

### HOMEWORK 3

Draft presentations due November 7

Oral presentations of Group problems, November 9, 14, 16

#### **Group A – Modeling marine magnetic anomalies**

Write a Matlab program to generate marine magnetic anomaly versus distance from a spreading ridge axis. Use the following equation relating the fourier transform of the magnetic anomaly to the fourier transform of the magnetic timescale.

$$A(k, z) = C\mu_0 2\pi |k| e^{-2\pi |k| z} e^{i\theta \operatorname{sgn}(k)} p(k)$$

$A$  - scalar magnetic anomaly (T)

$k$  - wavenumber  $1/\lambda$  ( $\text{m}^{-1}$ )

$z$  - average seafloor depth (m)

$\theta$  - skewness (start with  $\theta=0$ )

$C$  - unknown constant that is adjusted to match observed magnetic anomalies

$\mu_0$  - magnetic permeability =  $4\pi \times 10^{-7} \text{ T A}^{-1} \text{ m}$

$p(k)$ - fourier transform of magnetic timescale.

You will need a magnetic timescale and the start of a Matlab program (ftp topex.ucsd.edu cd pub/class/hw3 mget \*). Assume symmetric spreading about the ridge axis, constant spreading rate, and constant ocean depth.

Use the program and magnetic anomaly profiles across the Pacific-Antarctic Rise (NBP9707.xydm) and the Mid-Atlantic Ridge (a9321.xydm) to estimate the half-spreading rate at each of these ridges. You may need to vary the mean ocean depth and skewness to obtain good fits.

Describe some of the problems that you had fitting the data. Provide some estimates on the range of total spreading rate for each ridge.

### Group B – Heat flow across the San Andreas fault

(See Special Issue of *J. Geophys. Res.*, v.85, 1980: A) Turcotte, Tag, and Cooper, A Steady-State model for the distribution of stress and temperature on the San Andreas fault, p. 6224-6230; B) Lachenbruch and Sass, Heat flow and energetics of the San Andreas Fault Zone, p. 6158-6223)

The seismogenic zone extends from the surface to a depth of about 12 km. The shear stress on the locked fault should be some large fraction of the hydrostatic stress.

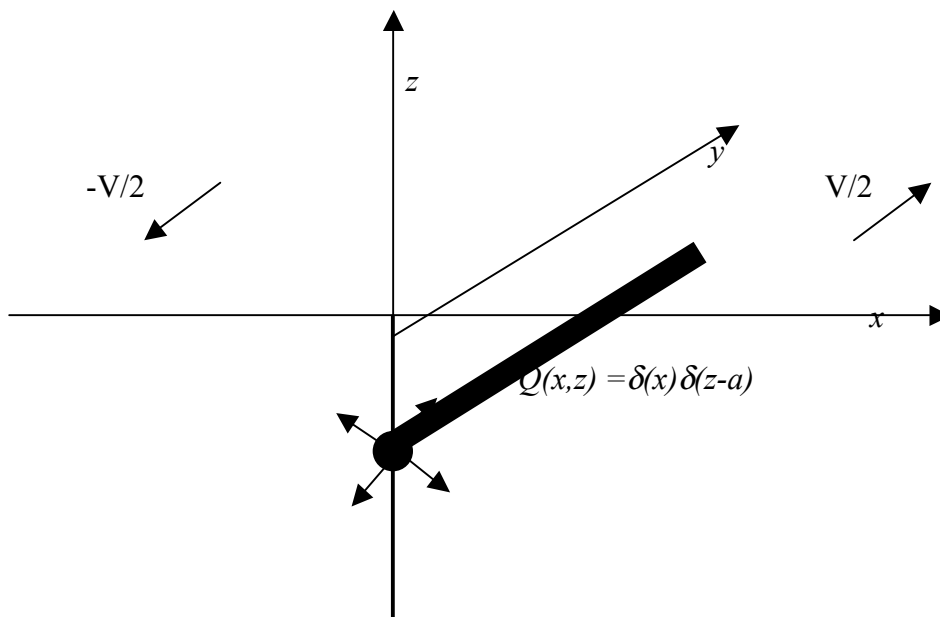
$$\tau(z) = f\rho_cgz$$

$f$	static coefficient of friction	$\sim 0.60$
$\rho_c$	crustal density	$2600 \text{ kg m}^{-3}$
$g$	acceleration of gravity	$9.8 \text{ m s}^{-2}$
$D$	depth of seismogenic zone	12 km

Compute the average shear stress on the fault and compare this with the typical stress drop during a major earthquake. Discuss the disagreement between these stresses.

The energy generated during an earthquake is equal to the shear stress times the fault area times the earthquake displacement. This energy is converted to seismic radiation (small fraction) and as heat (large fraction). If this heat energy is averaged over many earthquake cycles, then this average heat/area generated on the fault plane will appear as a heat flow anomaly on the surface having a similar heat/area as along the fault.

To calculate this heat anomaly for a variety of frictional heating models, first consider a line source of heat.



The differential equation and boundary conditions for a unit-amplitude, line source at depth  $-a$  is

$$\nabla^2 T = \frac{1}{k} Q(x, z) = \frac{1}{k} \delta(x) \delta(z + a)$$

$$T(x, 0) = 0$$

$$\lim_{|z| \rightarrow \infty} T(x, z) = 0$$

$$\lim_{|x| \rightarrow \infty} T(x, z) = 0$$

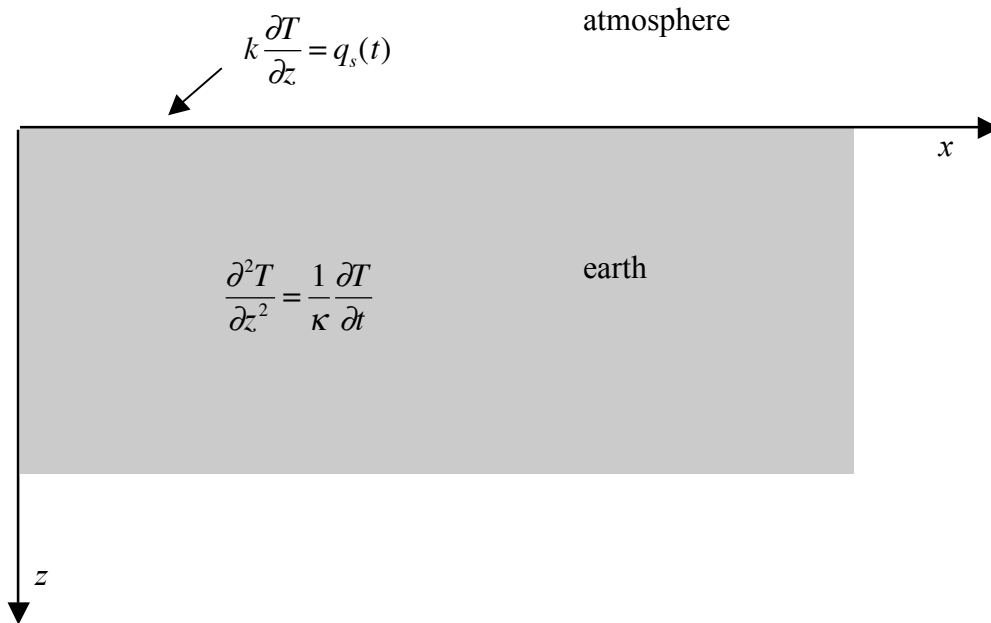
where  $T$  is the temperature anomaly,  $k$  is the thermal conductivity ( $3.3 \text{ W m}^{-1} \text{ K}^{-1}$ ), and  $Q$  is the heat generation in  $\text{W m}^{-3}$ . Use the method of images to satisfy the surface boundary condition. Solve for the temperature anomaly between the surface of the earth and the line source and calculate the surface heat flow. You now have the Green's function and can compute the heat flow for an arbitrary source distribution.

Assume that the stress follows the frictional sliding model  $\tau = f \rho_c g z$ . Calculate the surface heat flow and compare it with the measurements of *Lachenbruch and Sass* [1980]. This integral can be done analytically. What frictional coefficient provides the best fit to the measurements?

Suppose next that hydrothermal circulation penetrates the crust to a depth  $d$  and removes all the heat generated on the fault between 0 and  $d$ . Is it possible to satisfy the heat flow observations using a frictional coefficient of 0.6?

### Group C. – Periodic heating of the surface of the earth (Wine Cellar Problem)

Consider the land as a semi-infinite homogeneous half-space with constant thermal properties. The surface is subjected to periodic variations of solar radiation consisting of diurnal and annual variations. (see notes from remote sensing class <http://topex.ucsd.edu/rs/heat.pdf>)



Derive a formula that relates the temperature anomaly  $T$  at any time and depth to the fourier transform of the surface heat flux  $q_s$ . What constants determine the attenuation depth of the temperature anomaly. What is the attenuation depth of the periodic temperature variations due to the diurnal cycle, the annyal cycle, and the glacial cycle?

Find or construct a periodic time series of heat flux (anything interesting) and calculate the temperatures at a variety of depths.

### Group D. Temperature evolution of an oceanic fracture zone

An oceanic fracture zone is the boundary between lithosphere of different ages. Calculate the temperature as a function of depth  $z$ , time  $t$ , and distance  $x$  across the fracture zone. The differential equation, initial condition and boundary condition are

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{\kappa} \frac{\partial T}{\partial t}$$

$$T(x, z, t_o) = T_m \quad x < 0$$

$$T(x, z, t_o) = T_m \operatorname{erf}\left(\frac{z}{2\sqrt{\kappa t_o}}\right) \quad x > 0$$

$$T(x, 0, t) = 0$$

$$T(x, \infty, t) = T_m$$

where  $t_o$  is the age offset at the ridge-transform intersection. Derive an analytic solution to this problem? Here is a reference that may be helpful (Sandwell, D. T., and G. Schubert, Lithospheric Flexure at Fracture Zones, *J. Geophys. Res.*, 87, 4657-4667, 1982. <http://topex.ucsd.edu/sandwell/publications/5.pdf>)

Assuming local isostatic compensation, develop a formula for the topography across the fracture zone. How does real fracture zone topography differ from this model topography?

### Group E. Global oceanic heat flow

Derive the expression below relating the depth  $d$  and age  $A$  of the seafloor to the local heat loss of the thermal boundary layer. The equation has been derived before in 1-D by *Parsons and Mckenzie* [JGR 1978]. The 2-D expression is

$$\frac{\nabla d \cdot \nabla A}{\nabla A \cdot \nabla A} = \frac{-\alpha}{C_p (\rho_m - \rho_w)} (q_\infty - q_s)$$

where  $\alpha$  is the coefficient of thermal expansion,  $C_p$  is the heat capacity  $\rho_m$  is the mantle density and  $\rho_w$  is the seawater density. What are the main assumptions used for developing this formula?

Outline an approach for calculating the global oceanic heat loss. (Hint, graduate student Matt Wei could probably help you with this problem.)

### Group F. Heat flow due to plume

Birch [JGR, v. 80, no., 35, p. 4825-4827] developed a very simple model for the temperature in the lithosphere as it moves over a point heat source. A diagram is shown below. The derivation is given in Carslaw and Jaeger [*Conduction of Heat in Solids*, Oxford Univ. Press, 1959]. Setup the differential equation and derive it using our standard fourier transform method. Calculate the temperature and heat flow for a source that varies as a Gaussian function.

$$q(x, y, z) = A\delta(z - z_0)\exp\left(-\frac{x^2 + y^2}{2\sigma^2}\right)$$

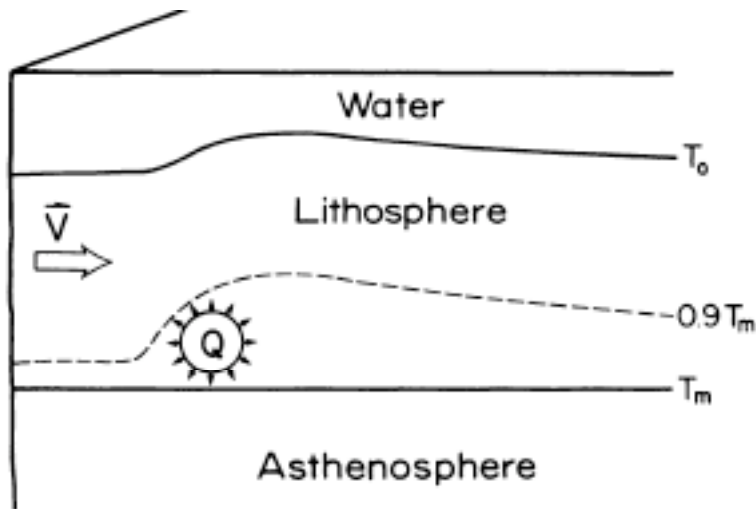


Fig. 1. As the lithosphere passes through a distributed heat source, at a velocity  $v$  it is heated, causing thermal expansion. The increase in surface heat flow far downstream from the source lowers the temperature perturbation, and the swell begins to subside. The base of the lithosphere, defined by the  $T_m$  isotherm, remains at a depth  $l$ .

(This is a new problem so please come and ask for help.)

## Group G. Frictional Heating During and Earthquake

During seismic rupture, most of the energy release due to an earthquake is spent on work against friction on a fault, and is ultimately converted into heat. The coseismically generated heat is removed from the slip surface by conduction. Because the fault thickness is much smaller than the fault's other dimensions, and because temperature variations across the fault are much higher than temperature variations along the fault, it is OK to assume that the heat transfer is governed by a 1-D diffusion equation (heat conduction in slip-parallel direction is negligible),

$$\frac{\partial T}{\partial t} = \kappa \frac{\partial^2 T}{\partial y^2} + \frac{Q}{c\rho}, \quad (1)$$

where  $T$  is rock temperature,  $y$  is the fault-perpendicular coordinate,  $\kappa$  is thermal diffusivity,  $\rho$  is density,  $c$  is heat capacity, and  $Q$  is the rate of frictional heat generation in the fault zone.

1) Show that the coseismic temperature increase  $\Delta T$  is given by

$$\Delta T(y, t) = \frac{1}{2c\rho\sqrt{\pi\kappa}} \int_0^t \int_{-\infty}^{\infty} \exp\left[-\frac{(y-\zeta)^2}{4\kappa(\tau-t)}\right] \frac{Q(\zeta, \tau)}{\sqrt{t-\tau}} d\zeta d\tau, \quad (2)$$

where  $t$  is duration of fault slip.

2) Assuming that the fault zone has a small but finite thickness  $2w$ , and slip velocity  $v$  varies linearly across the fault zone ( $\partial v/\partial y = \text{const}$ ), and does not vary along the fault, write down an expression for  $Q$ . Then, substitute it in equation (1) to obtain a solution for temperature distribution due to fault slip. Plot the temperature profile for an earthquake with the following parameters: slip velocity  $v = 1$  m/s, slip duration  $t = 5$  s, and fault zone thickness  $2w =$  (i) 1 mm, and (ii) 10 cm. The shear stress resisting fault slip is  $\mu\sigma_n$ , where  $\mu$  is the coefficient of friction, and  $\sigma_n$  is the fault-normal stress. Use  $\mu=0.6$ , and  $\sigma_n = 100$  MPa. Discuss your results.