COOLING OF THE OCEANIC LITHOSPHERE AND OCEAN FLOOR TOPOGRAPHY

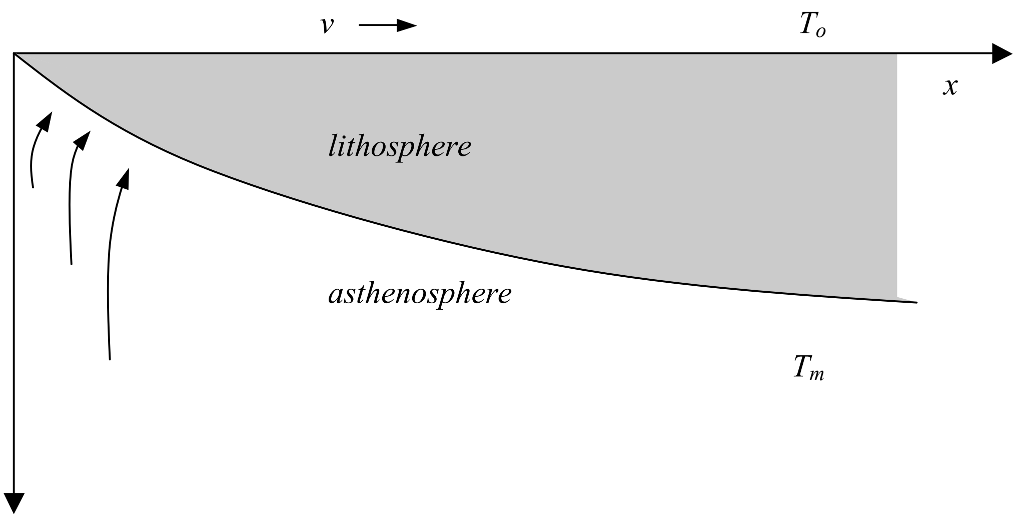
Introduction

This lecture is the development of the lithospheric cooling problem. For researchers in the areas of marine geology, marine geophysics, and geodynamics, this is the most important concept you can learn from this class. As noted in the original paper on the topic by *Turcotte and Oxburgh* [1967], convection of the mantle is primarily controlled by thin thermal boundary layers. The surface thermal boundary layer, or *oceanic lithosphere* is, the most important component of the convecting system because it represents the greatest temperature gradient in the earth and it also has a greater surface area than the second most important thermal boundary layer which is at the core-mantle boundary. As the lithosphere cools it becomes denser, the seafloor depth increases and ultimately the lithosphere founders (*subduction*). This subduction process both drives the convective flow and efficiently quenches the mantle.

These notes cover the same material as sections 4-15 and 4-16 in *Turcotte and Schubert* [1982] (pages 153-161). The main difference is the method of solution. Turcotte and Schubert solve the half-space cooling problem by guessing a similarity variable and then using this variable to reduce the time-dependent heat conduction equation from a partial differential equation to an ordinary differential equation that can be solved by integration. These notes provide an alternate solution to the problem by using the tools of fourier analysis. Basically, any type of heat conduction problem can be solved with the fourier approach [*Carslaw and Jaeger*, 1959]. This fourier approach is more than just a new way to solve an old problem. Many 3-D heat conduction problems, with complicated sources and boundary conditions, do not have complete analytic solutions but do have solutions in the fourier transform domain. In these cases, the FFT algorithms, coupled with modern computers, can be used to compute accurate results in seconds. Resorting to finite difference or other numerical schemes is error prone and the results are more difficult to interpret since the analytic foundation is gone. Thus the fourier approach is worth learning.

The basic model is shown in the following diagram that represents one half of a seafloor spreading system. The model assumptions and consequences are:

- lithospheric plates are rigid and move away from the spreading ridge axis at a uniform rate of v;
- hot, low-viscosity asthenosphere fills the void (passive);
- internal heat generation is much smaller than the other terms in the heat equation so it is neglected;
- there is a singular point at $x = z = 0$. (We'll let the "ridge scientists" deal with this issue.)



This is a 2-dimensional problem with no heat sources so the heat equation has only diffusive and advective terms

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial z^2} = \frac{v}{\kappa}\frac{\partial T}{\partial x} \tag{1}$$

where T is temperature and κ is the thermal diffusivity. The first term represents the lateral diffusion of heat, the second term represents the vertical diffusion of heat, and the third term (on the right side) is the advection of heat by the motion of the plate. Away from the ridge axis ($x >> 0$), one can show that the lateral heat diffusion is much smaller than the vertical heat diffusion. Dropping this term simplifies the differential equation although a solution can also be developed where the term is retained [ref]. Next we move from an Eulerian coordinate system to a Lagrangian system.

$$v = \frac{\partial x}{\partial t} \Rightarrow \frac{\partial T}{\partial x}\frac{\partial x}{\partial t} = \frac{\partial T}{\partial t} \qquad (2)$$

This reduces the problem to the half-space cooling problem.

$$\frac{\partial^2 T}{\partial z^2} = \frac{1}{\kappa}\frac{\partial T}{\partial t} \qquad (3)$$

The boundary and initial conditions are:

$$\begin{aligned} T(0,t) &= T_o \\ T(\infty,t) &= T_m \\ T(z,0) &= T_m \end{aligned} \qquad (4)$$

The infinite half-space has constant thermal diffusivity and an initially constant temperature T_m. At times greater than zero, the surface temperature is reduced to T_o. The temperature will evolve with time. Note for this problem, time also corresponds to the age of the cooling oceanic lithosphere. Define a dimentionless temperature as.

$$\theta = \frac{T - T_o}{T_m - T_o} \qquad (5)$$

Now the differential equation and boundary conditions become

$$\frac{\partial^2 \theta}{\partial z^2} = \frac{1}{\kappa}\frac{\partial \theta}{\partial t} \qquad (6)$$

$$\begin{aligned} \theta(0,t) &= 0 \\ \theta(\infty,t) &= 1 \\ \theta(z,0) &= 1 \end{aligned}$$

Turcotte and Schubert [2002, p. 154] introduce the following dimensionless quantity and use this to reduce this to an ordinary differential equation with two boundary conditions. They then integrate the differential equation twice and match the boundary conditions.

$$\eta = \frac{z}{2\sqrt{\kappa t}} \tag{7}$$

Suppose one did not know this trick or the problem was more complicated. An approach called *method of images* is straightforward. The model is expanded to a full-space with an initial step-function temperature distribution so the 0-temperature boundary condition is always matched. The problem becomes

$$\frac{\partial^2 \theta}{\partial z^2} = \frac{1}{\kappa}\frac{\partial \theta}{\partial t} \tag{8}$$

$$\theta(\infty,t) = 1$$
$$\theta(z,0) = 2H(z) - 1$$

where the definition of the step function is

$$H(z) \equiv \int_{-\infty}^{z} \delta(z)dz \tag{9}$$

Now take the fourier transform of (8) with respect to *z*. The differential equation becomes.

$$-\kappa(2\pi k)^2\,\Theta(k,t) = \frac{\partial \Theta}{\partial t} \tag{10}$$

The general solution is

$$\Theta(k,t) = C_o e^{-\kappa(2\pi k)^2 t} \tag{11}$$

Now take the fourier transform of the initial condition.

$$\Im[\Theta(k,0)] = \Im[2H(z)] - \Im[1] \tag{12}$$

We know that

$$\Im[1] = \delta(k). \tag{13}$$

Also using the derivative property we know that

$$\Im\left[\frac{\partial H}{\partial z}\right] = i2\pi k\Im[H(z)]. \tag{14}$$

Since the derivative of the step function is a delta function, the fourier transform of the initial condition is

$$\Theta(k,t) = \frac{1}{i\pi k} - \delta(k). \tag{15}$$

The solution that satisfies the initial condition is

$$\Theta(k,t) = \left[\frac{1}{i\pi k} - \delta(k)\right]e^{-\kappa(2\pi k)^2 t}. \tag{16}$$

Now we take the inverse fourier transform

$$\theta(z,t) = \int_{-\infty}^{\infty} \frac{e^{-\kappa(2\pi k)^2 t}}{i\pi k} e^{i2\pi kz} dk - \int_{-\infty}^{\infty} \delta(k) e^{-\kappa(2\pi k)^2 t} e^{i2\pi kz} dk \tag{17}$$

The second integral on the right side of (17) is equal to 1 since the delta function extracts the integrand at $k = 0$. The first integral on the right side of (17) is performed in two steps. First take the derivative with respect to z to note that

$$\frac{\partial\theta(z,t)}{\partial z} = 2\int_{-\infty}^{\infty} e^{-\kappa(2\pi k)^2 t} e^{i2\pi kz} dk \qquad (18)$$

This is the fourier transform of a Gaussian function. The following substitution puts the integral in the form that appears in *Bracewell* [1978].

$$k' = k\sqrt{4\pi\kappa t} \qquad \text{and} \qquad z' = \frac{z}{\sqrt{4\pi\kappa t}} \qquad (19)$$

The result is

$$\frac{\partial\theta(z,t)}{\partial z} = \frac{2}{\sqrt{4\pi\kappa t}} e^{\frac{-z^2}{4\kappa t}} \qquad (20)$$

Next integrate (20) over z. The introduction of the similarity variable based on equation (20) helps to identify the integral as the definition of the error function.

$$\eta = \frac{z}{2\sqrt{\kappa t}} \qquad \text{so} \qquad dz = 2\sqrt{\kappa t}\, d\eta \qquad (21)$$

The integral becomes

$$\theta(z,t) = \frac{2}{\sqrt{\pi}} \int_{-\infty}^{\eta} e^{-\eta^2} d\eta - 1 \qquad (22)$$

The right side of (22) is just the definition of the error function *erf(η)*. The final solution is

$$T(z,t) = (T_m - T_o)\, erf\left(\frac{z}{2\sqrt{\kappa t}}\right) + T_o \qquad (23)$$

Temperature versus depth and age

The thermal parameters and temperatures appropriate to the earth are given in the following table.

Parameter	Definition	Value
T_o	surface temperature	0˚C
T_l	temp. at base of thermal boundary later	1100˚C
T_m	mantle temperature	1300˚C
κ	thermal diffusivity	8×10^{-7} m^2 s^{-1}
k	thermal conductivity	3.3 W m^{-1} C^{-1}

If we define the base of the thermal boundary layer as some large fraction of the deep mantle temperature as in the table, one can calculate the thickness of the thermal boundary layer versus the age of the lithosphere.

$$\frac{T_l - T_o}{T_m - T_o} = 0.84 = erf\left(\frac{z}{2\sqrt{\kappa t}}\right) \tag{24}$$

or

$$z \cong 2\sqrt{\kappa t} \qquad \text{or} \qquad z(km) \cong 10\sqrt{age(Ma)} \tag{25}$$

The isotherms for this model are displayed on a following page.

Heat flow versus depth and age

The heat flow is the thermal conductivity times the temperature gradient.

$$q(z) = k\frac{\partial T}{\partial z} \tag{26}$$

To calculate the heat flow we take the derivative of the error function with respect to *z*.

$$\frac{\partial erf(\eta)}{\partial z} = \frac{\partial erf(\eta)}{\partial \eta}\frac{\partial \eta}{\partial z} = \frac{1}{\sqrt{\pi\kappa t}}e^{-\eta^2} \tag{27}$$

$$q(z,t) = \frac{k(T_m - T_o)}{\sqrt{\pi\kappa t}}e^{\frac{-z^2}{4\kappa t}} \tag{28}$$

In the limit as depth *z* goes to infinity, the heat flow is zero. So for this model, there is no heat transport into the base of the lithosphere. Later we'll compute seafloor depth versus age for this model and show that there are large deviations at old age (i.e. > 70 Ma). One way to flatten the depth versus age curve is to supply heat to the base of the lithosphere. There are a variety of ways to accomplish this:

- increasing basal heat flux with age corresponds to the plate cooling model *of Parsons and Sclater* [1977]. The physical mechanism for this basal heat input is small-scale convective rolls beneath old lithosphere.
- a constant basal heat flux with age corresponds to the CHABLIS cooling model of *Fletout and Dion* [1996]
- some papers (e.g., *Crough,* 1983) propose that mantle plumes re-heat the old lithosphere and eventually all old lithosphere encounters one or more plumes so re-heating is pervasive.

The surface heat flow is just equation (28) evaluated at the surface of the earth.

$$q(t) = \frac{k(T_m - T_o)}{\sqrt{\pi \kappa t}} \qquad (29)$$

The match to the observed heat flow is shown on the following page. For ages less than about 40 Ma, the surface heat flux is less than predicted by the model. This heat flow deficit occurs because cold seawater circulates deep into the crust and advects the heat so the temperature gradient will be less than predicted by a purely conductive model. At older ages, the heat flow is higher than expected. This could either be due to a non-zero basal heat flux or an incorrect estimate of thermal conductivity of the crust.

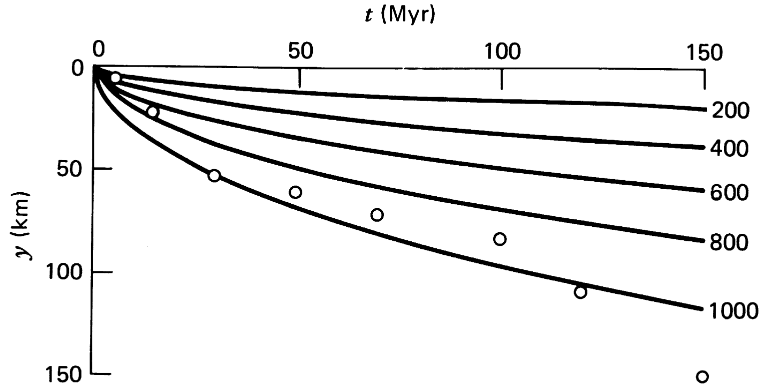


Figure 4-24 The solid lines are isotherms, $T - T_s$ (°K), in the oceanic lithosphere from Equation (4-125). The data points are the thicknesses of the oceanic lithosphere in the Pacific determined from studies of Rayleigh wave dispersion data. (From A. R. Leeds, L. Knopoff, and E. G. Kausel, Variations of upper mantle structure under the Pacific Ocean, *Science*, **186**, 141–143, 1974.)

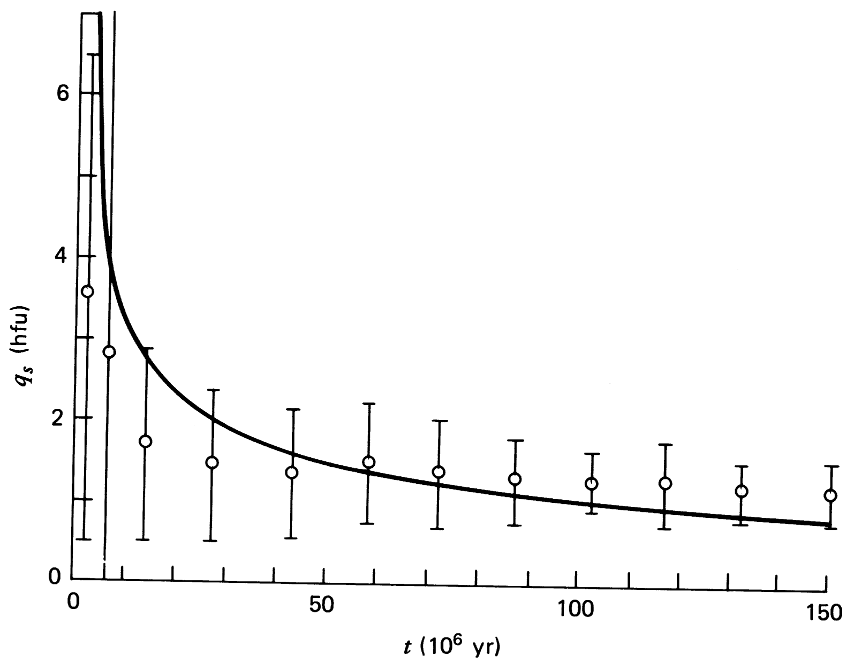
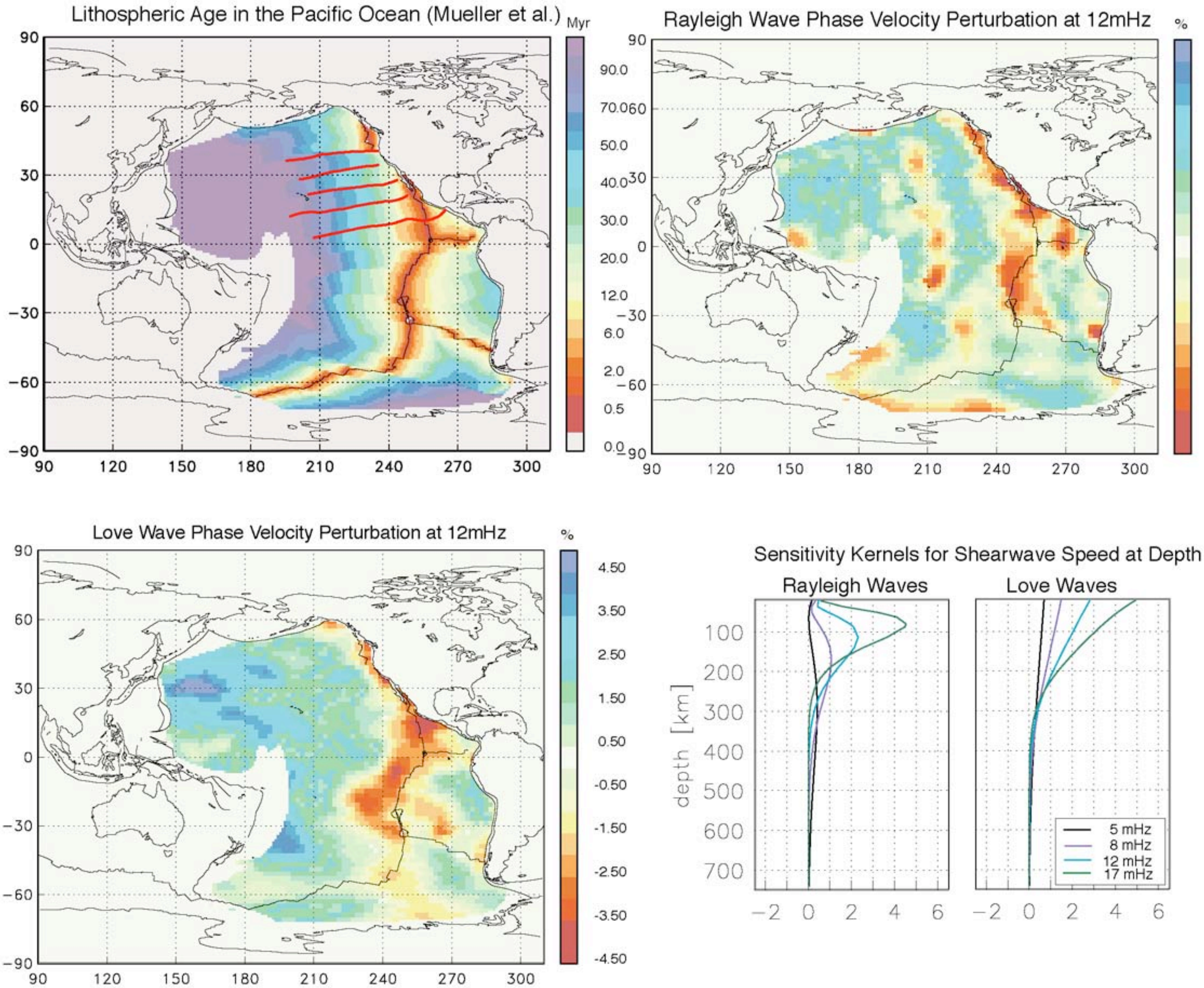
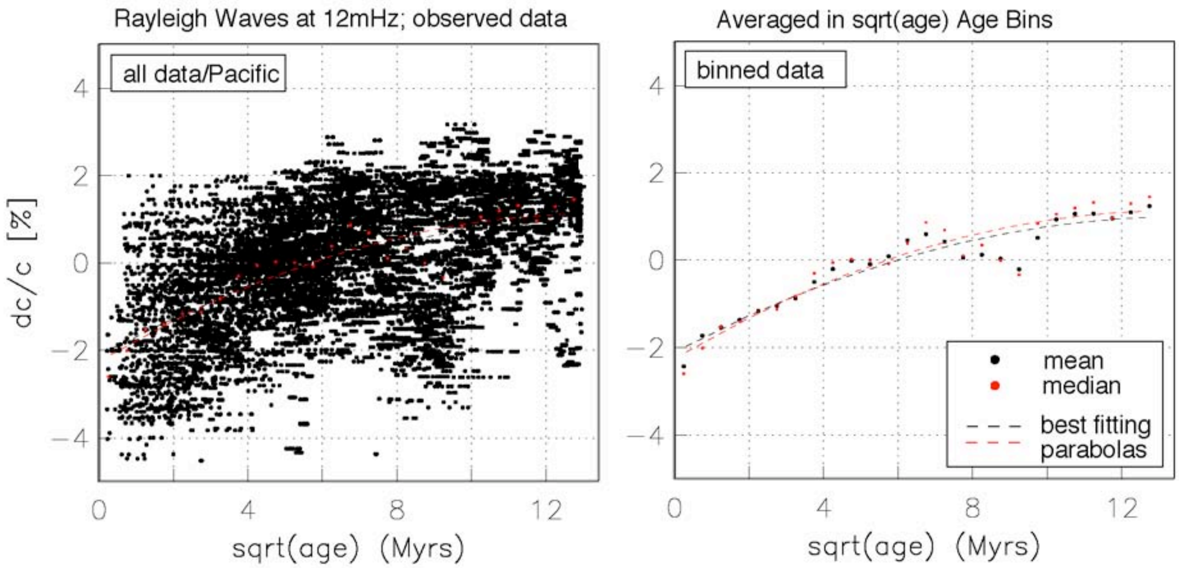


Figure 4-25 Mean values and standard deviations of ocean floor heat flow measurements as functions of age compared with Equation (4-127). Data from J. G. Sclater, C. Jaupart, and D. Galson, The heat flow through oceanic and continental crust and the heat loss of the Earth, *Reviews of Geophys. and Space Physics*, **18**, 269–311, 1980.

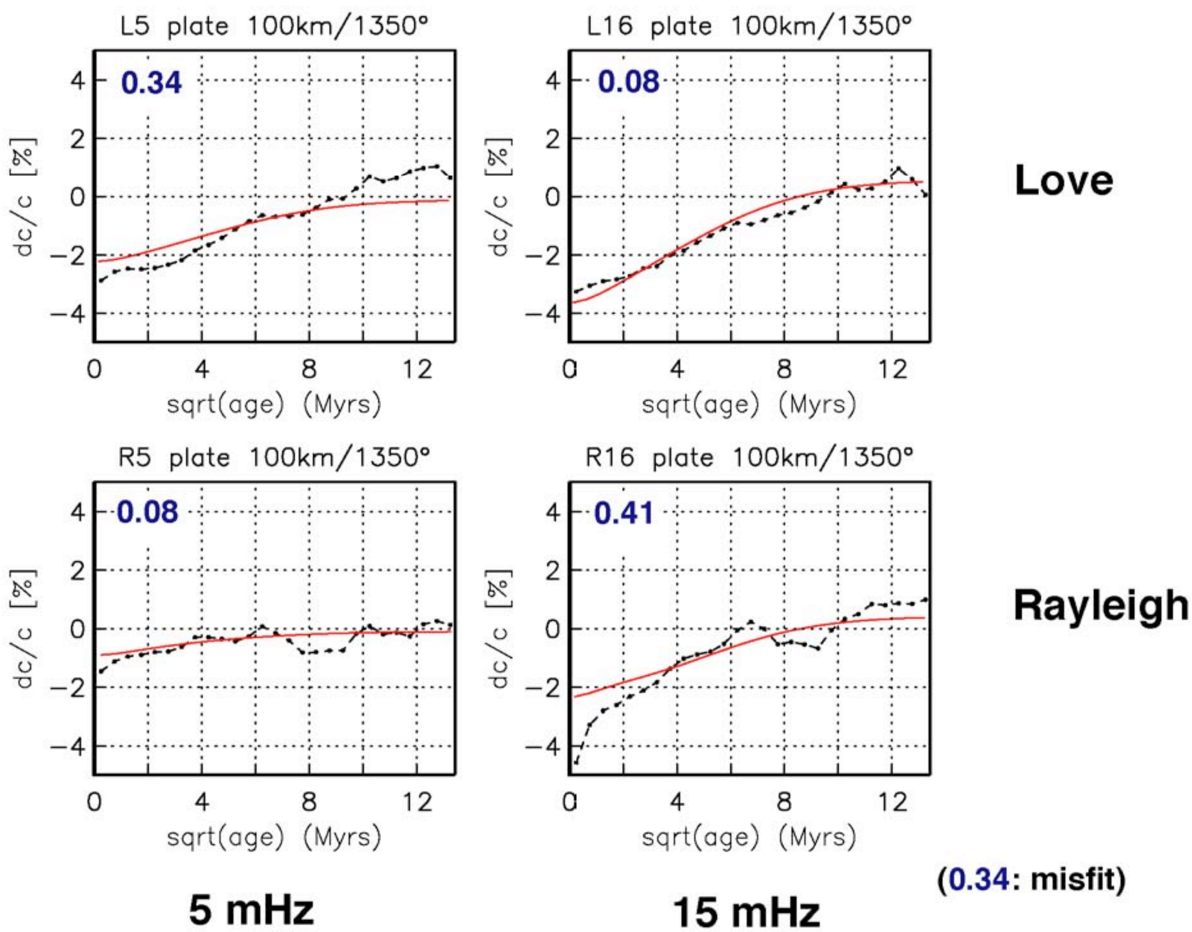


Hogg, C. and G. Laske, The cooling oceanic lithosphere as constrained by surface wave dispersion data, SOS Trans. AGU, Fall AGU Meeting, December, 2003.

Observed Data (raw and binned)



Fit of a cooling plate (100km thick, basal T=1350ºC)



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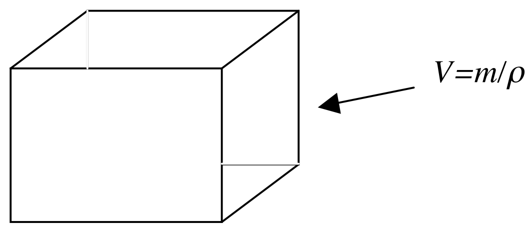
Thermal Subsidence

As the oceanic lithosphere cools by conductive heat loss, it contracts. This thermal contraction causes the average density of the lithosphere to increase. The seafloor depth increases with age and eventually the lithosphere becomes so dense it founders at a subduction zone. To develop a linear relationship between density and temperature, consider a cube of volume V, mass m, and density ρ, at temperature T_o under a confining pressure P_o.

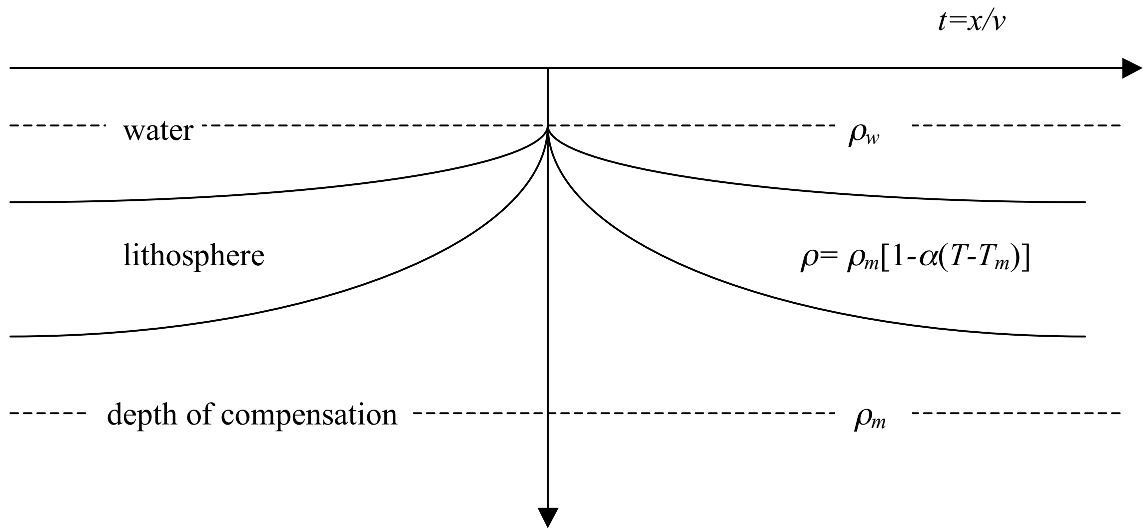


Changes in both temperature and pressure will produce changes in the volume of the cube.

$$dV = \left(\frac{\partial V}{\partial T}\right)_{P_o} dT + \left(\frac{\partial V}{\partial P}\right)_{T_o} dP \qquad (30)$$

The two terns in (30) are related to the volumetric coefficient of thermal expansion $\alpha = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_{P_o}$ and the isothermal compressibility is $\beta = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_{T_o}$. Since $\rho = mV^{-1}$ it is easy to show that $\frac{\partial \rho}{\rho} = -\frac{\partial V}{V}$ so the coefficient of thermal expansion becomes $\alpha = -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_{P_o}$. We are considering the lithosphere that slides laterally across the surface of the earth, so there are no significant pressure variations. Thus we need only the first term in (30). If ρ_m is the density of the lithosphere at a temperature of T_m, then a reduction in temperature will cause an increase in density.

$$\rho(T) = \rho_m\left[1 - \alpha\left(T - T_m\right)\right] \qquad (31)$$



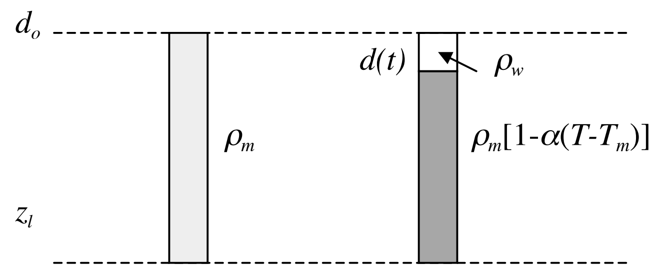
The diagram illustrates the thermal subsidence of the oceanic lithosphere as it spreads from the ridge axis at a velocity of v. There are three layers in the model. The ocean has a density of ρ_w and a depth of d_0 at the ridge axis. This depth increases with age/distance from the ridge axis. We will use the principles of thermal contraction and isostasy to determine the increase in seafloor depth with increasing age $d(t)$. The density of the lithosphere depends on temperature according to equation (31). The asthenosphere behaves as a fluid on geological timescales so the lithosphere floats on the mantle.

The major assumptions are:

- The pressure at the depth of compensation is a constant value and depends only on the weight of the rock and water directly above (i.e., isostatic equilibrium).
- The crust has uniform thickness so it has no effect on the overall isostatic balance.
- The thermal diffusivity, κ is isotropic and independent of P and T.
- The thermal expansion coefficient α is isotropic and independent of P and T.
- Heat is transferred by conduction so hydrothermal circulation is not important. This is a poor assumption at the ridge axis.
- Heat conducts only vertically. This is also a poor assumption at the ridge axis.
- There are no heat sources in the crust or lithosphere.
- No heat flows into the base of the lithosphere See *Doin and Fleitout* [EPSL, 1996] for s discussion of alternate models with basal heat input.

An additional assumption is that the lithosphere is free to contract in all three dimensions. Since the lithosphere is thin in relation to its horizontal dimension, free contraction in the vertical dimension is a good assumption. Contraction of the plate in the direction perpendicular to the ridge axis is probably valid as well. However, contraction in the ridge-parallel direction will produce significant shear strain, which will result in thermoelastic stress. We will neglect this for now but this is an interesting area of research.

As the lithosphere cools and contracts, its vertically-integrated density increases which will increase the pressure at its base. To maintain isostatic balance (i.e., constant pressure at constant depth z_l), ocean depth must increase to replace high density rock with lower density water. The increase in depth is determined by the following isostatic balance between a ridge-axis column and an off-axis column.



The mathematical statement of isostatic balance is

$$g\int_o^{z_l} \rho_m dz = g\int_o^{d} \rho_w dz + g\int_d^{z_l} \rho_m\left[1-\alpha\left(T-T_m\right)\right]dz \qquad (32)$$

where g is the acceleration of gravity.

After subtracting the standard ridge-axis column from both sides and dividing through by g we get.

$$0 = \int_o^{d} \left(\rho_w - \rho_m\right)dz - \int_d^{z_l} \rho_m \alpha\left(T - T_m\right)dz. \qquad (33)$$

Now we'll use the solution to the half-space cooling problem (equation 23) to define $T(t,z)$. Note this solution has temperature perturbations at infinite depth so we must extend the depth integration from the seafloor to infinity.

$$d(\rho_m - \rho_w) = \rho_m \alpha (T_m - T_o) \int_d^{\infty} 1 - erf\left(\frac{z-d}{2\sqrt{\kappa t}}\right) dz \qquad (34)$$

By setting $z'=z-d$, and solving for $d(t)$ we find

$$d(t) = \frac{\rho_m \alpha (T_m - T_o)}{(\rho_m - \rho_w)} \int_o^{\infty} erfc\left(\frac{z}{2\sqrt{\kappa t}}\right) dz \qquad (35)$$

To integrate this function let $\eta = \frac{z}{2\sqrt{\kappa t}}$ so $dz = 2\sqrt{\kappa t}\ d\eta$

$$d(t) = \frac{2\rho_m \alpha (T_m - T_o)}{(\rho_m - \rho_w)} \sqrt{\kappa t} \int_o^{\infty} erfc(\eta) d\eta \qquad (36)$$

After performing the definite integral of $\int_o^{\infty} erfc(\eta) d\eta = 1/\sqrt{\pi}$ and adding the ridge axis depth d_o, we find depth depends on material constants times the square root of seafloor age.

$$d_{tot}(t) = d_o + \frac{2\rho_m \alpha (T_m - T_o)}{(\rho_m - \rho_w)} \left(\frac{\kappa t}{\pi}\right)^{1/2} \qquad (37)$$

Now lets plug in some numbers to get an estimate of how seafloor depth varies with age.

Parameter	Definition	Value
T_o	surface temperature	0˚C
T_m	mantle temperature	1300˚C
κ	thermal diffusivity	8 x 10^{-7} m^2 s^{-1}
k	thermal conductivity	3.3 W m^{-1} C^{-1}
α	thermal expansion coefficient	3.1 x 10^{-5} C^{-1}
ρ_w	seawater density	1025 kg m^{-3}
ρ_m	mantle density	3300 kg m^{-3}
d_o	ridge axis depth	2500 m

A good approximation for the depth-age relation is

$$d = 2500m + 350\sqrt{age(Ma)} \quad (38)$$

To test this model of the cooling oceanic lithosphere we need, seafloor depth, seafloor age, and sediment thickness [*Renkin and Sclater, JGR, 93,* p.2919-2935, 1988].

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