HEAT FLOW PARADOX

(See Special Issue of *J. Geophys. Res., v.85*, 1980: A) Turcotte, Tag, and Cooper, A Steady-State model for the distribution of stress and temperature on the San Andreas fault, p. 6224-6230; B) Lachenbruch and Sass, Heat flow and energetics of the San Andreas Fault Zone, p. 6158-6223; Scholz, C. H., *The Mechanics of earthquakes and Faulting*, Cambridge University Press, Cambridge England, 1990.)

Paradox - The seismogenic zone extends from the surface to a depth of about 10 km. According to Byerlee's law, the shear stress on the fault should be some large fraction of the hydrostatic stress.

$$\tau(z) = f(\rho_c - \rho_w)gz \qquad (1)$$

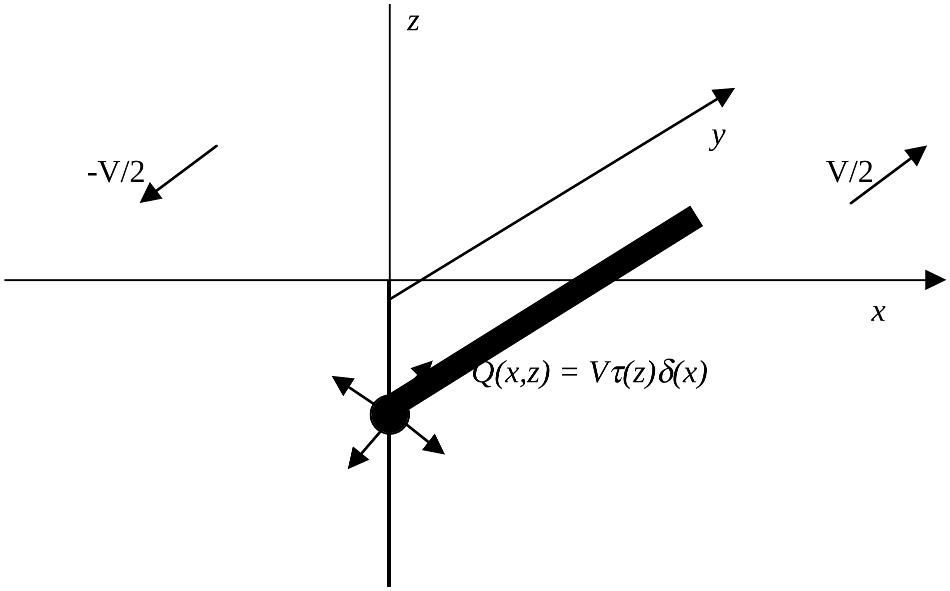
f	static coefficient of friction	~ 0.60
ρ_c	crustal density	2600 kg m^{-3}
ρ_w	water density	1000 kg m^{-3}
g	acceleration of gravity	9.8 m s^{-2}
D	depth of seismogenic zone	12 km

This assumes that water percolates to 12 km depths to lower friction on the fault. We can compute the average shear stress on the fault.

$$\bar{\tau} = \frac{1}{D}\int_o^D f(\rho_c - \rho_w)gzdz \ = \ \frac{1}{2} f(\rho_c - \rho_w)gD \ = \ 56 \text{ MPa} \qquad (2)$$

The observed stress drop during an earthquake ranges from 0.1 to 10 MPa with a typical value of 5 MPa which is about 10 times smaller than the average stress from Byerlee's Law. This implies that only a fraction of the total stress is released during an earthquake. The average stress during the earthquake times the earthquake displacement produces energy both as seismic radiation (small fraction) and as heat (large fraction). If this heat energy is averaged over many earthquake cycles, then this average heat/area generated on the fault plane will appear as a heat flow anomaly on the surface having a similar heat/area as along the fault.

To calculate this heat anomaly for a variety of frictional heating models, first consider a line source of heat.



The differential equation and boundary conditions for a unit-amplitude, line source at depth -*a* is

$$\nabla^2 T = \frac{1}{k} Q(x,z) = \frac{1}{k}\delta(x)\delta(z+a) \qquad (3)$$

$$T(x,0) = 0$$
$$\lim_{|z|\to\infty} T(x,z) = 0$$
$$\lim_{|x|\to\infty} T(x,z) = 0$$

where T is the temperature anomaly in ˚K, k is the thermal conductivity (3.3 $Wm^{-1\circ}K^{-1}$), and Q is the heat generation in Wm^{-3}. Note this is the same differential equation as equation (5) of the last section. The only difference is the surface boundary condition. The surface stress problem has vanishing shear stress at the surface (i.e., vertical derivative of displacement is v is zero) so we introduced a positive image source to force the displacement field to be symmetric about $z = 0$. In this heat flow case, we have vanishing temperature anomaly at the surface so we introduce a negative line heat source at $z = a$ to form an anti-symmetric temperature function. The solution to the full-space problem is identical to equation (14) of the previous section.

$$T(x,z) = \frac{-1}{2\pi k} \ln\left[x^2 + (z+a)^2\right]^{1/2} \qquad (4)$$

After including the image source, the result is

$$T(x,z) = \frac{-1}{2\pi k}\left\{\ln\left[x^2 + (z+a)^2\right]^{1/2} - \ln\left[x^2 + (z-a)^2\right]^{1/2}\right\} \qquad (5)$$

Note that this is similar to equation (23) in the *Turcotte et al.*, [1980]. The quantity of interest is the surface heat flow versus distance from the fault.

$$q(x,z) = -k\frac{\delta T}{\delta z} = \frac{1}{2\pi}\frac{\delta}{\delta z}\left\{\ln\left[x^2+(z+a)^2\right]^{1/2} - \ln\left[x^2+(z-a)^2\right]^{1/2}\right\} \qquad (6)$$

After a little algebra one arrives at the heat flow.

$$q(x,z) = \frac{1}{2\pi}\left\{\frac{(z+a)}{x^2+(z+a)^2} - \frac{(z-a)}{x^2+(z-a)^2}\right\} \qquad (7)$$

Thus the surface heat flow for a line source of unit strength at depth *a* is

$$q(x) = \frac{1}{\pi}\quad\frac{a}{x^2+a^2} \qquad (8)$$

For an arbitrary shear stress distribution with depth $\tau(z)$ the surface heat flow is

$$q(x) = \frac{\mathrm{V}}{\pi}\int_{o}^{\infty}\frac{z\tau(z)}{x^2+z^2}dz \qquad (9)$$

Now lets assume that the stress follows equation (1), Byerlees's law (i.e. high stress and high heat flow). Also allow hydrothermal circulation to extend from the surface to some depth *d* which effectively removes all the heat produced between the surface and that depth. The integration is

$$q(x) = \frac{f(\rho_c - \rho_w)g\mathrm{V}}{\pi}\int_{d}^{D}\frac{z^2}{x^2+z^2}dz \qquad (10)$$

This integral is done with help from the table of integrals.

$$\int\frac{x^2}{a+bx^2}dx = \frac{x}{b} - \frac{a}{b}\int\frac{1}{a+bx^2}dx \qquad (11)$$

After some algebra one arrives at the following analytic formula for the heat flow

$$q(x) = \frac{f(\rho_c - \rho_w)g\mathrm{V}}{\pi}\left\{(D-d) + \left(x\tan^{-1}\frac{d}{x} - x\tan^{-1}\frac{D}{x}\right)\right\} \qquad (12)$$

It is interesting to compare this heat flux to the heat flux at a mid-ocean ridge for the same total opening rate V (see figure on next page). The formula is

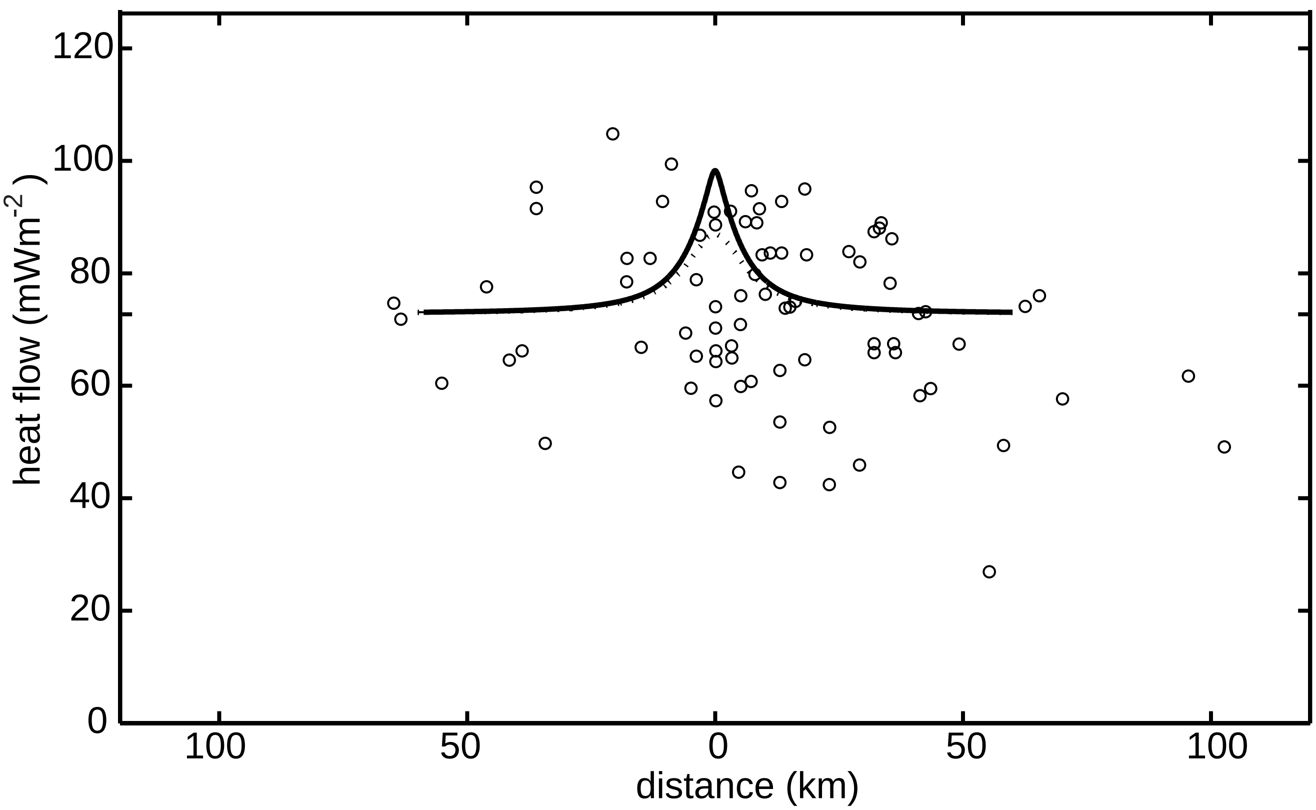
$$q(x) = k(T_m - T_o)(2\pi\kappa x / \mathrm{V})^{-1/2} \qquad (13)$$

T_m	mantle temperature	1600 ˚K
T_o	surface temperature	273 ˚K
k	thermal conductivity	3.3 $Wm^{-1}{}^{\circ}K^{-1}$
κ	thermal diffusivity	8. x 10^{-7} m^2 s^{-1}

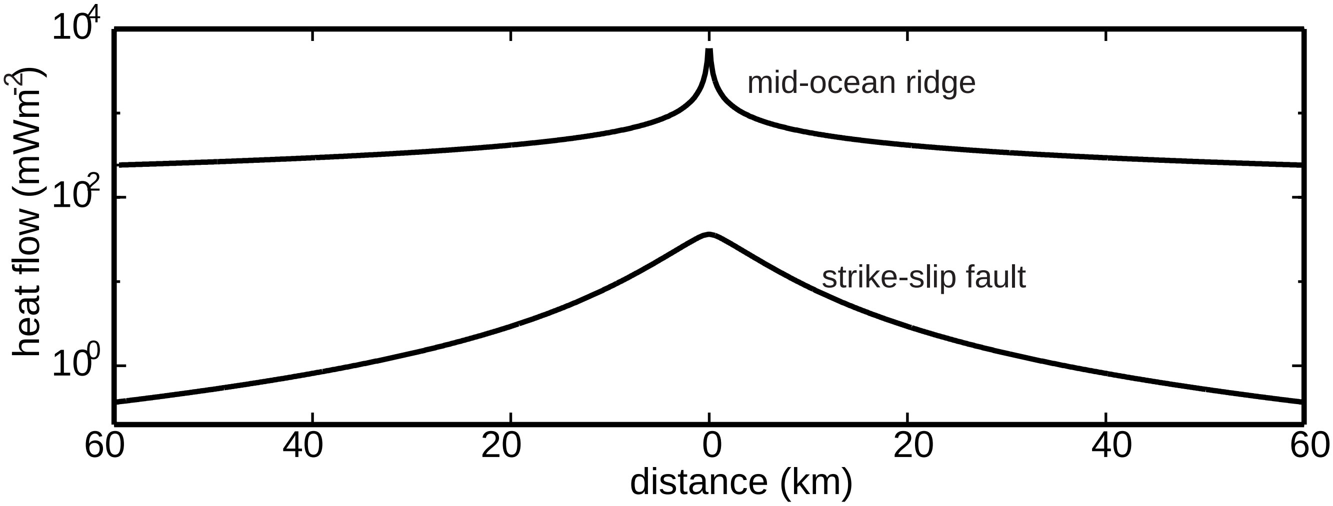
Matlab Example

The following is a Matlab program simulates a high-stress fault (i.e., Byerlee's Law) extending to a depth of 12 km and sliding at a rate of 30 mm/yr. Two cases are considered; the first case (solid curve on next page) has hydrothermal heat removal extending to a depth of 1 km while the second (dotted curve) has heat removal to a depth of 5 km. These models are compared with the heat flow measurements across the San Andreas Fault [*Lachenbruch and Sass*, 1980]. It is clear that the shallow heat removal model is inconsistent with the data. However, the deep heat removal model is not precluded by the observations, especially if the background level of the model heat flow is allowed to vary from the spatial average. One argument against hydrothermal removal of heat is the absence of hot springs along the fault with sufficient vigor to remove this heat. Hydrothermal circulation is the dominant heat removal mechanism at the mid-ocean ridges and hydrothermal vents are common. However, as shown in the following figure, heat generation along a strike-slip fault is 2-3 orders of magnitude less than a mid-ocean ridge so it is not clear that the same mechanism should operate at a fault. Even if heat loss is concentrated in small areas it may be difficult to detect at the surface.

```
%
%  program to calculate the surface heat flux due to frictional heating on a
%  strike-slip fault
D=12;
d1=1; d5=5;
rc=2600; rw=1000; g=9.8;
V=.03/3.15e7; f=.60;
q0=1.e6*f*(rc-rw)*g*V/pi;
%
%  calculate the heat flow for the two models of shallow and deep heat removal
%
x=-60:.1:60;
q1=q0*((D-d1)+x.*atan(d1./x)-x.*atan(D./x));
q5=q0*((D-d5)+x.*atan(d5./x)-x.*atan(D./x));
%  plot the results
plot(x,q1+73,x,q5+73,':');xlabel('distance  (km)'); ylabel('heat flow (mWm-2)')
axis([-120,120,0,167]);
```



120
100
80
60
40
20
0
heat flow (mWm-2)
100
50
0
50
100
distance (km)



10^4
10^2
10^0
heat flow (mWm-2)
mid-ocean ridge
strike-slip fault
60
40
20
0
20
40
60
distance (km)

MOMENT PARADOX: Seismic Moment versus Tectonic Saturation Moment

The *Moment Paradox* described next is really part of the heat-flow paradox except it is expressed in a different way. As discussed in a previous lecture, and in *Brace and Kohlstedt* [1980], measurements of stress difference in the uppermost crust to depths of several kilometers are consistent with a yield strength model following Byerlee's law. The static frictional resistance to sliding is related to a coefficient of friction f of about 0.60 times the overburden pressure of $\Delta\rho g z$. This leads to differential stress difference of 140 MPa at a depth of only 10 km. We also found that these high stresses are required to support the 5000 m elevation of Tibet relative to India. This isostatic model is the minimum stress needed to support topography so it is clear that high stresses exist at shallow depths in the crust. Similarly, one can calculate the bending moment needed to support the trench and outer rise topography at a subduction zone. The moment calculation is model-independent [McNutt and Menard, Constraints on yield strength in the oceanic lithosphere derived from observations of flexure, *Geophs. J. R. astr. Soc., 71*, p. 363-394, 1982;]

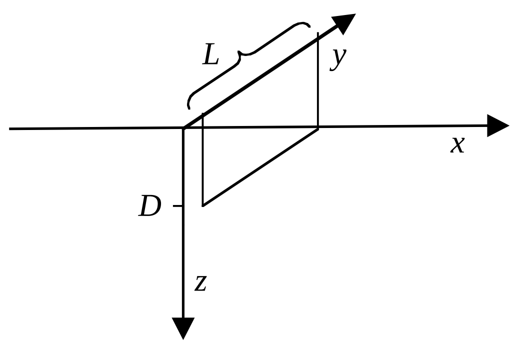
$$M(x_o)/L = \int_{x_o}^{\infty} \Delta\rho w(x)\left(x - x_o\right)dx \qquad (1)$$

where M is the moment per unit length along the trench, $\Delta\rho$ is the mantle-to-seawater density contrast, is the height of the outer rise above the normal depth and x-x_o is the distance between the first zero crossing of the trench flexure profile and some point out on the outer rise. The integral converges because $w(x)$ goes to zero exponentially with distance. Observed bending moments at outer rises vary from 5 x 10^{16} N for young lithosphere (10 Ma) to 3 x 10^{17} at old oceanic lithosphere (140 Ma) [Levitt and Sandwell, Lithospheric bending at subduction zones based on depth soundings and satellite gravity, *J. Geophys. Res., 100*, p. 379-400, 1995]. Next we'll compare these numbers to typical seismic moments of large earthquakes using the Alaska 1964 and Landers 1992 event as examples. The Landers rupture was about 70 km long so we'll divide its moment by length for the comparison with models. The results are provided in the table below.

tectonic example	moment per length (N)
outer rise flexure (10 Ma)	5×10^{16} N
outer rise flexure (140 Ma)	3×10^{17} N
Alaska flexure	1.2×10^{17} N
Alaska 1964 earthquake, M9.2	1.1×10^{17} N = 8.2×10^{22}/750 km
Landers 1992 earthquake, M7.4	1.4×10^{15} N geodetic/length 2.8×10^{15} N seismic/length
Byerlee's criterion (0 - 12 km only)	1.3×10^{16} N m

This comparison highlights two issues: first, the moment of the Alaska 1964 earthquake was sufficient to cause a collapse of the outer rise(??); second, the seismic/geodetic moment of the Landers 1992 earthquake is 10-20 times smaller than the moment estimated next using a the simple elastic dislocation model where stress is limited only by Byerlee's law.

Seismic Moment Released During an Earthquake



The moment released during an earthquake can be estimated in two ways, either by analysis of the seismic radiation pattern or by the geodetic analysis of the geodetic ground motion. They usually provide similar values; although in the case of the Landers 1992 rupture, the seismic moment estimate is about 2 times the geodetic moment estimate. The moment is defined as

$$M_s = \mu L D \Delta y \qquad (2)$$

f	static coefficient of friction	~ 0.60
ρ_c	crustal density	2600 kg m^{-3}
ρ_w	water density	1000 kg m^{-3}
g	acceleration of gravity	9.8 m s^{-2}
μ	shear modulus	2.6×10^{10} Pa
L	length of rupture	70 km
D	depth of rupture	12 km

Δy	rupture offset	4.5 m
V	plate velocity	0.015 m/yr
t	earthquake recurrence interval	

Now we can do some simple calculations. First, a check on the geodetic moment 1.4 x 10^{15} N provides a match to the published value. The recurrence interval of $\Delta y /V$ = 300 years seems OK for a fault out in the Mojave Desert away from the San Andreas Fault. So everything seems consistent. Next lets assume that the stress on the fault, as a function of depth, matches Byerlee's law for the case of hydrostatic pore pressure. We'll compare this saturation moment and recurrence interval with the observations from earthquakes.

Tectonic Saturation Moment

Assume that the simple half-space solution (developed above) provides the stress and strain field for a fault locked from the surface to a depth D. Further, assume that the maximum stress that can be maintained on a fault is given by Byerlee's law

$$\tau(z) = f\left[-(\rho_c - \rho_w)gz + \tau_n\right] \tag{3}$$

where τ_n is the additional tectonic normal stress applied to the fault plane. The tectonic moment per unit length is given by

$$M_T / L = \Delta y \int_{-D}^{0} \mu(z)dz \tag{4}$$

What is $\mu(z)$? This is the effective shear modulus needed to keep the stress below the upper bound provided by Byerlee's law so

$$\mu(z) = \frac{\tau(z)}{\varepsilon(z)} \quad \text{where} \quad \varepsilon(z) = \frac{\partial v(x,z)}{\partial x} \tag{5}$$

Now assume that $v(x,z)$ is provided by the interseismic strain solution developed above (equation 19). It is left as an exercise to finish the problem. You will find that $\varepsilon(z)$ is proportional to Δy so this factor cancels in equation (4). The final result is

$$\frac{M_T}{L} = f\pi D^2\left[\frac{(\rho_c - \rho_w)gD}{4} + \frac{2}{3}\tau_n\right] \tag{6}$$

Using the values in the table above and for zero normal stress, we find the saturation moment per unit length is 1.3 x 10^{16}N. Again, this is 10 times larger than the observed moment. Given the fault parameters above, this moment implies an potential seismic offset of 45 m and a recurrence time of 3000 years; a giant earthquake indeed!

There are only two ways to understand this delemma.

A) Faults are somehow lubricated (*f*~.05) so the average stress on the fault is 10-20 times smaller than predicted by Byerlee's law. In this case one has the difficulty of maintaining the elevation of the topography in California. For example, San Jacinto Mountain, which is less than 25 km from the San Andreas Fault, has a relief of about 3000 m which implies stresses of 80 MPa (16 times the stress drop in an earthquake).

B) Faults are strong as predicted by Byerlee's law. In this case, faults are always very close to failure and each earthquake relieves only a small fraction (~10%) of the tectonic stress. As we saw in the last section, this model implies a large amount of energy dissipation along the fault; friction from both aseismic creep and seismic rupture will generate heat. It has been proposed that perhaps during the earthquake, the coefficient of friction drops from 0.60 to say 0.05 to temporarily disable the heat generation. However, is seems that such a slippery fault would release all of the elastic energy during an earthquake (~60 m of offset). Another possibility is that heat is generated but a large fraction of the heat is advected to the surface by circulation of water in the upper couple km of crust. The unfortunate implication of this high-stress model is that since faults are always close to failure, it will be almost impossible to predict earthquakes.