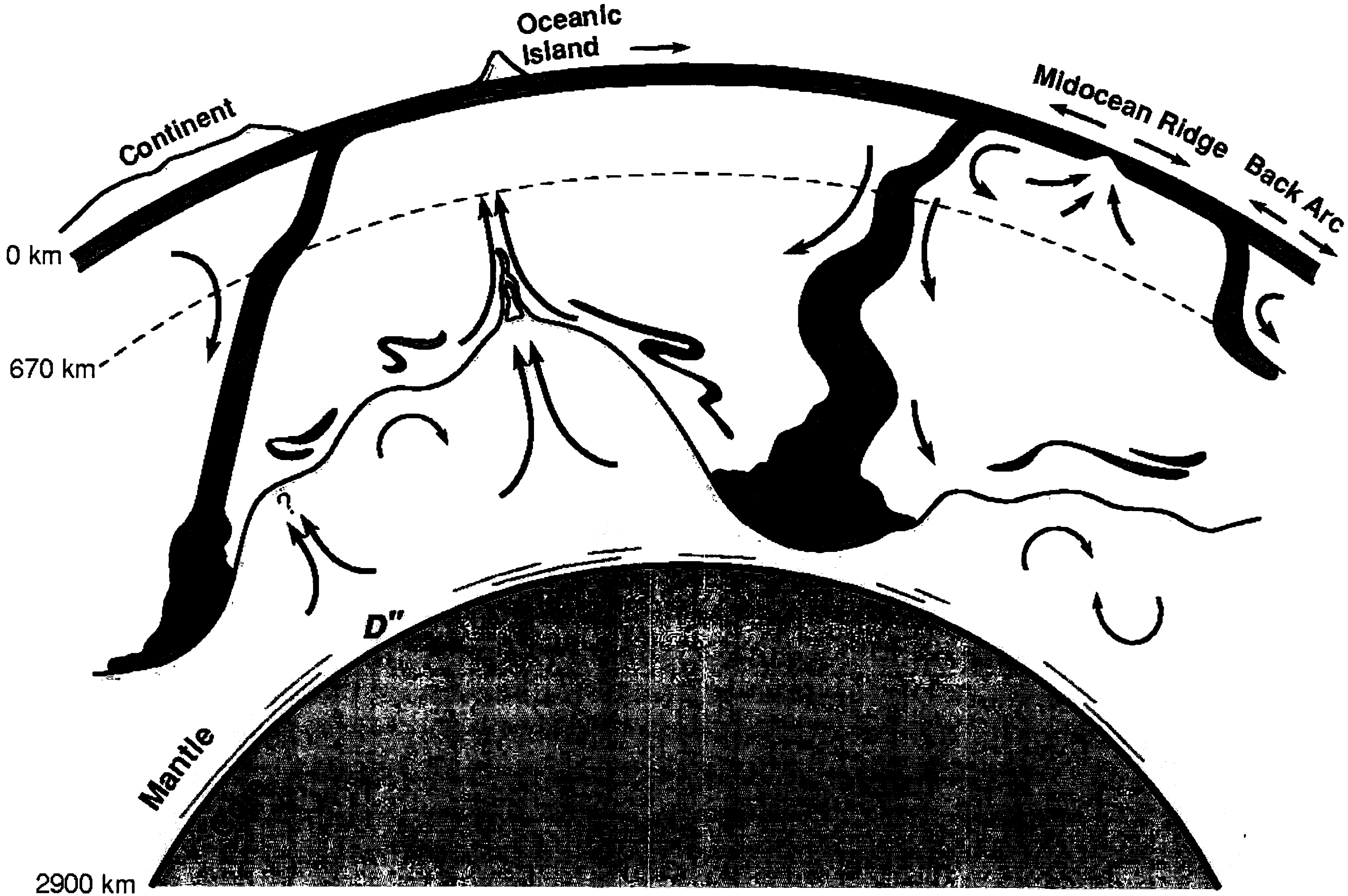
Heat Flow Considerations

(From Jerry Schubert's lecture at IGPP, Fall, 2000)

- Kellogg et al., [1999] have argued that the MORB source region is so depleted in radioactivity (heat production) that were it identical to the entire mantle, it could not supply the heat observed to be escaping through the earth's surface.

- According to this argument there must be an isolated region of the mantle with a higher concentration of Ur, Th, and K compared to the MORB source region.

- Kellogg et al., [1999] hypothesize that this region is a compositionally distinct (undepleted, rich in radiogenic elements, ^{3}He and ^{40}Ar), variable thickness, separately convecting layer buried at the bottom of the lower mantle. However, the constraints on mantle radioactivity are not so certain.


Oceanic
Island
Continent
Midocean Ridge
Back Arc
0 km
670 km
D"
Mantle
2900 km

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