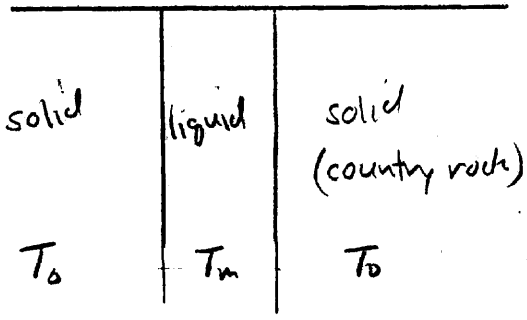
Stefan Problem

Several important problems in geology and geophysics include the solidification of magmas that are either injected into the crust or extruded onto the surface.

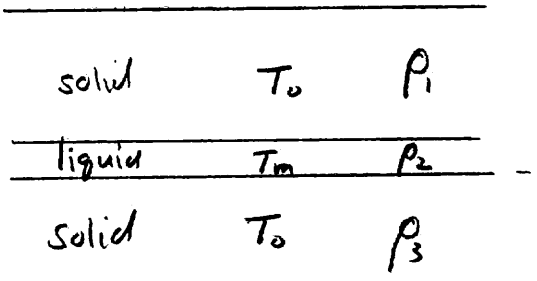
Three common geometries:

Dike



Examples:
- mid-ocean ridge
- radial dike swarm

Sill

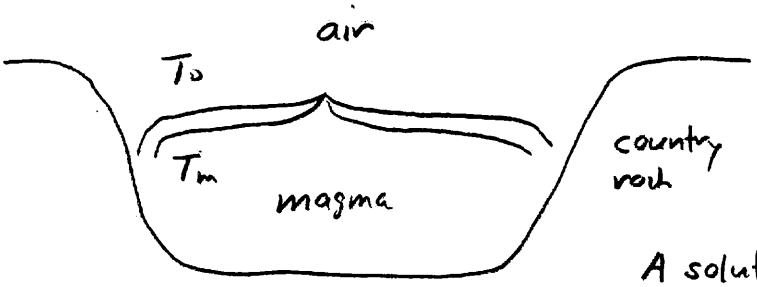


$$\rho_1 < \rho_2 < \rho_3$$

Sills usually occur when a dense magma is injected into a layered strata where ρ increases with depth.

examples: ice thickness vs time
magma ocean on moon.
Kilauea Lava Lake

Freezing Pond or Lava Lake



A solution to the heat diffusion equation with a moving liquid to solid phase boundary was first obtained by Stefan in 1891 and applied to the thickening of ice on a lake.

Solution to general heat conduction problem:

$$\theta = \operatorname{erf}(\eta) \qquad \text{but } \theta(\lambda_1) = 1 \quad \text{so}$$

$$\theta = \frac{\operatorname{erf}(\eta)}{\operatorname{erf}(\lambda_1)}$$

Now we need to determine λ_1. It is determined by requiring that the latent heat released at the moving phase boundary is conducted vertically upward

$$\rho L \frac{\partial z_m}{\partial t} = k \left.\frac{\partial T}{\partial z}\right|_{z_m}$$

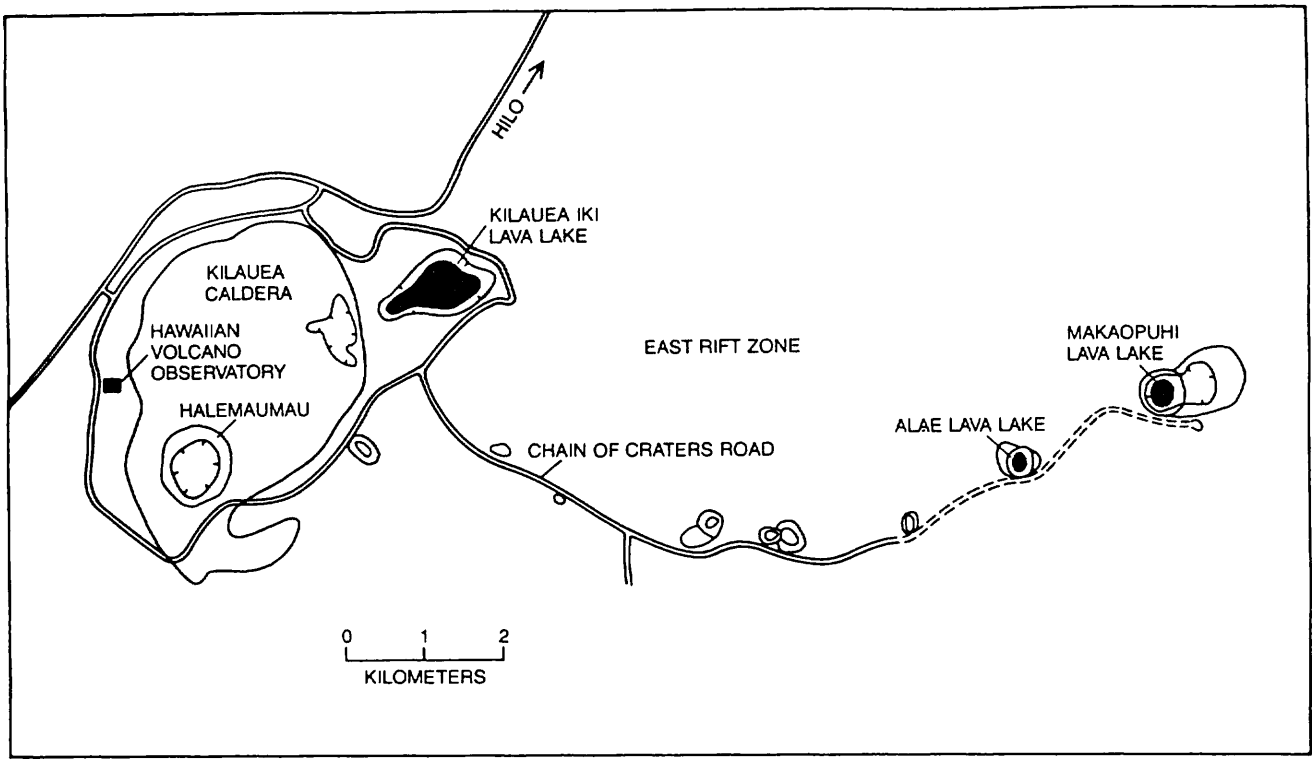
↑ latent heat ↑ heat conducted

but $\dfrac{\partial z_m}{\partial t} = \lambda_1 \sqrt{\dfrac{\kappa}{t}}$ and $\left.\dfrac{\partial T}{\partial z}\right|_{z_m} = \dfrac{(T_m - T_0)}{2\sqrt{\kappa t}} \dfrac{2}{\sqrt{\pi}} e^{-\lambda_1^2} \dfrac{1}{\operatorname{erf}(\lambda_1)}$

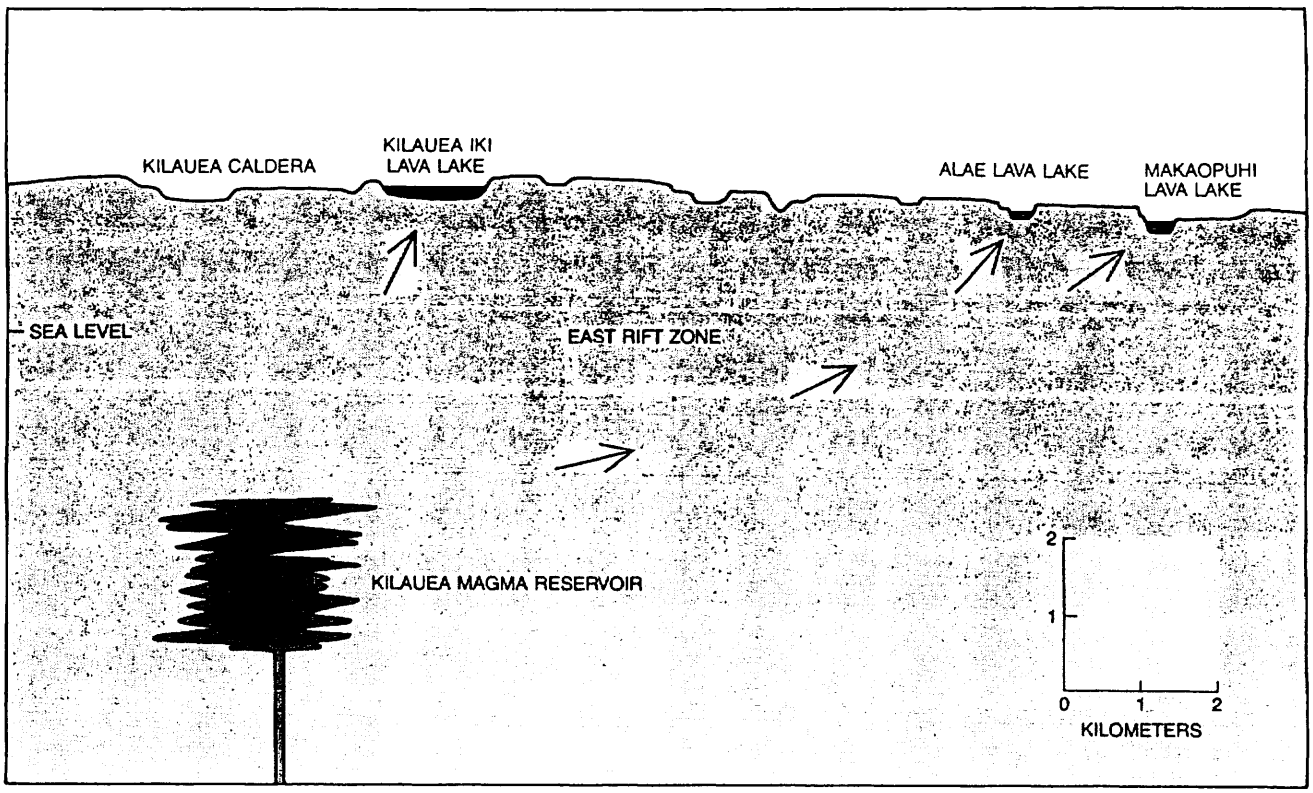
The equation becomes:

$$\frac{L\sqrt{\pi}}{c_p(T_m - T_0)} = \frac{e^{-\lambda_1^2}}{\lambda_1 \operatorname{erf} \lambda_1}$$

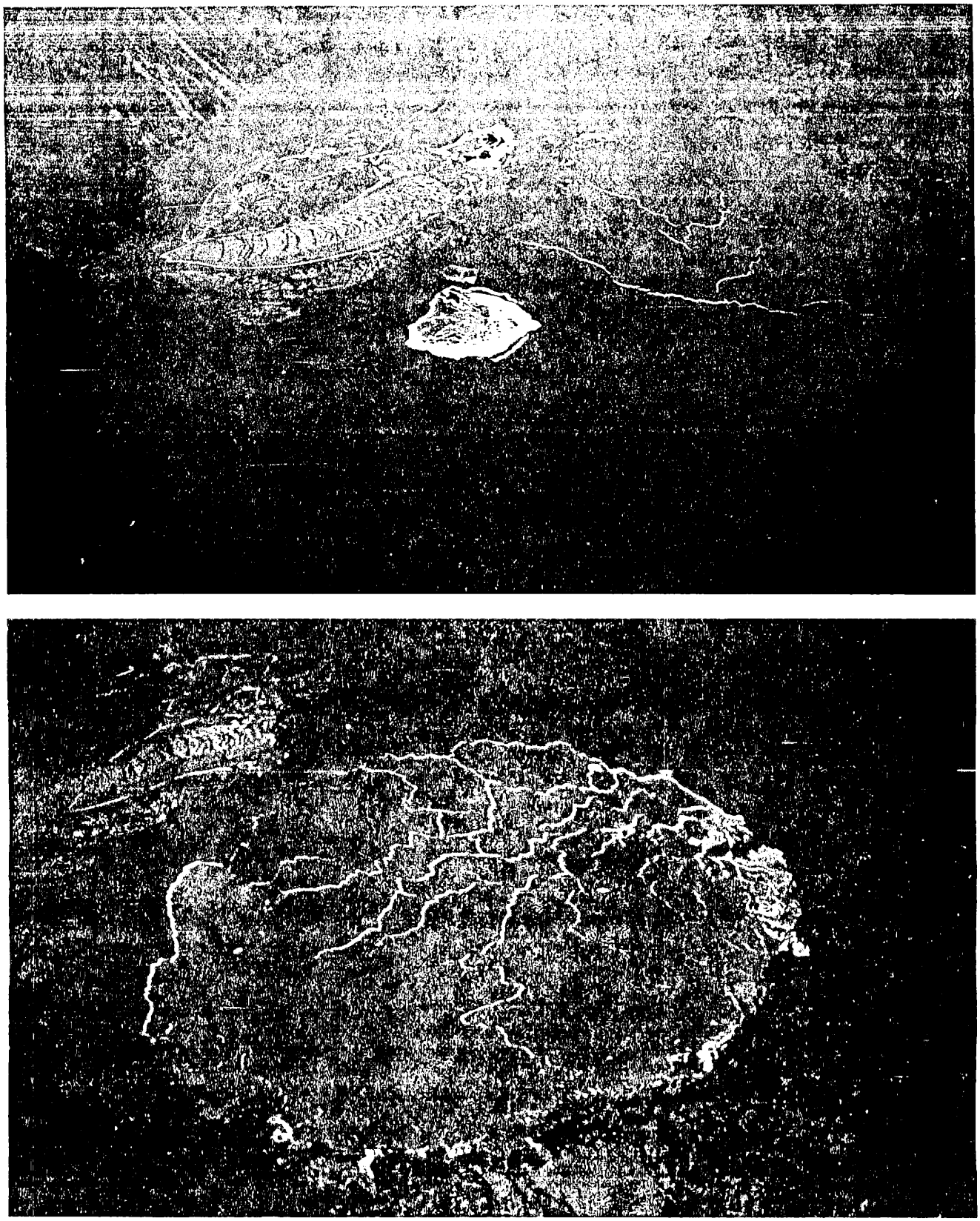
transcendental equation (solved by iteration)



MAP OF THE EAST RIFT ZONE of Kilauea shows the location of the three lava lakes the authors investigated: one formed in 1959 in Kilauea Iki Crater, a second formed in 1963 in Alae Crater and a third formed in 1965 in Makaopuhi Crater. Halemaumau is the fire pit in the caldera at the summit of the volcano, which is shaped like an inverted saucer. Over the past 10 years fresh lava flows have covered Alae and Makaopuhi lava lakes and the part of the Chain of Craters road indicated by the broken lines. Kilauea Iki lake was not covered.



MAGMA RESERVOIR lying about four kilometers below the summit of Kilauea is the source of the lava that erupted to the surface and flowed into depressions to form the lava lakes. Kilauea Iki lava lake, which reaches depths of 120 meters, was formed in a spectacular event characterized by 17 separate eruptions over 36 days. Makaopuhi lava lake, which has a peak depth of 83 meters, was formed in an eruption that lasted for 18 days. Alae lava lake, which is 15 meters deep, was created in an eruption that lasted for three days.



FOUNDERING OF SOLIDIFIED LAVA at the surface of Makaopuhi lava lake on Kilauea is shown in these photographs made in 1965. Gases exsolved from the underlying molten lava rose toward the surface and were trapped under the solidified but hot crust of the lake. In the top photograph the trapped gas gave rise to a gravitationally unstable situation that resulted in the foundering of a small piece of crust near the center of the lake's surface. The sunken lava was replaced by glowing molten lava (*center*). In the bottom photograph, made 10 minutes later, the foundering has extended to all the margins of the active part of the lake except the area at the upper right. The lava above 1,200 degrees Celsius is yellow. Between 900 and 1,100 degrees the lava is red, and below 900 degrees it is dark red or black.

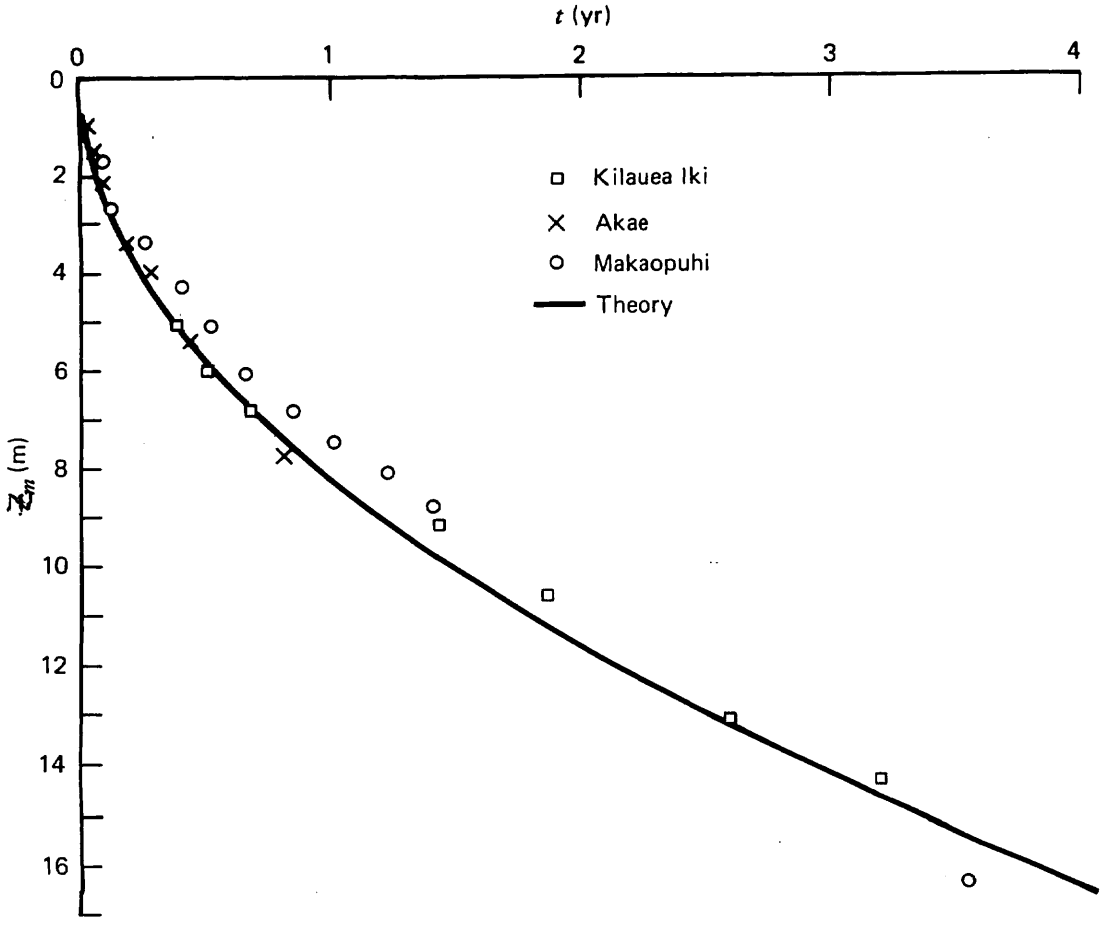


Figure 4-33 The thicknesses of the solidifying crusts on the lava lakes in the three pit craters Kilauea Iki (1959), Alae (1963), and Makaopuhi (1965) on the volcano Kilauea, Hawaii. (Data from T. L. Wright, D. L. Peck, and H. R. Shaw, Kilauea lava lakes: Natural laboratories of study of cooling, crystallization and differentiation of basaltic magma, in *The Geophysics of the Pacific Ocean Basin and its Margin*, G. H. Sutton, M. H. Manghnani, and R. Moberly, eds., pp. 375–390, American Geophysical Union, Washington, D.C., (1976). The theoretical curve is from Equations (4-129) and (4-134).