

Post Glacial Rebound

①

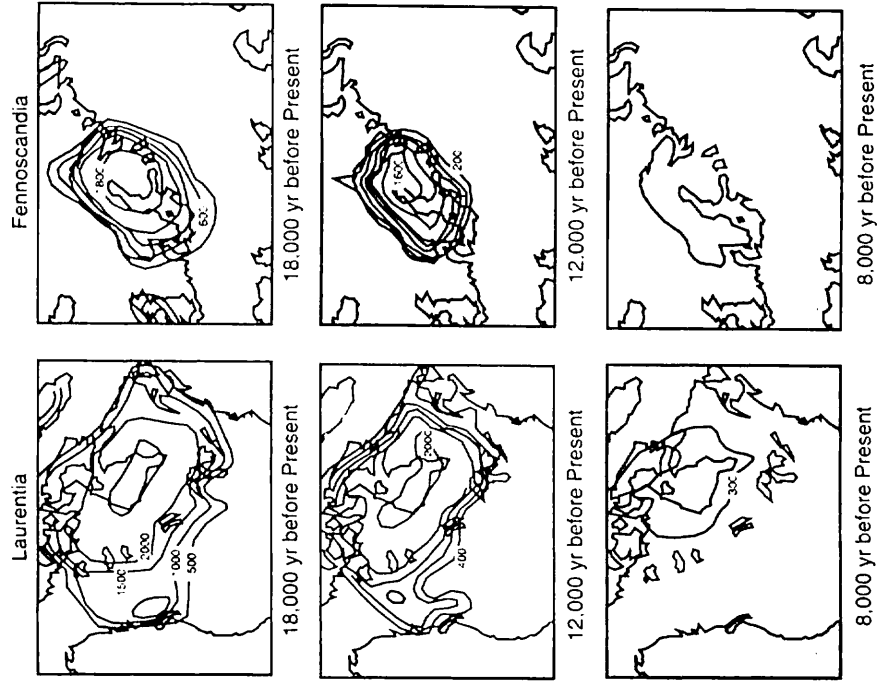
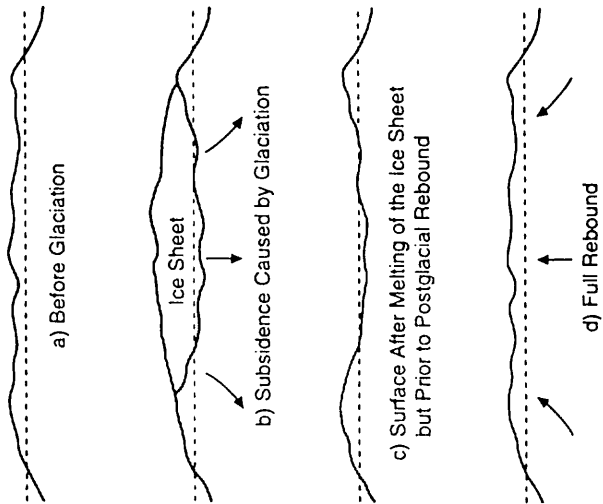
During the past few million years, the polar regions of the Earth have undergone glacial cycles with a period of about 40,000 years. A large ice cap still covers the continental crust of Greenland and Antarctic.

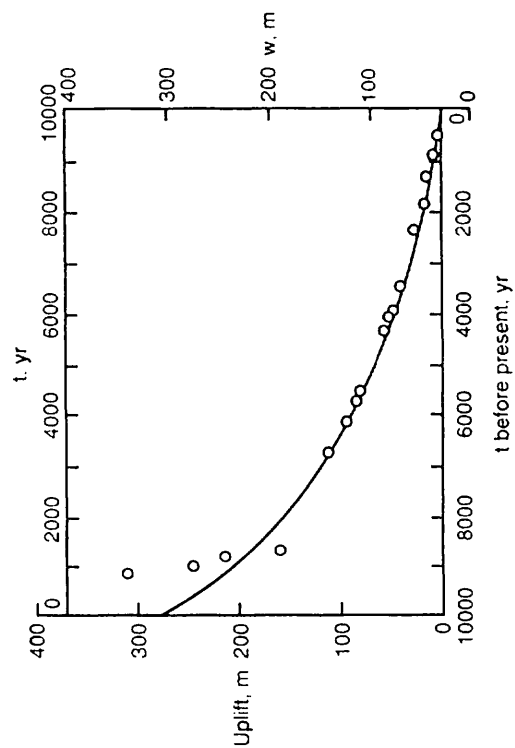
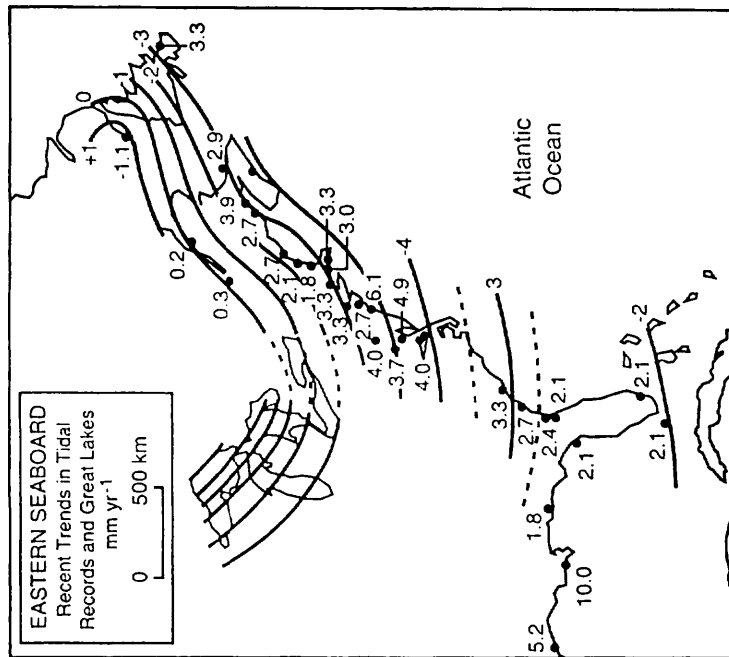
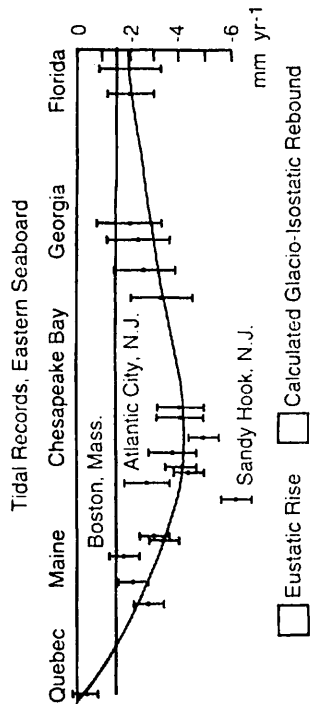
Just 10,000 years ago there were also ice caps centered at Scandinavia, and Hudsons Bay. The Scandinavian ice sheet is sometimes called the Fennoscandia ice sheet.

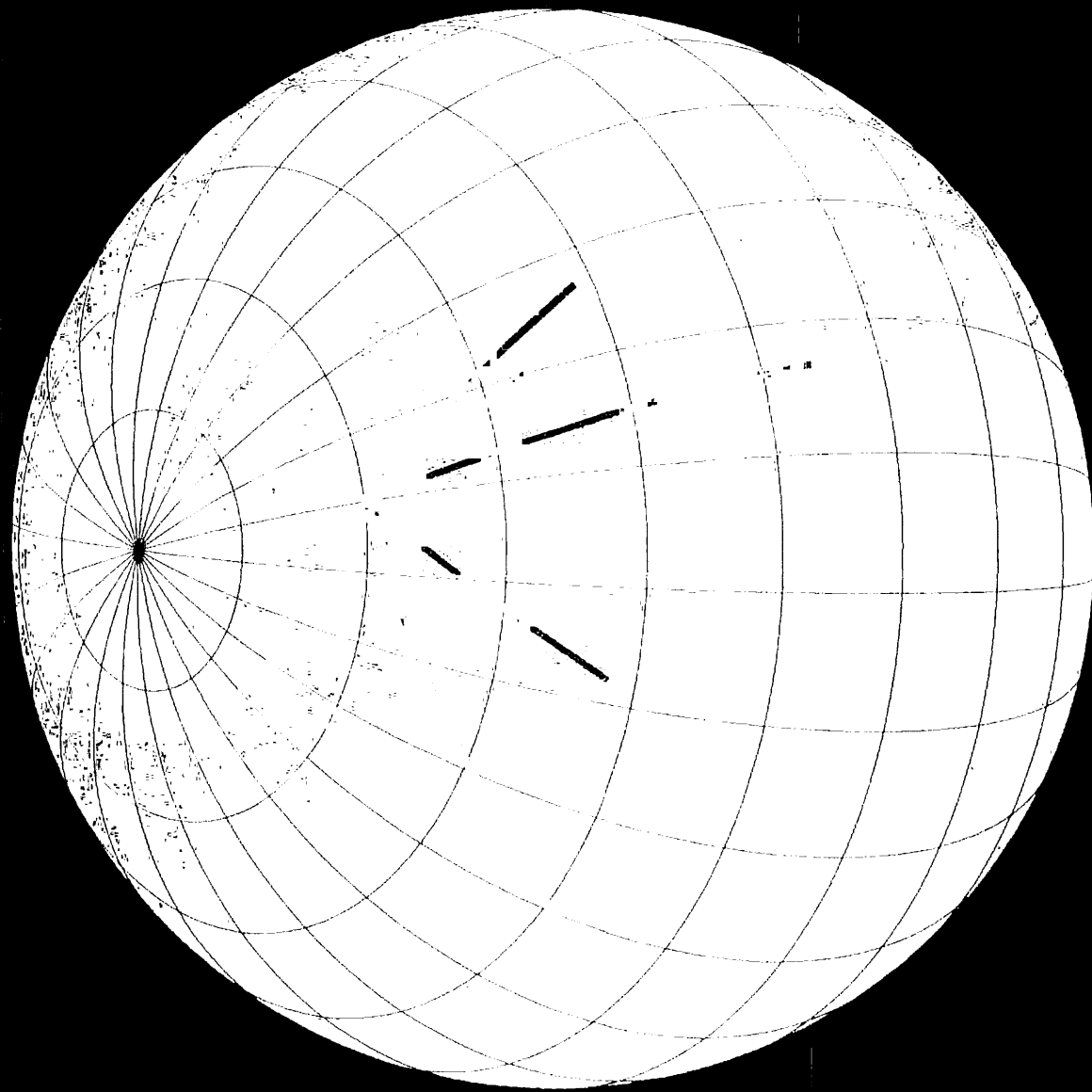
* Show glacial cycle diagram.

When the large ice sheets melt a number of things happen:

- 1) sea level rises by about 200m
 - 2) continental areas beneath the load rebound resulting in elevated beach terraces.
 - 3) the peripheral buldge of the ice sheet collapses
 - 4) the gravity field of the earth changes in response to 1-3. The most important change is J_2
- Like to go over the simple half-space glacial rebound problem.
 - To do the problem correctly one must go to spherical co-ordinates and calculate changes in potential, SL,

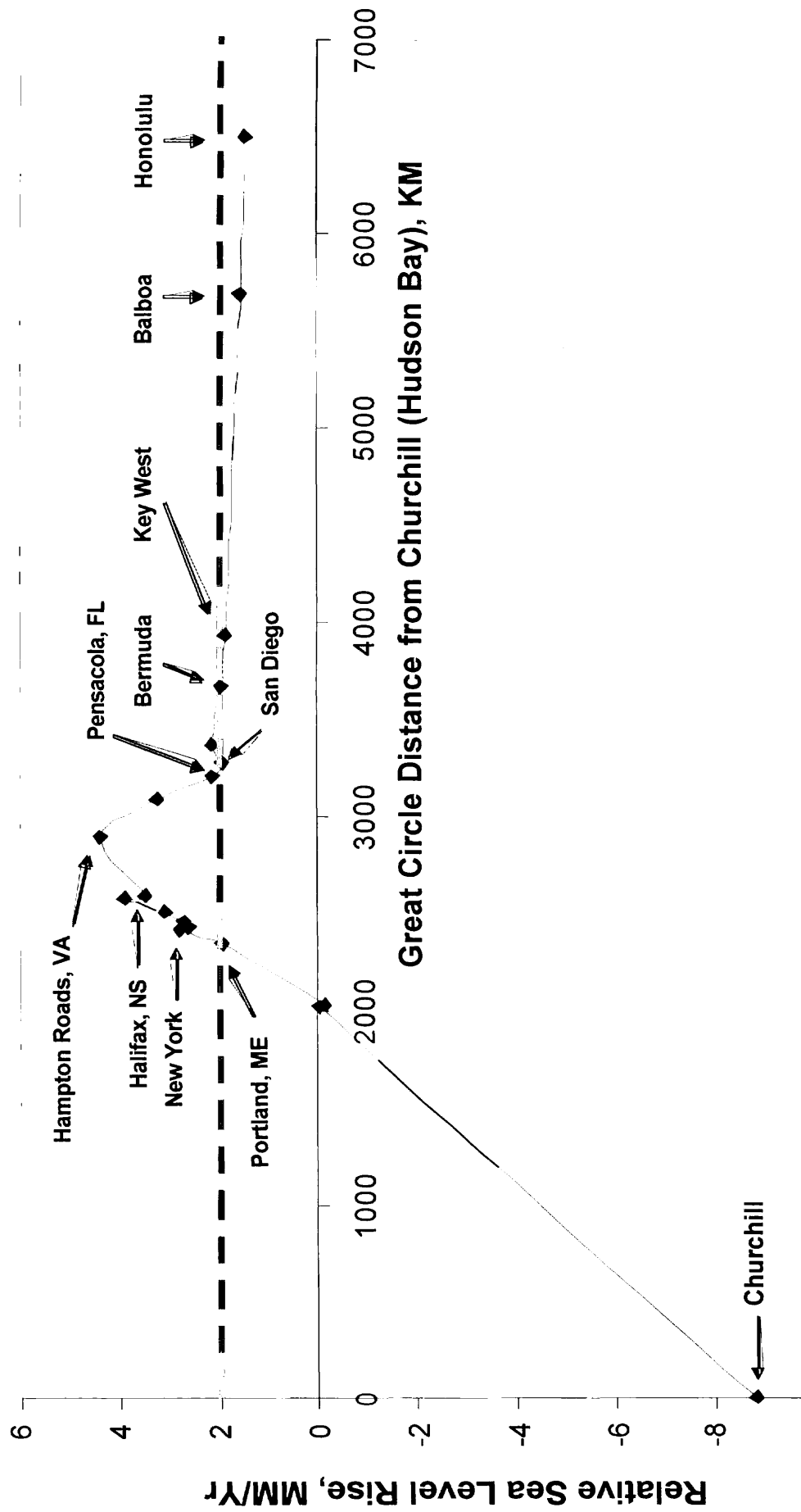




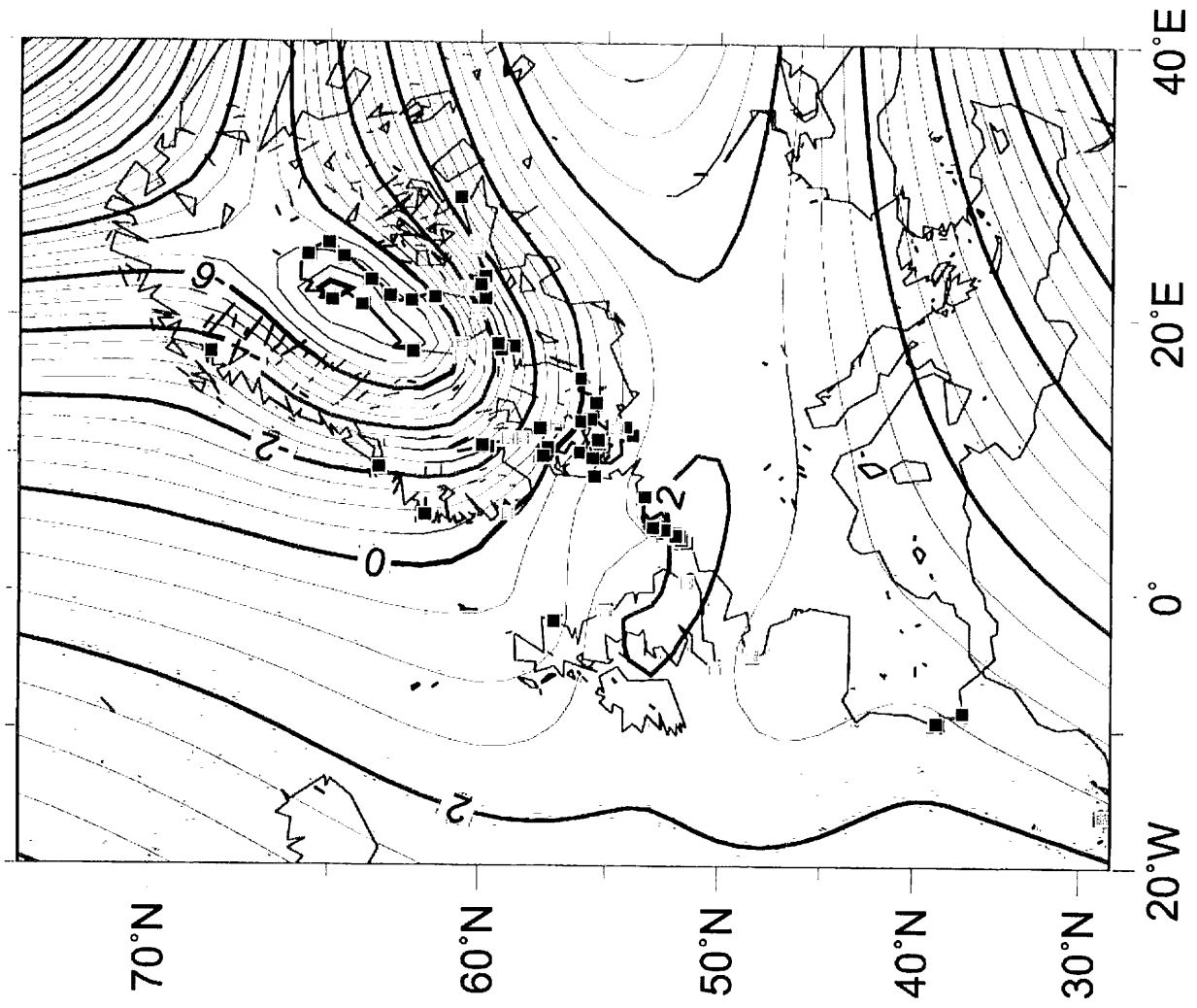


Great Circle Distances from Hudson Bay to Near and Far Field Tide Gauges

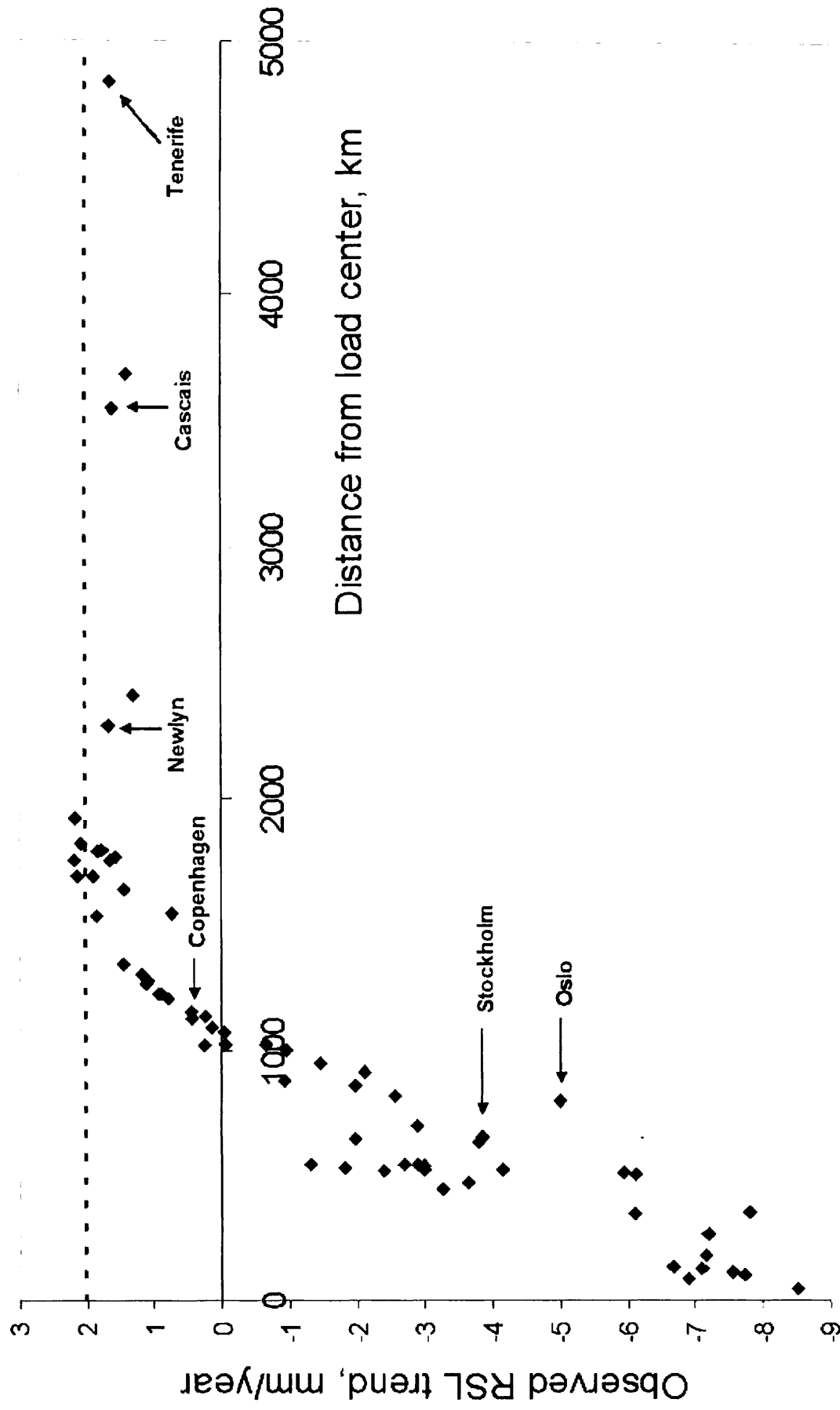
Relative Sea Level Trends and Distance from Hudson Bay



RSL Rise in Europe



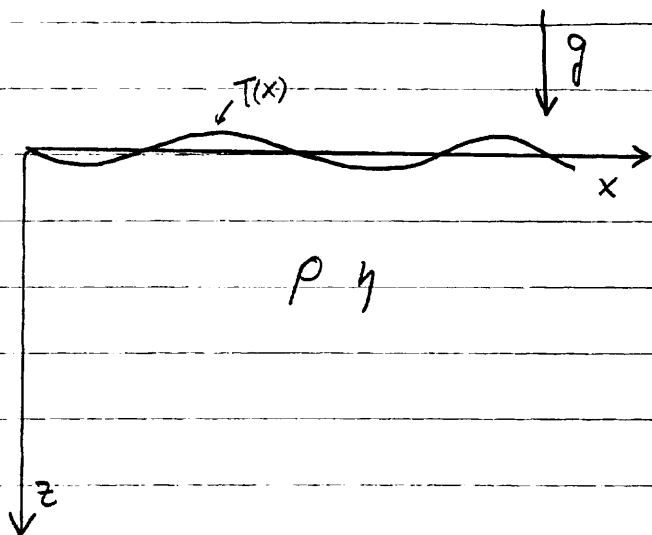
20th Century Relative Sea Level Trends in Europe



④

Postglacial Rebound

$T(x)$ - initial topography m
 η - viscosity Pa s
 u - x-velocity $m s^{-1}$
 w - z-velocity $m s^{-1}$
 ρ - density $kg m^{-3}$



assume incompressibility $\frac{\partial u}{\partial x} = -\frac{\partial w}{\partial z}$

assume 2-D $u = -\frac{\partial \psi}{\partial z}$ $w = \frac{\partial \psi}{\partial x}$

$$\tau_{xx} = 2\eta \frac{\partial u}{\partial x} = -2\eta \frac{\partial^2 \psi}{\partial x \partial z} \quad \tau_{zz} = 2\eta \frac{\partial^2 \psi}{\partial z \partial x} \quad \tau_{xx} = -\tau_{zz}$$

$$\tau_{xz} = \eta \left(\frac{\partial^2 \psi}{\partial x^2} - \frac{\partial^2 \psi}{\partial z^2} \right)$$

$$w = w_0 e^{-\frac{z}{\lambda}} \quad \text{GUESS}$$

$$\tau = \frac{\eta}{g \rho \lambda} \quad \begin{matrix} F m^2 s \\ F m^3 \end{matrix}$$

$$\tau = 4000$$

$$\nabla^4 \psi = 0$$

$$\tau_{xz}|_0 = 0$$

$$\lim_{z \rightarrow \infty} \psi = 0$$

$$\frac{\partial p}{\partial x}|_0 = -\rho g \frac{\partial T}{\partial x}$$

$$\eta = 1.6 \times 10^{22}$$

Note that uniform topography $T(x)$ does not drive any flow. The flow is driven by the horizontal pressure gradient setup by the topography gradient.

(5)

take fourier transform w.r.t. x

$$(i2\pi k)^4 \psi + 2(i2\pi k)^2 \frac{\partial^2 \psi}{\partial z^2} + \frac{\partial^4 \psi}{\partial z^4} = 0$$

guess solution

$$\psi(k, z) = A(k) e^{-2\pi |k| z} + B z e^{-2\pi |k| z} + C e^{2\pi |k| z} + D z e^{2\pi |k| z}$$

solution must vanish as $z \rightarrow \infty$ $C = D = 0$

$$\psi = (A + B z) e^{-2\pi |k| z}$$

shear traction must vanish at the surface

$$\tau_{xz}|_0 = \eta \left(\frac{\partial^2 \psi}{\partial x^2} - \frac{\partial^2 \psi}{\partial z^2} \right)$$

$$\frac{\partial \psi}{\partial z} = [B - 2\pi |k| (A + B z)] e^{-2\pi |k| z}$$

$$\frac{\partial^2 \psi}{\partial z^2} = [-2B(2\pi |k|) + (2\pi |k|)^2 (A + B z)] e^{-2\pi |k| z}$$

$$\tau_{xz}|_0 = -(2\pi k)^2 A - [(2\pi k)^2 A - 2B(2\pi |k|)] \quad \boxed{B = 2\pi |k| A}$$

$$\psi(k, z) = A(k) (1 + 2\pi |k| z) e^{-2\pi |k| z} \quad \frac{\partial^2 \psi}{\partial z^2} = -A(2\pi k)^2 (1 - 2\pi |k| z) e^{-2\pi |k| z}$$

$$\frac{\partial^3 \psi}{\partial z^3} = -A(k) (2\pi k)^2 z e^{-2\pi |k| z} \quad \frac{\partial^3 \psi}{\partial z^3} = A(2\pi |k|)^3 (2 - 2\pi |k| z) e^{-2\pi |k| z}$$

(6)

solution

$$\psi = A (1 + 2\pi |k| z) e^{-2\pi |k| z}$$

$$u = A (2\pi |k|)^2 z e^{-2\pi |k| z}$$

$$w = A (i 2\pi k) (1 + 2\pi |k| z) e^{-2\pi |k| z}$$

Now we need to match the pressure gradient boundary condition.

$$\left. \frac{\partial P}{\partial x} \right|_0 = -\rho g \frac{\partial T}{\partial x}$$

use the x force balance for $\frac{\partial p}{\partial x} = -\eta \left(\frac{\partial^3 \psi}{\partial x^2 \partial z} + \frac{\partial^3 \psi}{\partial z^3} \right)$

$\left. \frac{\partial \psi}{\partial z} \right|_0 = 0$ so only one term remains. Also take

fourier transform.

$$\eta \frac{\partial^3 \psi}{\partial z^3} = \rho g (i 2\pi k) T(k)$$

$$\left. \frac{\partial^3 \psi}{\partial z^3} \right|_0 = 2A (2\pi |k|)^3 \Rightarrow A = \frac{(i 2\pi k) \rho g T(k)}{2\eta (2\pi |k|)^3}$$

$$w = \frac{-\rho g}{2\eta (2\pi |k|)} T(k)$$

but $w = \frac{\partial T}{\partial t}$

⑦

$$\frac{\partial T}{\partial t} = - \frac{\rho g}{4\pi(k)\eta} T(k)$$

$$T(k,t) = T(k,0) e^{-\frac{\rho g}{4\pi(k)\eta} t}$$

$$\tau_r = \frac{4\pi\eta}{\rho g \lambda}$$

Fennoscandia $\lambda = 3000 \text{ km}$ $\rho = 3300 \text{ kg m}^{-3}$ $g = 9.8 \text{ m s}^{-2}$

$$\tau_r = 4400 \text{ yr} = 1.38 \times 10^8 \text{ sec}$$

$$\eta = 1.1 \times 10^{21} \text{ Pa s}$$

This model shows long λ responds faster than short λ .

Should use 3-D but cannot use stream function

Should do on a sphere.

Should account for re-distribution of water over the sphere and the change in the Earth's gravity field due to the redistribution of mass.

momentum $\eta \nabla^2 \vec{u} - \nabla p + \rho \nabla \phi = 0$

continuity $\nabla \cdot \vec{u} = 0$

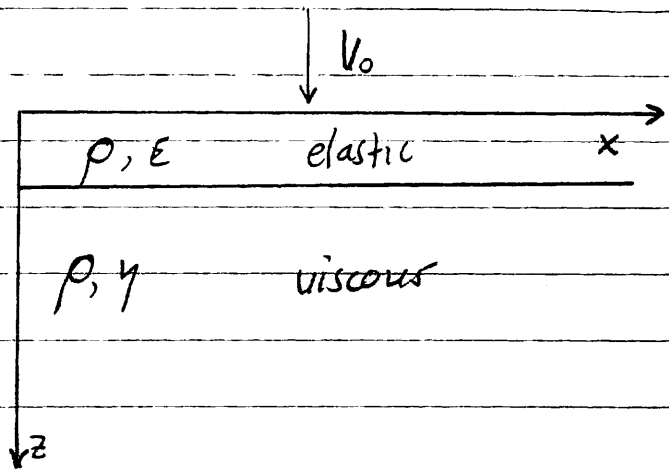
Poisson $\nabla^2 \phi = -4\pi G \rho$

Use global shoreline data plus $\dot{C}_{em}$ and $\dot{S}_{em}$

(8)

ADD ELASTIC LITHOSPHERE

- 1) Apply a load V_0 for an infinite time. This is the solution to the flexure problem



- 2) Remove the load and observe the rebound

$$D = \frac{E h^3}{12(1-\nu^2)}$$

1) $D \frac{d^4 w}{dx^4} + \rho g w = V_0 \delta(x)$

$$D (i2\pi k)^4 w(k) + \rho g w = V_0 \Rightarrow w(k) = \left[1 + \frac{D(2\pi k)^4}{\rho g} \right]^{-1} \frac{V_0}{\rho g}$$

- 2) Use this as w_0

$$w(k, t) = \frac{V_0}{\rho g} \left[1 + \frac{D(2\pi k)^4}{\rho g} \right]^{-1} e^{-\frac{t}{\tau_r}}$$

$$h = 100 \text{ km}$$

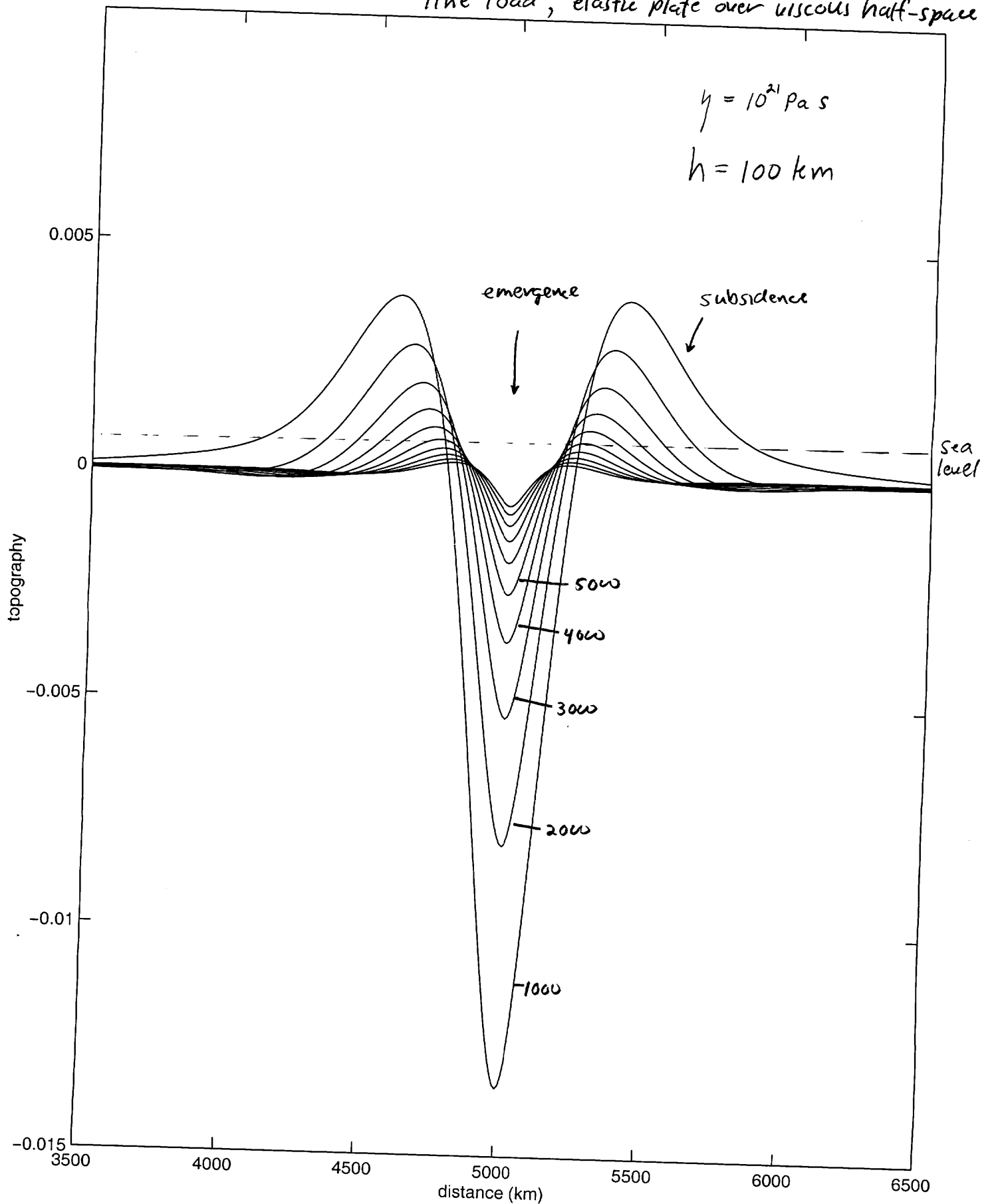
$$\nu = .25$$

$$E = 7 \times 10^{10} \text{ Pa}$$

$$\rho = 3300 \text{ kg m}^{-3}$$

$$\eta = 10^{24} \text{ Pa s}$$

⑨
line load, elastic plate over viscous half-space



ICE4G GIA (mm per year) vs latitude

