Abyssal recipes

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Abstract—Vertical distributions in the interior Pacific (excluding the top and bottom kilometer) are not inconsistent with a simple model involving a constant upward vertical velocity $w \approx 1\cdot2$ cm day^{-1} and eddy diffusivity $\kappa \approx 1\cdot3$ cm^2 sec^{-1}. Thus temperature and salinity can be fitted by exponential-like solutions to $[\kappa \cdot d^2/dz^2 - w \cdot d/dz] T, S = 0$, with $\kappa/w \approx 1$ km the appropriate " scale height." For Carbon 14 a decay term must be included, $[\quad]^{14}C = \mu\,^{14}C$; a fitting of the solution to the observed ^{14}C distribution yields $\kappa/w^2 \approx 200$ years for the appropriate " scale time," and permits w and κ to be separately determined. Using the foregoing values, the upward flux of Radium in deep water is found to be roughly $1\cdot5 \times 10^{-21}$ g cm^{-2} sec^{-1}, as compared to 3×10^{-21} g cm^{-2} sec^{-1} from sedimentary measurements by GOLDBERG and KOIDE (1963). Oxygen consumption is computed at $0\cdot004$ (ml/l) year^{-1}. The vertical distributions of $T, S, {}^{14}C$ and O_2 are consistent with the corresponding south–north gradients in the deep Pacific, provided there is an average northward drift of at least a few millimetres per second.

How can one meaningfully interpret the inferred rates of upwelling and diffusion ? The annual freezing of $2\cdot1 \times 10^{19}$ g of Antarctic pack ice is associated with bottom water formation in the ratio 43 : 1, yielding an estimated 4×10^{20} g year^{-1} of Pacific bottom water; the value $w = 1\cdot2$ cm day^{-1} implies 6×10^{20} g year^{-1}. I have attempted, without much success, to interpret κ from a variety of viewpoints: from mixing along the ocean boundaries, from thermodynamic and biological processes, and from internal tides. Following the work of Cox and SANDSTROM (1962), it is found that surface tides are scattered by the irregular bottom into internal modes with an associated energy flux of 4×10^{-6} ergs g^{-1} sec^{-1} (one sixth the total tidal dissipation). Such internal modes can produce shear instability in the Richardson sense. It is found that internal tides provide a marginal but not impossible mechanism for turbulent diffusion in the interior oceans.

INTRODUCTION

IN THE course of preparing for a program of measuring oceanographic variables *from the bottom up*, I have reviewed various models for the distribution of T, S, ^{14}C, O_2 etc. in the deep sea. In this connection the box models of the radio chemists are of little use. But a very simple model of diffusion and advection, coupled with the appropriate mechanism of decay when required. leads to a set of distribution functions which fit the observations rather better than one would expect. The model is not new; it was used by WYRTKI (1962) in a discussion of the oxygen minimum, and in various forms goes back to oceanographic antiquity; nor are the numerical results much different from those obtained by Stommel and his collaborators in their manifold attacks on the abyssal problem (STOMMEL, 1958; ROBINSON and STOMMEL, 1959; STOMMEL and ARONS, 1960a and 1960b; BOLIN and STOMMEL, 1961); and by WOOSTER and VOLKMANN (1960) and KNAUSS (1962).

The following discussion is limited to the central Pacific between 1 and 4 km because the situation there is relatively simple.

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TEMPERATURE – SALINITY

Figures 1 and 2 show the potential temperature – salinity relation at stations in the Mindanao Deep occupied 21 years apart. The observed temperature has been corrected at each depth for the adiabatic increase, the vertical gradient of which is given by
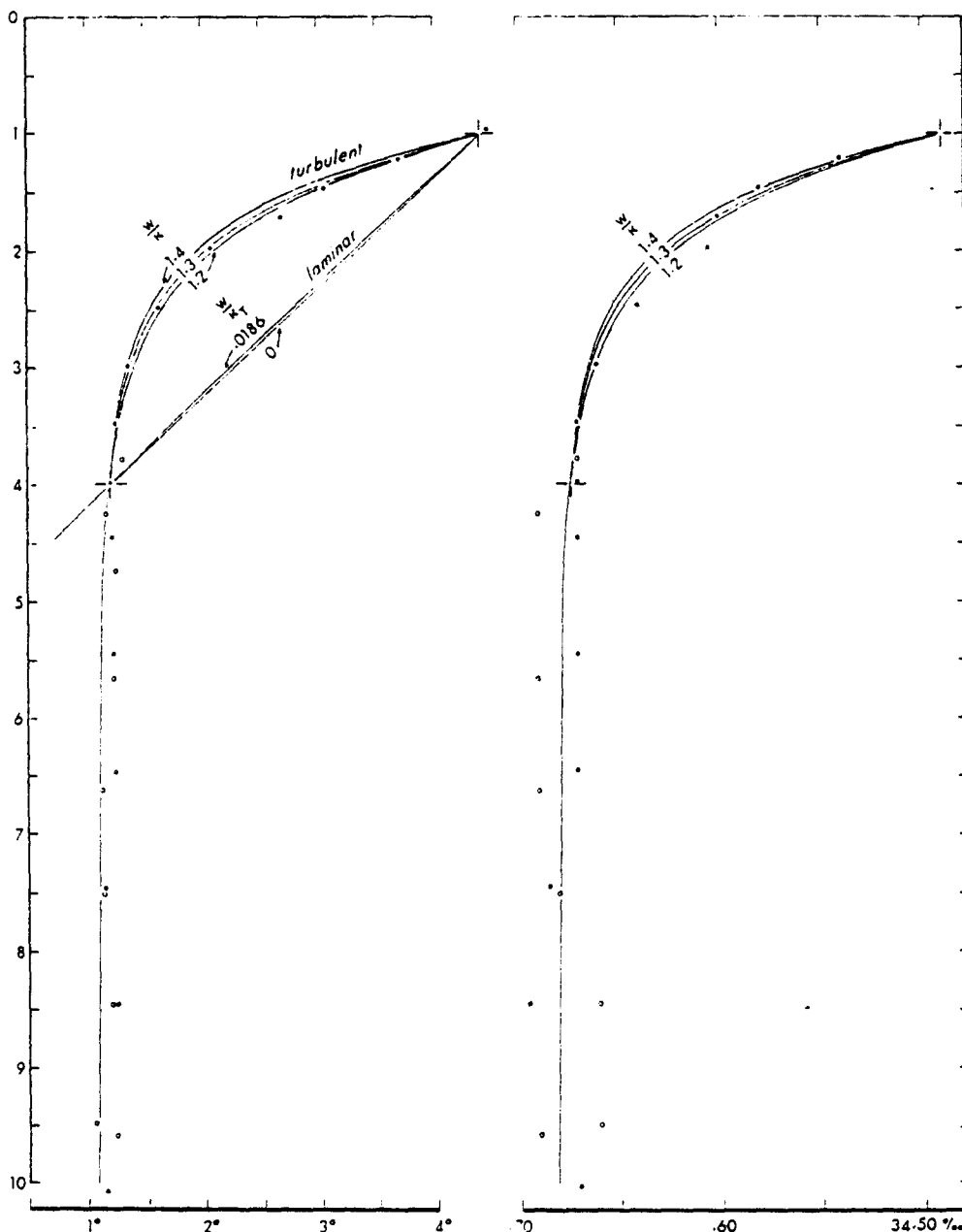

Fig. 1. Potential temperature and salinity as functions of depth (km) at stations *Snellius* 1930 : ≠ 262, 9° 41′N, 126° 51′E, (closed circles) and *Galathea* 1951 : ≠ 433, 9° 51′N, 126° 51′E, (open circles). Curves labeled w/κ (in units km⁻¹) are based on equations (1) and (2) for turbulent and laminar diffusion, respectively.
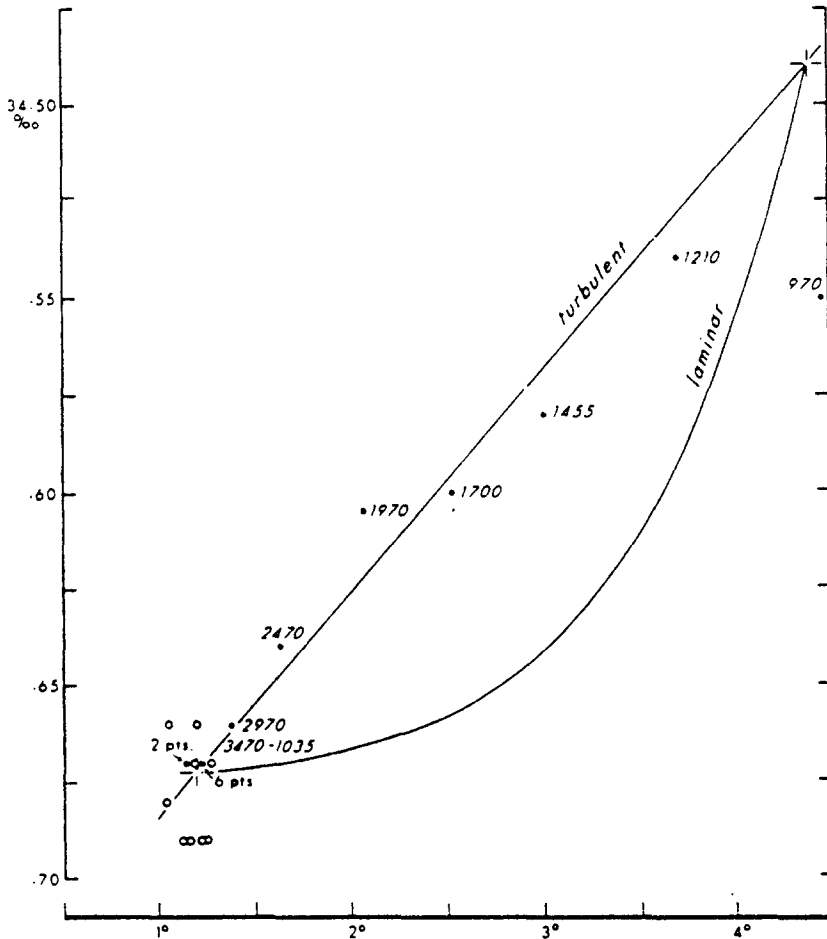
Fig. 2. The potential temperature – salinity relation for the *Snellius* and *Galathea* stations. Depth is indicated in meters.

$$dT_A/dz = T'_A = - \frac{g\,(\bar{\gamma} - 1)}{ac^2} = - \, 0\cdot00623\,(T + 7\cdot0 + 2P)\,^{\circ}\text{C km}^{-1}$$

in accordance with the equations 4–6, 7, 8, 9 and 7–8 of ECKART (1960); here z is positive upwards, $\bar{\gamma}$ the ratio of specific heats, a the thermal expansion, c the velocity of sound, T the temperature in centigrade, and P the pressure in units of 10^2 bars (or roughly the depth in km). The effect of salinity is ignored. Typically at 4 km we have $T = 1\,^{\circ}\text{C}$, and $T'_A = - \, 0\cdot10\,^{\circ}\text{C km}^{-1}$, subject to an uncertainty of perhaps 10% because of our imperfect knowledge concerning the equations of state of sea water at large pressure. Within these limits and those imposed by observational error, the temperature is adiabatic beneath 4 km, and the salinity is constant. The important question as to whether conditions are slightly sub-adiabatic or super-adiabatic cannot be answered.

We now regard the distribution as steady (there were no significant alterations in 21 years) and governed by a balance between vertical diffusion, and the effect of slowly rising water (due to the production of Bottom Water) :

$$\text{turbulent}: \quad \kappa\,\hat{T}'' - w\hat{T}' = 0; \quad \kappa\,S'' - wS' = 0 \tag{1a, b}$$

$$\text{laminar}: \quad \kappa_T\,T'' - w\hat{T}' = 0; \quad \kappa_S\,S'' - wS' = 0 \tag{2a, b}$$

For the case of turbulent diffusion the same Austausch coefficient κ applies to the temperature* $\hat{T} = T - T_A$ and salinity S, thus leading to identical depth distributions and to a linear relation $T(S)$. For constant w/κ,

$$\frac{\hat{T} - \hat{T}_1}{\hat{T}_2 - \hat{T}_1} = \frac{e^{\gamma\xi} - 1}{e^{\gamma} - 1}, \quad \frac{S - S_1}{S_2 - S_1} = \frac{e^{\gamma\xi} - 1}{e^{\gamma} - 1} \tag{3a, b}$$

where

$$\gamma = \frac{(z_2 - z_1)\,w}{\kappa}, \quad \xi = \frac{z - z_1}{z_2 - z_1}.$$

A fair fit is found for the values $\gamma = 3\cdot9$ with $z_1 = -4\,\text{km}$, $z_2 = -1\,\text{km}$; the ratio $\kappa/w = 0\cdot77\,\text{km}$ is the pertinent scale height. We do not pretend that the observed exponential-like distribution confirms the vertical diffusion-advection model, but it is not inconsistent with it.

It is by no means obvious that the deep sea should be in a state of turbulent flux. The usual criterion for shear-induced turbulence in a stably stratified fluid is that the Richardson number $Ri = N^2/u'^2$ should be less than $\frac{1}{4}$; the Väisälä (or stability) frequency N is typically $10^{-3}\,\text{c/s}$ (ECKART 1960, p. 68) so that the vertical shear, u', should exceed $2 \times 10^{-3}\,\text{sec}^{-1}$ ($2\,\text{m sec}^{-1}$ per km) to give turbulence. The observed magnitude of horizontal flow in the deep sea is 1–$10\,\text{cm sec}^{-1}$. We shall return to this question.

The laminar case differs from the turbulent case in two ways: (i) thermal molecular diffusion applies to T'', not $\hat{T}''$ (the final diffusive state is isothermal, not adiabatic), and (ii) thermal and haline diffusivities differ by a factor

$$\kappa_\theta/\kappa_S = \gamma_S/\gamma_\theta = 1\cdot4 \times 10^{-3}/2 \times 10^{-5} = 70.$$

Suppose that $S(z)$ is governed by (3b) with γ replaced by $\gamma_S = (z_2 - z_1)\,w/\kappa_S$. This implies that $w = (w/\kappa_S)\,\kappa_S = 1\cdot3 \times 10^{-5} \cdot 2 \times 10^{-5} \approx 10^{-3}\,\text{cm year}^{-1}$, and so $\gamma_\theta = \gamma_S/70 = 0\cdot055$, with $z_2 - z_1 = 3\,\text{km}$. The solution to (2a) is

$$\frac{T - T_1}{T_2 - T_1} = \frac{\hat{T} - \hat{T}_1}{\hat{T}_2 - \hat{T}_1} = \frac{e^{\gamma_\theta\xi} - 1}{e^{\gamma_\theta} - 1} \approx \xi - \tfrac{1}{2}\gamma_\theta\,(\xi - \xi^2). \tag{4}$$

To a first order, $T(z)$ is linear with depth and hence in conductive equilibrium (Fig. 1). To a second order, the vertical motion introduces a curvature γ_θ. The observations are incompatible with the laminar model, and one can conclude that molecular processes contribute less than 1% to the total diffusion. Similar conclusions can be reached from the uniformity in certain isotopic ratios (CRAIG and GORDON, in press) and in the chlorinity/salinity ratio. Briefly the argument is as follows: with $w \approx 10^{-3}\,\text{cm sec}^{-1}$ as implied by molecular processes, the " residence time " $(z_2 - z_1)/w$ is 100 million years. But gravitational differentiation would lead to

*Here $T_A = \int_z^0 T'_A\,[T(z), P(z)]\,dz$, where $T(z)$ is the local observed temperature, not the temperature of a parcel lowered adiabatically from the surface. I refer to $T - T_A$ as potential temperature, though this is not strictly in accord with the usual definition.
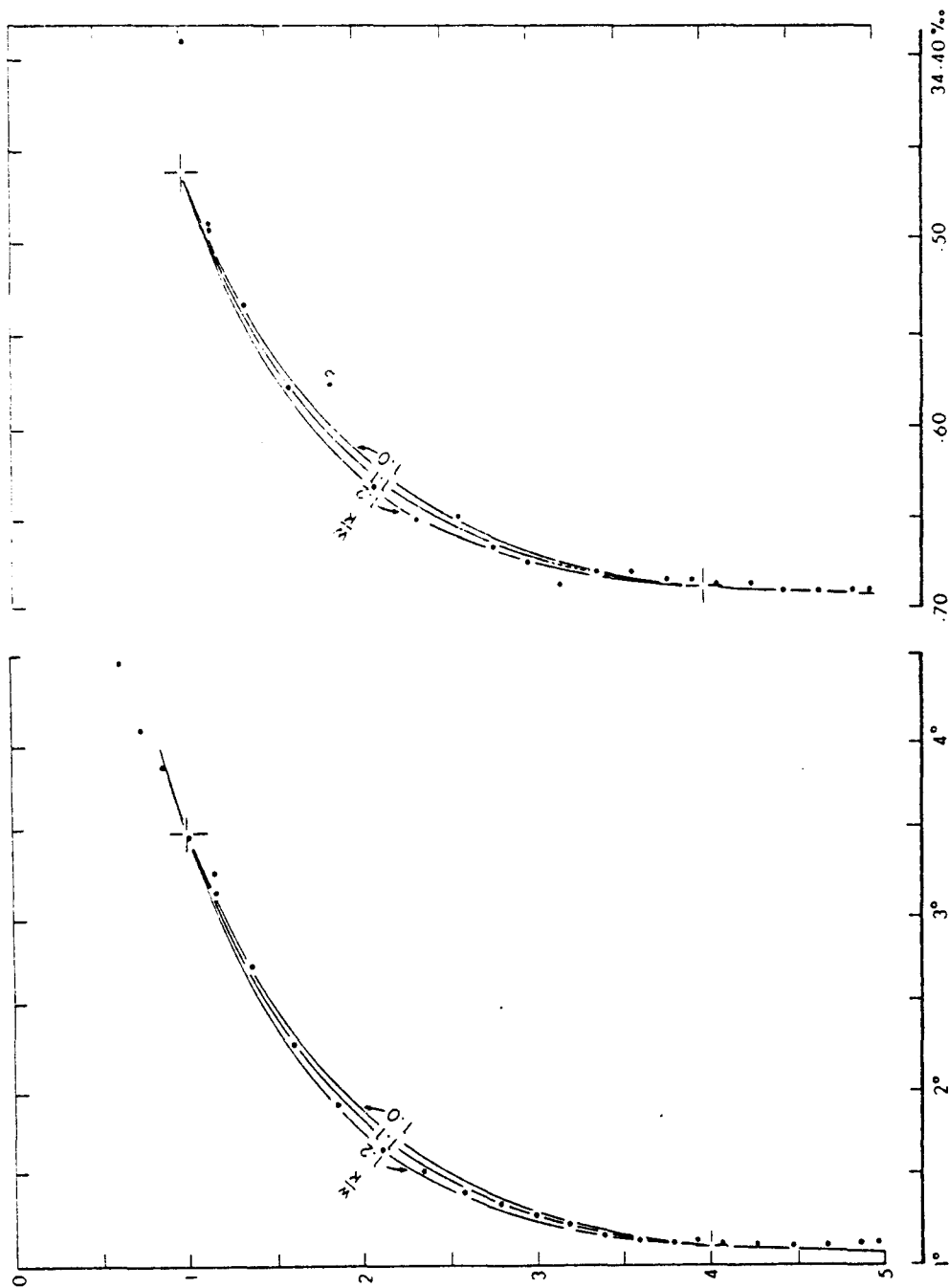
Fig. 3. Potential temperature and salinity as functions of depth (km) at station *Calcofi* 1964: # 60·190, 33° 17'N, 132° 42·5'W (salinity at depth 1859 m was questioned in the original observations). Curves labeled w/κ (in units km^{-1}) are based on equation (1).
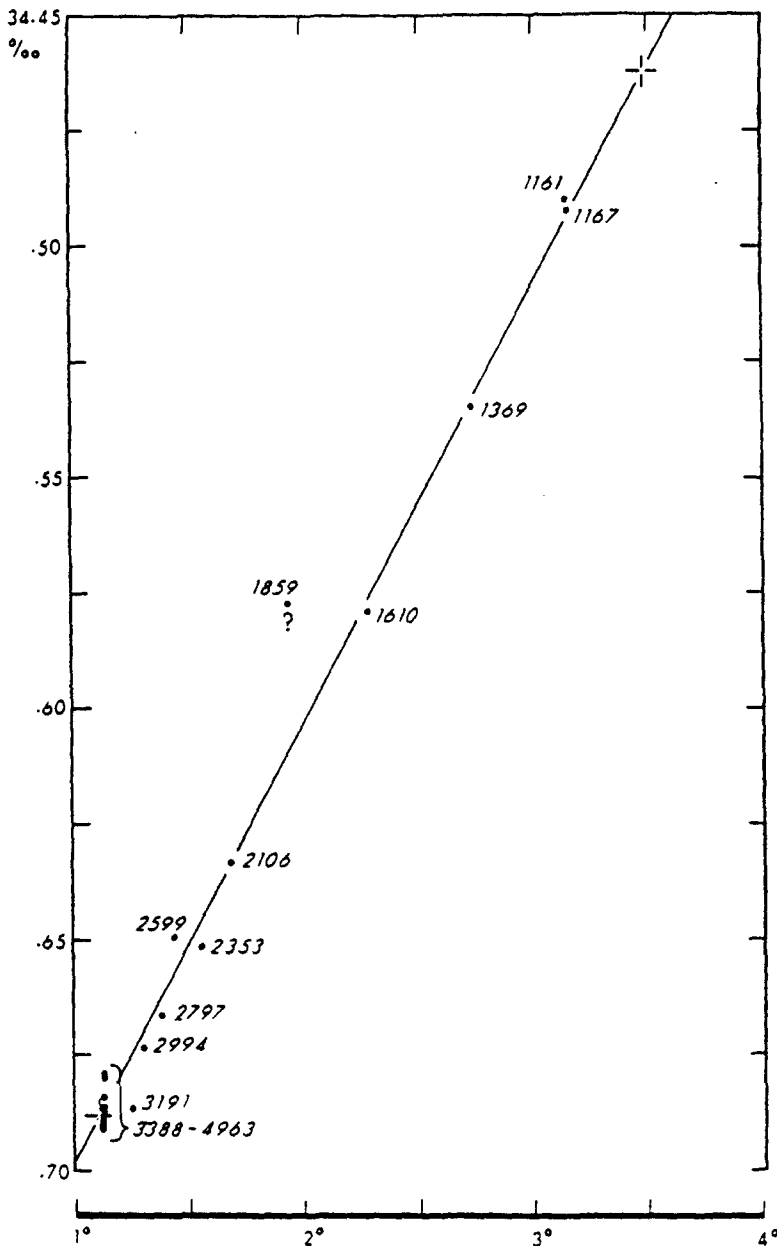
Fig. 4. The potential temperature – salinity relation for station *Calcofi* 60·190, with depth
indicated in meters.

observable nonuniformity after only a million years. Thus w must be at least 100 times
larger, and for a fixed w/κ so must κ. The problem merits further consideration.

The conclusion is that the Mindanao distributions of T and S are consistent with
a model involving vertical advection and diffusion characterized by a scale height
$\kappa/w = 0.8$ km, and the additional constraint that molecular processes contribute
less than 1 % to the diffusion. Figures 3 and 4 show a similar situation with regard

to observations off California, giving $\gamma = 3.3$, hence $w/\kappa = 1.1$ km^{-1}. Here the use of a salinometer has resulted in a reduction in the salinity scatter as compared to the Mindanao titrations.

CARBON 14 AND RADIUM 226

The distribution of ^{14}C is governed by the equation[*]

$$\kappa \, (^{14}\mathrm{C})'' - w \, (^{14}\mathrm{C})' = \mu_C \, {}^{14}\mathrm{C} \tag{5}$$

$\mu_C = 0.693/5568 = 1.24 \times 10^{-4}$ years^{-1} = 3.93×10^{-12} sec^{-1} is the decay constant. The solution is

$$^{14}\mathrm{C} = C^+ \, e^{\frac{1}{2}\gamma(1+\lambda)\,\xi} + C^- \, e^{\frac{1}{2}\gamma(1-\lambda)\xi}, \tag{6}$$

$$\lambda_C^2 = 1 + 4x_C, \quad x_C = \kappa\mu_C/w^2,$$

where the constants

$$C^{\pm} = \pm \frac{{}^{14}C_2 - {}^{14}C_1 \, e^{\frac{1}{2}\gamma(1 \mp \lambda)}}{e^{\frac{1}{2}\gamma(1+\lambda)} - e^{\frac{1}{2}\gamma(1-\lambda)}}$$

are determined by the concentrations $^{14}C_1$, $^{14}C_2$ at z_1, z_2 ($\xi = 0, 1$) respectively. Pacific measurements[†] by BIEN, RAKESTRAW and SUESS (1965) are badly scattered
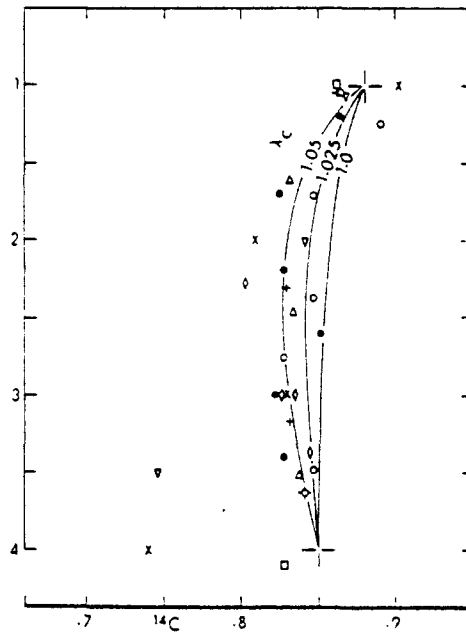

Fig. 5. ^{14}C (relative units) as function of depth (km) (from BIEN, RAKESTRAW and SUESS, 1965). Symbols represent various Pacific stations. Curves are based on equation (6) for $\gamma = 3.3$, and for indicated values of λ_C.

(Fig. 5); we shall adopt $\lambda_C = 1.05$, hence $w^2/\kappa = 1.54 \times 10^{-10}$ sec^{-1}. With w/κ and w^2/κ given, w and κ can now be determined. From here on we use the values

[*]Ignoring a possible transport by organisms (LAL, 1962).
[†]The plotted values are $1 + 10^{-3} \delta {}^{14}$C, where $\delta {}^{14}$C is the measured anamoly relative to some standard reference. A similar plot for $\Delta {}^{14}$C (corrected for ^{13}C/^{12}C ratios) gives essentially the same results. For our purpose the uncorrected $\delta {}^{14}$C anomalies provide a more direct comparison with the diffusion-advection model.

$$\gamma = 3{\cdot}3, \quad w = 1{\cdot}4 \times 10^{-5} \,\text{cm sec}^{-1}, \quad \kappa = 1{\cdot}3 \,\text{cm}^2\,\text{sec}^{-1}.$$

The estimates are in line with those given by STOMMEL and others.

Observations of ^{226}Ra ($\mu_R = 0{\cdot}693/1620 = 4{\cdot}27 \times 10^{-4}$ years$^{-1} = 13{\cdot}6 \times 10^{-12}$ sec^{-1}) are badly scattered (Fig. 6), but do not rule out $\lambda_R = 1{\cdot}17$ which leads to the previous selection of w^2/κ. Radium observations by PETTERSSON (1955) are even more scattered, but not inconsistent with this choice of parameters.
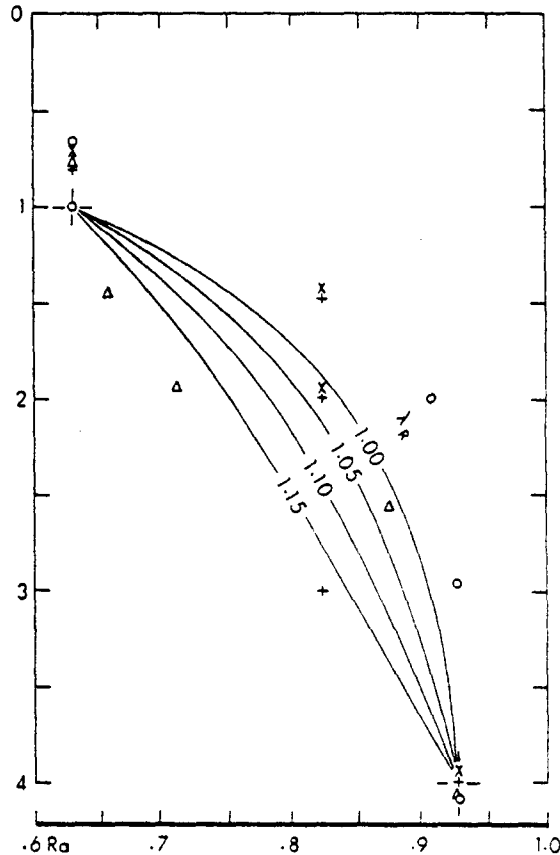

Fig. 6. Radium (in 10^{-13} g/l) as function of depth (km) at four Pacific stations (from MIYAKE and SUGIMURA, 1964). Values have been adjusted to agree at 1 and 4 km. Curves are based on equation (6) for $\gamma = 3{\cdot}3$, and for indicated values of λ_R.

The upward flux of radium R near the bottom can be evaluated from (6):

$$(wR - \kappa R')_{t=0} \approx w\left\{(1 + \alpha_R)\,R_1 + e^{-\gamma(1 + x_R)}\,(R_1 - R_2)\right\}$$

The flux is $1{\cdot}4 \times 10^{-21}$ g cm^{-2} sec^{-1} for $x_R \approx \tfrac{1}{2}(\lambda_R - 1) = 0{\cdot}085$, $R_1 = 0{\cdot}93 \times 10^{-16}$ g cm^{-3}, $R_2 = 0{\cdot}63 \times 10^{-16}$ g cm^{-3}. This can be compared with an estimate of the upward flux *through the sediments* based on measurements by GOLDBERG and KOIDE (1963). The sedimentary radium balance is given by

$$\frac{\partial R}{\partial t} = \kappa R'' - \mu_R R + \mu_T T$$

under the assumptions that $T (= {}^{230}\text{Th})$ does not diffuse in the sediments, and that the accumulating sediments have a constant concentration of ^{230}Th. GOLDBERG and KOIDE assume steady-state conditions relative to an instantaneous bottom rising at the rate c cm $\sec^{-1}$ due to sedimentation. Accordingly they replace z by $z - ct$ and $\partial/\partial t$ by $\partial/\partial t - c (\partial/\partial z) = - c (\partial/\partial z)$. The upward flux is then given by

$$- \kappa R' - cR = \int_0^{-\infty} (\mu_T T - \mu_R R) \, dz.$$

In the published tables, ^{230}Th and ^{210}Pb are given in disintegrations per minute per gram of sediment (density $\rho \approx 1.5$ g cm^{-3}). The disintegrations of ^{210}Pb and ^{226}Ra can be taken as equal. The analyses are for each 3 cm of sediment thickness. Accordingly we write

$$- \kappa R' - cR = \frac{(3 \text{ cm}) (1.5 \text{ g cm}^{-3})}{(60 \text{ sec/min}) \, 6.02 \times 10^{23} \text{ atoms/mole}} [230 \, \Sigma^{230}\text{Th} - 226 \, \Sigma^{226}\text{Ra}]$$

$$\approx 2.8 \times 10^{-23} \, \Sigma (^{230}\text{Th} - {}^{226}\text{Ra}) \text{ g cm}^{-2} \text{ sec}^{-1}.$$

The difference between the ^{230}Th and ^{216}Pb counts vanish beneath 21 cm of core depth, and the summations are over the upper seven analyses in the table. The results for stations Monsoon 49 G, 57 G and 68 G (adjusted to 21 cm) are 3.1, 5.3 and 2.1×10^{-21} g cm^{-2} sec^{-1}, as compared to 1.4×10^{-21} g cm^{-2} sec^{-1} for the radium flux in the water at a 4-km depth*. In view of the many uncertainties we consider the agreement to be remarkably close.

CONSUMPTION OF OXYGEN

The consumption of O_2 by organisms and bacteria is believed to be limited by factors other than the O_2 concentration. The simplest formulation is to assume a constant rate of consumption by v ml/l per second. Accordingly

$$\kappa O''_2 - w O'_2 = v, \quad \frac{O_2 - (O_2)_1}{(O_2)_2 - (O_2)_1} = \frac{e^{\gamma \xi} - 1}{e^{\gamma} - 1} (1 + \beta) - \beta \xi \qquad (7, 8)$$

where $\beta = v/(w\overline{O'_2})$, and $\overline{O'_2} = [(O_2)_2 - (O_2)_1]/[z_2 - z_1]$ is the mean gradient. There is an oxygen minimum at

$$\xi = \frac{1}{\gamma} \ln \left[\frac{\beta}{1 + \beta} \frac{e^{\gamma} - 1}{\gamma} \right]$$

provided $(- \beta) > 1$. Figures 7 and 8 show the distribution at the *Calcofi* station previously discussed. A fit can be made with

$$\gamma = 3.3, \quad - \beta = 0.6$$

for $\overline{O'_2} = - 0.97$ (ml/l) km^{-1}. This gives $v/w = + 0.6$ (ml/l) km^{-1}. By an equivalent method, WYRTKI (1962) finds $w/\kappa = 10^{-5}$ cm^{-1} and $v/w = 1.2$ (ml/l) km^{-1} for the Coral Sea. The consumption of oxygen is accompanied by a production of phosphate in the ratio $0.37 \, \mu$g at/l of phosphate per ml/l of O_2 (REDFIELD, 1958). The phosphate distribution is then represented by equations (7, 8) with v replaced by $- 0.37 \, v$.

*Allowing for the radium decay in the slowly rising water between station depths and 4 km increases this value by 10%.
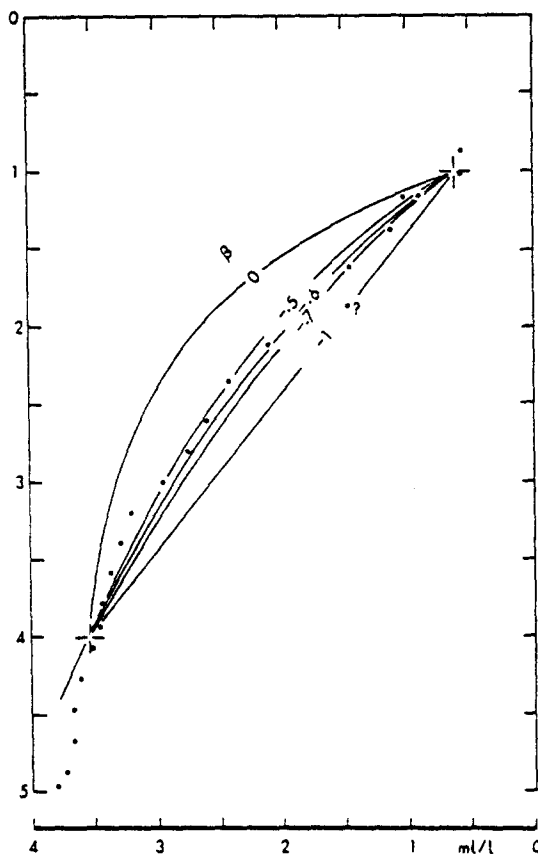
Fig. 7.   Oxygen in ml/l as function of depth (km) of station *Calcofi* ≠ 60·190 (see Fig. 3 for temperature and salinity profiles). Curves are based on equation (8) for $\lambda = 3.3$, and for indicated values of β.

The distributions differ in shape, and the extremes (if they occur) are at different depths. All this has been pointed out by WYRTKI. (For better fitting in the upper layers, WYRTKI assumes the oxygen consumption to decrease exponentially with depth).

With the previous value for w, the oxygen consumption is estimated at

$$\nu = 0.0027 \text{ to } 0.0053 \text{ (ml/l) year}^{-1}.$$

RILEY (1951) has estimated 0·002 (ml/l) year^{-1} for the deep Atlantic.

HORIZONTAL GRADIENTS

An order-of-magnitude comparison of our results with measurements of south-north gradients near the bottom is desirable. We assume a northward flow of bottom water within a layer of thickness h, neglecting all east–west assymetries. Then the velocity, salinity, ^{14}C, O_2 and temperature, averaged over this layer, diminish northward at an average rate

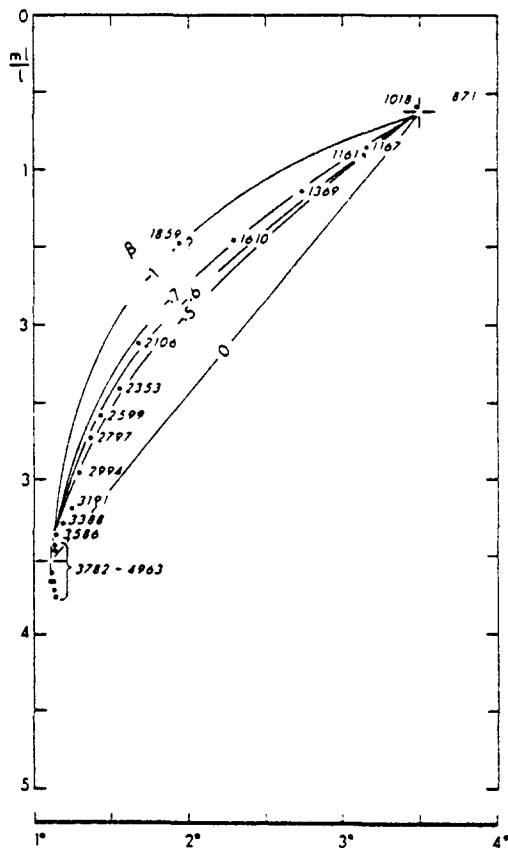$$- \rho \frac{\partial}{\partial x} (vh) = \rho w \tag{9}$$

Fig. 8. The potential temperature-oxygen relation for station *Calcofi* # 60·190, with depth indicated in meters.

$$- \rho \frac{\partial}{\partial x} (vhS) = \rho (wS - \kappa S') \tag{10}$$

$$- \frac{\partial}{\partial x} (vh \,^{14}C) = w \,^{14}C - \kappa \,^{14}C' + h \, \mu \,^{14}C \tag{11}$$

$$- \frac{\partial}{\partial x} (vhO_2) = wO_2 - \kappa O'_2 + hv \tag{12}$$

$$- \rho c \frac{\partial}{\partial x} (vhT) = \rho c (wT - \kappa T') - F \tag{13}$$

where x and v are positive northward. Terms on the right-hand-side represent the loss by vertical advection and diffusion through the top of the bottom layer, and by decay (or generation), if any.

The difference in northward water transport between Antarctic and Arctic ($l \approx 10^4$ km) is then

$$(vh)_S - (vh)_N \approx (vh)_S = wl. \tag{14}$$

Salinity

Subtracting (9) from (10) gives $- vh \, (\partial S/\partial x) = - \kappa S'$. According to equation (3)

$$S' = (S_2 - S_1) \frac{w}{\kappa} \frac{e^{\gamma\xi}}{e^\gamma - 1}$$

Let $\partial S/\partial x = (S_N - S_S)/l$, $vh = \frac{1}{2}wl$, and identify (somewhat arbitrarily) the upper boundary of the bottom layer with $\xi = 0$ (depth 4 km). Then

$$S_S - S_N = -2\,(\kappa/w)\,S'_{\xi=0} \approx 2e^{-\gamma}\,(S_1 - S_2) \approx 0.02\text{‰}, \tag{15}$$

compared with an observed difference of roughly $0.03‰$ between southern and northern abyssal salinities (WOOSTER and VOLKMANN, 1960; KNAUSS, 1962).

Carbon-14

In a similar manner, we find

$$^{14}C_S - {}^{14}C_N = -2\,(\kappa/w)\,{}^{14}C'_{\xi=0} + v^{-1}\,l\,\mu_C\,{}^{14}C_1 \tag{16}$$

where

$$-2\,(\kappa/w)\,{}^{14}C'_{\xi=0} = (-1 + \lambda\coth\tfrac{1}{2}\gamma\lambda)\,C_1 - (\lambda\,e^{-\frac{1}{2}\gamma}\operatorname{csch}\tfrac{1}{2}\gamma\lambda)\,C_2$$

$$\approx 2\varkappa\,{}^{14}C_1 + 2e^{-\gamma(1+\varkappa)}\,({}^{14}C_1 - {}^{14}C_2).$$

Equation (16) is to be interpreted as follows: in addition to the radio-active decay $\mu\,{}^{14}C$ in the northward moving water there is a loss by vertical diffusion; if diffusion is ignored, the northern water will appear too " old," and the computed velocities too small. The need for this correction was pointed out by KNAUSS (1962). Letting $^{14}C_1 = 0.85$, $^{14}C_2 = 0.88$, $^{14}C_S = 0.820$, $^{14}C_N = 0.775$, $l = 10^9$ cm, gives

$$\frac{^{14}C_S - {}^{14}C_N}{^{14}C_1} = 0.056, \quad 2\,\frac{\kappa}{w}\,\frac{^{14}C'_{\xi=0}}{^{14}C_1} = -0.052 \tag{17}$$

so that $l\,\mu_C\,v^{-1} = 0.056 - 0.052 = 0.004$ is a small difference between two uncertain values. This gives $v = l\,\mu_C/0.004 \approx 1$ cm sec^{-1}. The uncorrected value, $v = l\,\mu_C/0.056 \approx 0.07$ cm sec^{-1}, corresponds to the latest estimate by BIEN, RAKESTRAW and SUESS (1965). The conclusion is that the observed northward " aging " of bottom water is in part, and perhaps largely, the result of vertical diffusion, and can yield only a lower limit to the permissible velocity.

The effective thickness of the bottom current has an upper limit

$$h = \frac{vh}{v} = \frac{1}{2}\,\frac{wl}{v} \approx 1 \text{ km} \tag{18}$$

for $v = 0.07$ cm sec^{-1}.

Oxygen

For O_2 we find

$$(O_2)_S - (O_2)_N = -2\,(\kappa/w)\,(O_2)'_{\varkappa=0} + v^{-1}lv = -2\big[e^{-\gamma}(1+\beta)\big]\big[(O_2)_2 - (O_2)_1\big] + v^{-1}lv$$

$$\approx (1.52 + 0.07)\text{ ml/l to } (1.52 + 0.70)\text{ ml/l} \tag{19}$$

for $v = 1$ cm sec^{-1} and 0.1 cm sec^{-1}, respectively, using $\gamma = 3.3$, $\beta = -0.6$, $(O_2)_2 - (O_2)_1 = -3.88$ ml/l, $\nu = 0.7 \times 10^{-10}$ ml/l sec^{-1} and $l = 10^9$ cm. This compares reasonably well with the estimates by WOOSTER and VOLKMANN (1960).

We note that a significant fraction of the northward decrease is due to diffusion into the upper layers.

Heat

Proceeding as before, we find

$$T_N - T_S = 2F/(\rho c w) + 2(\kappa/w)T' = 2F/(\rho c w) + 2e^{-\gamma}(T_2 - T_1) = (0.14 \div 0.24)\,°C \quad (20)$$

using $F = 10^{-6}$ cal cm^{-2} sec^{-1}, $\rho c = 1$ cal deg C^{-1} cm^{-3}, $T_2 - T_1 = 3.20$ °C. The observed northward warming near the bottom is of this order (WOOSTER and VOLKMANN, 1960), suggesting that the entire bottom layer (50–500 m thick, depending on v) is convectively stirred. The problem requires further consideration.

<div align="center">UPWELLING</div>

The vertical distribution of variables is consistent with a model that calls for an upwelling by 1.2 cm day^{-1}. The area of the Pacific (excluding marginal seas) is 1.65×10^{18} cm^2. Probably we wish to exclude the area south of the Antarctic Convergence, and this leaves 1.37×10^{18} cm^2. The upward flux of mass in the Pacific is then 6.0×10^{20} g year^{-1}.

Can this be reconciled with the rate of formation of bottom water ? The formation of sea ice involves the seasonal thickening by roughly 1 m at the bottom of the permanent Antarctic ice sheet (area 4×10^{16} cm^2) as well as the seasonal extension of the ice cover by 16×10^{16} cm^2 with a typical thickness of 1 m. From direct observations as well as considerations of the heat budget of an ice sheet, UNTERSTEINER (1964, 1966) estimates that 2.3×10^{19} cm^3 $\pm 20\%$ of Antarctic sea ice is formed each year. W. I. WITTMANN (personal communication) estimates 2.5×10^{19} cm^3 from U.S. Navy Hydrographic Office Publication HO 705 (Part 1, Antarctic), 2.2×10^{19} cm^3 on the basis of pack-ice boundaries from *Nimbus I* satellite views, 28 August to 22 September 1964, and 2.55×10^{19} cm^3 from his estimate of the minimum, and that of BUINITSKII (1956) for the maximum.

For a density of 0.9 g cm^{-3}, the yearly ice formation may be taken as 2.1×10^{19} g. The salinity of new sea ice is 4–5‰. In the process of ice formation the residual salty water sinks and entrains additional water, eventually forming bottom water.

Let M_i, S_i designate the mass and salinity of the ice formed each year, M_0, S_0 refer to the surface water from which ice and bottom water are formed, and M_b, S_b to the resulting bottom water. Then

$$M_0 = M_i + M_b, \quad M_0 S_0 = M_i S_i + M_b S_b$$

and so
$$\frac{M_b}{M_i} = \frac{S_0 - S_i}{S_b - S_0} \quad (11)$$

grams of bottom water is formed for each gram of frozen ice. Salinities of the surface water and ice are typically 34‰ and 5‰, respectively. If the residual water consisted of 150‰ brine, then the ratio is $(34‰–5‰)/(150‰–34‰) \approx 0.25$. In fact the ratio is much larger than 1, because of the entrainment of surrounding water. Taking the observed value $S_b = 34.67‰$, the " entrainment ratio " equals 43.

ÇRAIG and GORDON (1965, Fig. 12) have determined the ^{18}O–S values in the Weddell Sea from measurements on *Eltanin* Cruise 12. Shallow and deep salinities are 34‰ and 34.67‰ respectively; the ^{18}O anomaly remains fixed at $- 0.45‰$ for

the entire water column, in support of the assumption that the bottom water is locally formed by vertical mixing.

The production of bottom water is accordingly estimated at $2.1 \times 10^{19} \times 43 = 9 \times 10^{20}$ g year^{-1}, and perhaps half goes into the Pacific. This compares to 6×10^{20} g year^{-1} from the vertical mass flux.

There are many uncertainties in our estimate of the production of bottom water. We have neglected processes other than freezing. In the Arctic it is known that in open water areas of high salinity and near-uniform density the loss of surface heat and the resulting deep convection is a principal factor, and in the Antarctic it may be a significant factor (LEDENEV, 1960).

Our model has ascribed to the surface water the exclusive role of being the water from which the ice is crystallized, and the water with which the sinking residual is diluted to form bottom water, thus ignoring the dilution with, and formation of, intermediary water. To go further one will need some understanding of the processes involved in the formation of bottom water. Here one can take two extreme views. One is that the heavy residual water, formed just under the ice, " rains " downward in filaments into a reservoir of bottom water which slowly empties to the north (Fig. 9). A suitable starting point in the analysis of filaments and their entrainment of surrounding water is the analysis of plumes from maintained sources of buoyancy (such as household chimneys) as given by MORTON, TAYLOR and TURNER (1956) and subsequent work by ELLISON and TURNER (1959) on turbulent entrainment in stratified flows. The formation of residual water over the continental shelf, and the subsequent flow down the continental slope with some entrainment of the overlying water is a special case of the *filament hypothesis*. Perhaps the differential in the rates of diffusion of salt and heat leads to an " inverted salt fountain " in the sense of STOMMEL *et al.* (1956) and STERN (1960). L. COACHMAN'S (private communication) laboratory experiments indicate the presence of penetrating filaments.

The other view is that of *convective stirring* of the entire water column between ice and bottom. Laboratory results concerning convection between flat plates (see Appendix) lead to the conclusion that except for a boundary layer of negligible thickness, one should expect the formation of a uniform water type (salinities within $0.1\%_0$). I am not enthusiastic about this analogy with laboratory convection, for the ice sheet is anything but a uniform flat plate, and the system is not confined by lateral walls but in communication with the open sea to the north. (Could the stirred water mass be geostrophically confined ?).

In the case of the filament formation one should expect a young surface and bottom layer, with older water in between. Convective stirring would lead to a gentle but monotonic increase of " age " with depth, and thus be associated with anomalously (as compared to typical Pacific profiles) old surface water and anomalously young bottom water. The ^{14}C measurements of BIEN *et al.* (1965, Fig. 9) clearly indicate anomalously old surface water south of 40°S, (450 years as compared to typically 150 years), and anomalously young and uniform water between 1 km and the bottom (1400 years as compared to typically 2000 years). This would favour the convection hypothesis. The situation is very confusing, and I doubt whether any substantial progress can be made until one understands something about penetrative convection. This point has been made by STEWART (1962) who emphasizes also the need for direct measurements under the ice.
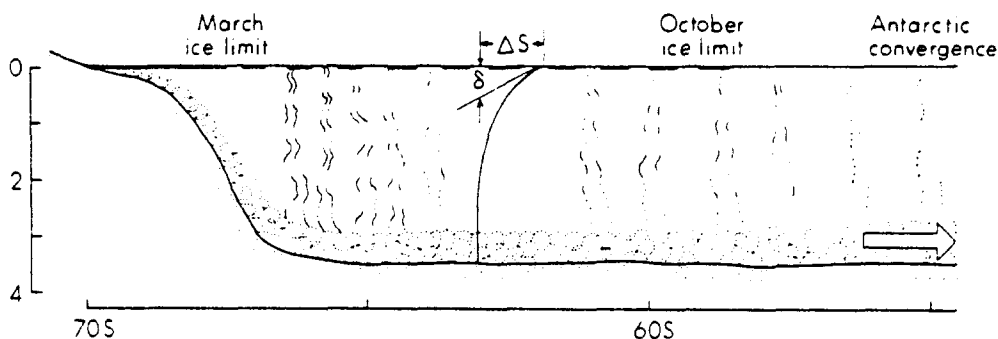
Fig. 9. Schematic diagram of Antarctic profile, with minimum and maximum of ice cover indicated. The curve shows a convective salinity distribution, with an increase by ΔS largely within a surface boundary layer δ. Sinking of saline water in vertical filaments and down the continental slope is indicated schematically.

I would conclude that, regardless of the degree of penetration and entrainment, the simple argument (11) concerning the formation of bottom water supports the previous estimate of an upwelling by 1 cm day^{-1} over the interior Pacific. The heat flow of 0·1 cal cm^{-2} day^{-1} from the Earth's interior can maintain the bottom water 0·1 °C above what it would be otherwise. If the formation of cold bottom water were to cease (possibly because of the absence of sea ice), then the deep oceans would be heated from the bottom up, and the temperature of the entire water column would rise at the average rate of 1 °C in 10,000 years; the resultant thermal expansion raises the sea level by about 1 m in 10,000 years. If all the geothermal heat were to be used in the melting of ice, the resulting rise in sea level would be 40 m in 10,000 years.

DIFFUSION

The estimate $\kappa \approx 1$ cm^2 sec^{-1} provides no insight into the processes responsible for vertical mixing in the deep interior, unless it can be reconciled with physical considerations. We consider four models in the order of increasing strangeness.

1. *Boundary mixing*

ISELIN (1939) has noted the remarkable resemblance of the vertical T–S relation in the Sargasso Sea with the T–S relation of the North Atlantic surface water; similar conditions obtain in the Pacific. In the schematic presentation of Fig. 10, the T–S relations along OA and OB are thus similar. ISELIN's interpretation is that the water attains its T–S characteristics by surface processes, and these characteristics are readily communicated along surfaces of constant potential density $\bar{\rho}$ into the interior. The profile of $\bar{\rho} = 1.027$ outcrops at high latitudes, and we may expect the distribution above this surface to be strongly influenced by surface mixing. At mid-latitudes this surface extends to a depth of almost 1 km and this is the reason why we have confined our attention to deeper water.

If the surface boundary is vital to the properties of the upper 1 km, then perhaps the lateral boundaries are vital to the deeper distribution. One can envisage strong mixing along the ocean boundaries and islands due to current shear and internal wave breaking, together with an effective communication into the interior along
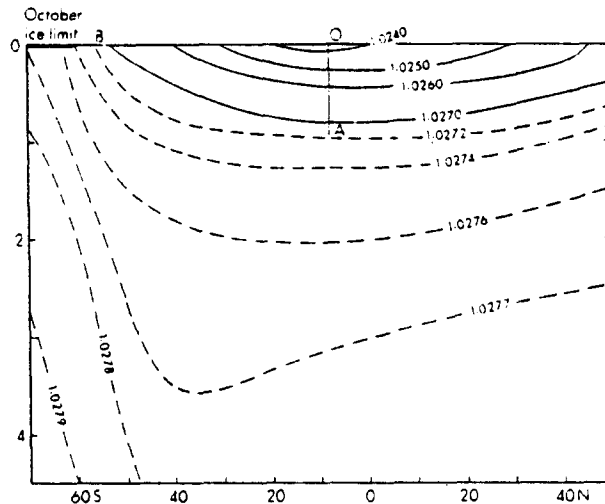
Fig. 10. Schematic presentation of potential density in a north-south section of the Pacific.

$\hat{\rho}$-surfaces. The interpretation of κ then ultimately involves consideration of lateral circulation as well as boundary processes. A preliminary attempt in this direction is given in the adjoining note (SCHIFF, 1966).

2. *Thermodynamic mixing*

Under this topic we include any effects associated with the fact that the oceans are a two-phase system, potential density being determined by potential temperature and salinity. Differences in the respective rates of diffusion may lead to instabilities (STOMMEL *et al.*, 1956; STERN, 1960; TURNER and STOMMEL, 1964; TURNER, 1965; P. K. WEYL, personal communications).

In STERN's experiments relatively light, warm, salty water was floated over heavy, cold, fresh water. Vertical filaments develop across the interface within an hour. In TURNER's experiments relatively light, cold, fresh water was floated over heavy, warm, salty water. Turbulent convection and progressive layering resulted. In these experiments the layering was stable, with T' and S' either both positive or both negative. In the central Pacific the layering is stable with T' positive and S' negative. The foregoing considerations do not apply, but Stommel (personal communication) has suggested the possibility of some filament – convection interaction that might conceivably lead to vertical mixing. The observed *microstructure* in $T(z)$, $S(z)$ is far from monotonic and this may play a role in diffusion.

Another aspect of thermodynamic mixing is a process called *caballing* associated with the curvature of $\hat{\rho}$-lines on a $\hat{T}$, S-grid. Thus when two water types, $\hat{T}_1$, S_1 and $\hat{T}_2$, S_2, with identical densities mix, the resultant mixture is denser than the two parent types and tends to sink. (FOFONOFF 1956).

3. *Shear mixing*

Interior Väisälä frequency. A statically stable fluid, $-\rho' > 0$, becomes dynamically unstable in the presence of shear, provided $u'^2 > 4g\,(-\hat{\rho}')/\hat{\rho}$ (MILES 1961,

1963; HOWARD, 1961). A convenient measure of the static stability is the Väisälä frequency N (ECKART, 1960, p. 68), which can be written as

$$N^2 = g\,(-\hat{\rho}')/\hat{\rho} = g\,(\alpha\,\hat{T}' - \beta\,S')$$

where

$$\alpha = 1{\cdot}3 \times 10^{-4}\ \text{deg C}^{-1}, \quad \beta = 8 \times 10^{-4}\ {}^{\circ}\!/_{00}{}^{-1}$$

are representative values of the thermal and haline coefficients of cubic expansion in the depth range 1–4 km. Referring back to the exponential distributions (3a, b), we can express $\hat{T}'$ and S' in terms γ, ξ, and the mean gradients in potential temperature and salinity between 1 and 4 km, $\bar{\hat{T}'} = 1{}^{\circ}\text{C km}^{-1}$, $\bar{S'} = -0{\cdot}07{}^{\circ}\!/_{00}\ \text{km}^{-1}$. The result is $N^2 = N^2\,(0)\,e^{\gamma\xi}$, with

$$N^2\,(0) = \frac{g\gamma}{e^\gamma - 1}\,(\alpha\,\bar{\hat{T}'} - \beta\,\bar{S'}) = (1{\cdot}6 - 0{\cdot}8)\,10^{-7}\ \text{radians sec}^{-1}$$

so that temperature and salinity in the interior Pacific contribute in the ratio 2 : 1 to the static stability. For the present purposes the dimensionless quantities γ and ξ are not convenient. Let $\bar{\xi} = z - z_1$ be the elevation above $z_1 = -4$ km, $\bar{\gamma} = w/\kappa = 1{\cdot}1\ \text{km}^{-1}$ the reciprocal scale height of stratification, so that $\gamma \cdot \xi = (z_2 - z_1)\,\bar{\gamma} \cdot \bar{\xi}/(z_2 - z_1) = \bar{\gamma} \cdot \bar{\xi}$. Write z for $\bar{\xi}$. We find

Depth (km)	0	1	2	3	4
z (km)	4	3	2	1	0
$10^3\,N$ (radians sec^{-1})	4·4*	2·6	1·5	0·94	0·49
N (c/hr)	2·52*	1·46	0·84	0·54	0·28

Internal tide waves. For instability to occur, the shear, u', should exceed $2\,N \approx 0{\cdot}003\ \text{sec}^{-1}$ (3 m sec^{-1} per km) at 2-km depth. Shear associated with the general circulation is less than one tenth this value. We may however regard u' as an r.m.s. shear associated with oscillatory currents, provided their period of oscillation, $2\,\pi\omega^{-1}$, is much larger than the growth time of the instabilities. For this purpose it may be sufficient to require that $\omega \ll N$. In terms of the dimensionless shear $S = u'/N$ (Richardson's Number is S^{-2}), the conditions for dynamic instability can then be written†

$$S > 2, \quad \omega \ll N. \tag{12}$$

Let

$$X\,(z)\,e^{i\,(\alpha x - \omega t)}, \quad Z\,(z)\,e^{i\,(\alpha x - \omega t)}$$

designate the horizontal and vertical displacements associated with internal waves. Then approximately (see for example TOLSTOY, 1963)

$$Z'' + \frac{\alpha^2\,N^2\,(z)}{\omega^2 - f^2}\,Z = 0, \quad X = i\alpha^{-1}\,Z' \tag{13a, b}$$

where $f = 2$ (Earth's angular velocity) sin (latitude) is the Coriolis frequency‡.

*The extrapolation to the surface does not represent actual conditions.

†R. Stewart has warned me against taking too seriously the case for a critical Richardson number. Under certain conditions turbulence may obtain even at large Richardson numbers (ELLISON, 1957: STEWART, 1958).

‡As $\omega \to f$ there is also an appreciable component of X in phase with Z' and at right angles to the direction of wave propagation: we shall pay no attention to two-dimensional aspects.

Hence

$$u' = i\omega X' = -(\omega/\alpha) Z'' = \omega\alpha N^2 (\omega^2 - f^2)^{-1} Z$$

and

$$S = u'/N = \omega N (\omega^2 - f^2)^{-1} \alpha Z \qquad (14)$$

This is an interesting result: off hand one expects u' to be large for large N, for in the limit of an inviscid, sharply *layered* ocean, internal waves are associated with infinite shear at the density discontinuities. But it is not immediately clear whether the ratio $S = u'/N$ should increase or decrease with N. Evidently the former case applies, so that zones for which the static stability N is largest are the most likely to be unstable dynamically. Instability is proportional to the wave slope αZ, as one might have expected.

So far nothing has been prescribed about $N(z)$. For the case $N^2 = N^2(0) e^{\tilde\gamma z}$ the differential equation (13a) has the Bessel solution

$$Z_n(z) = J_0(\phi_n) + a_n N_0(\phi_n),$$

with

$$\phi_n = \frac{2N(0)}{c_n \tilde\gamma \mathscr{C}} e^{\frac{1}{2}\tilde\gamma z} = b_n e^{\frac{1}{2}\tilde\gamma z}$$

where $c_n = \omega/\alpha_n$ is the phase velocity, and $\mathscr{C} = (1 - f^2/\omega^2)^{\frac{1}{2}}$ is a Coriolis correction. The parameters a_n, b_n are determined by the boundary conditions* $Z_n = 0$ at $z = 0, H$ (or at $\phi_n = b_n$, kb_n where $k = e^{\frac{1}{2}\tilde\gamma H} = 9.025$). This requires

$$\begin{vmatrix} J_0(b_n) & N_0(b_n) \\ J_0(kb_n) & N_0(kb_n) \end{vmatrix} = 0.$$

The mode number n designates the number of intermediary levels at which $Z_n(z)$ vanishes.

For large values of b_n, the determinant is proportional to $\sin b_n (k-1)$, and accordingly

$$b_n = \frac{n\pi}{k-1}, \quad a_n = -\cot b_n,$$

$$Z_n(z) = \sqrt{\left(\frac{2}{\pi\phi_n}\right)} \sqrt{[2(a_n^2 + 1)]} \cos(\phi_n - \tfrac{1}{4}\pi - b_n)$$

$$= 2\sqrt{\left(\frac{1}{\pi b_n}\right)} e^{-\frac{1}{4}\tilde\gamma z} \operatorname{cosec} b_n \cos\left[b_n(e^{\frac{1}{2}\tilde\gamma z} - 1) - \tfrac{1}{4}\pi\right]$$

$$= A_n e^{-\frac{1}{4}\tilde\gamma z} \cos\left[n \frac{e^{\frac{1}{2}\tilde\gamma z} - 1}{e^{\frac{1}{2}\tilde\gamma H} - 1} - \tfrac{1}{2}\right]\pi \qquad (16)$$

with A_n designating the representative amplitude of the n'th mode. The associated wave velocities are

$$c_n = \frac{2N(0)}{b_n \tilde\gamma \mathscr{C}} = \frac{2(k-1)N(0)}{n\pi\tilde\gamma \mathscr{C}}. \qquad (17)$$

*To the present approximation it is permissible to replace the condition of a free upper boundary at $z = H$ by a fixed boundary. However, the extrapolation of $N^2(z) = N^2(0) e^{\tilde\gamma z}$ to the surface may not be justified.

Finally

$$S_n\,(z) \;=\; \frac{\pi n A_n\,\bar{\gamma}}{2\,(k-1)\,\mathscr{C}}\;e^{\frac{1}{2}\bar{\gamma}\,z}\cos\left[n\,\frac{e^{\frac{1}{2}\bar{\gamma}\,z}-1}{e^{\frac{1}{2}\bar{\gamma}H}-1}-\tfrac{1}{2}\right]\pi. \tag{18}$$

For any given mode, the shear instability increases by a factor $e^{\frac{1}{2}\bar{\gamma}H} = 3\cdot00$ between a 4-km depth and the surface*, and for any given depth it increases with mode number. Because of the factor $\mathscr{C}^{-1}$, the instability associated with diurnal tides is amplified at polar latitudes, that associated with semidiurnal tides is amplified near latitudes 30°.

Suppose that at a depth of 1 km the internal wave energy is evenly distributed between modes $n = 10$ to $n = 19$, and zero for other modes. Then the mean-square amplitude and shear are, respectively,

$$\bar{Z}^2 \;=\; A^2\,e^{-\frac{1}{2}\bar{\gamma}\,z}\cos^2\big[\quad\big]\,\pi\cdot 10$$

$$\bar{S}^2 \;=\; A^2\,\mathscr{C}^{-1}\,e^{\frac{1}{2}\bar{\gamma}\,z}\cos^2\big[\quad\big]\,\pi\left[\frac{\bar{\gamma}\pi}{2\,(k-1)}\right][10^2 + 11^2 + \ldots + 19^2]$$

$$=\; \mathscr{C}^{-1}\,e^{\bar{\gamma}\,z}\left[\frac{\pi}{2\,(k-1)}\right]^2 218\,\bar{\gamma}^2\,\bar{Z}^2$$

For a r.m.s. internal wave amplitude of 30 m, the dimensionless r.m.s. shear is 0·5, as compared to the critical value 2 [but see footnote to equation (12)]. To achieve instability requires concentration of internal wave energy among very high modes with typical velocities of the order $c_{30} = 10$ cm sec^{-1}, and corresponding wave lengths (at tidal frequencies) of 5–10 km†.

Internal planetary waves. For frequencies $\omega < f$ we must refer to the propagating quasi-horizontal current systems called " planetary waves." M. Rattray has pointed out that measurements with Swallow floats have indicated that interior horizontal velocities are largely associated with frequencies well below tidal frequencies. The observed currents are of the order 10 cm sec^{-1} and apparently uncorrelated at different levels. So perhaps planetary wave shear is a more promising source of instability than internal tide shear ?

For modal number n, the vertical wave number is $\kappa_n = \pi n/H$. In order for instability to occur, we require $u' = \kappa u$ to exceed $2N$, and hence $n > HN/u$. Letting $H = 4$ km, $N = 10^{-3}$ sec^{-1} and $u = 10$ cm sec^{-1} gives $n > 40$. As in the case of internal tides, this requirement for high modes may be too severe.

Mode conversion. One possible source of high-n internal wave energy is the conversion of long surface waves to internal waves over an irregular sea bottom. We shall consider this mechanism in some detail, simply because it can be done. Cox and SANDSTROM (1962, their equation 54) give the fractional conversion per unit distance $\mathscr{L}_{0n}$, from surface mode 0 to internal mode n, assuming the reverse conversion to be negligible. Multiply $\mathscr{L}_{0n}$ by the energy flux of the surface tides, $\tfrac{1}{2}\rho g A_0{}^2\,c_0 = \tfrac{1}{2}\rho g A_0{}^2\,\omega\,(\omega^2 - f^2)^{-\frac{1}{2}}\,(gH)^{\frac{1}{2}}$, and divide by the mass of the unit column, ρH. The resultant energy flux

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*For the case $N(z) = N(0)\,(1 + mz)^{3/2}$, $m = (3/\sqrt{2})\,N(0)\,c_n{}^{-1}\,\mathscr{C}^{-2}$ we obtain a solution $Z(z) \sim N^{-1}(z)$ for which S is independent of depth (i.e. uniform instability), but it is then impossible to satisfy boundary conditions.

†This requirement may be too severe. Perhaps it will do if the critical value is exceeded in some places and some times. C. Cox has pointed out to me the analogy with whitecap formation : the critical surface acceleration is $-g$, yet instability first occurs during a Beaufort 4 wind when the r.m.s. acceleration is only about 0·06 g.

$$\epsilon_{0n} = \frac{\frac{1}{4}\pi^2\, gH^{-2}\,\omega^{-1}\,(\omega^4 - f^4)\, Z'_n{}^2\,(0)\, S\,(\alpha_n) A_0{}^2}{\displaystyle\int_0^H N^2\,(z)\, Z_n{}^2\,(z)\,\mathrm{d}z} \tag{19}$$

is proportional to Z'^2 (and hence to the mean square horizontal velocity) of mode n at the sea bottom, to the spectral density $S\,(\alpha_n)$ of bottom roughness evaluated at the wave number α_n of mode n, and to the mean square surface tides. In evaluating the denominator we can replace $\cos^2[\quad]$ in equation (16) by its mean value, $\frac{1}{2}$. The result is

$$\epsilon_{0n} = \tfrac{1}{16}\,\pi^4\,g\,(k-1)^{-3}\,n^2\,\gamma^3\,H^{-2}\,A_0{}^2\,\omega^{-1}\,(\omega^4 - f^4)\,N^{-2}\,(0)\,S\,(\alpha_n) \tag{20}$$

Some estimates by Cox (1959) based on sonic profiles across the Atlantic suggest $S\,(\alpha) = B\alpha^{-3}$ over the range $10^{-4}\,m^{-1} < \alpha < 10^{-3}\,m^{-1}$ (waves lengths $2\,\pi$ to $20\,\pi$ km), with $B = 1$ m. The mean-square elevation within this spacial " frequency band " is $\displaystyle\int_0^{2\pi}\int_{\alpha_1}^{\alpha_2} S\,(\alpha)\,\alpha\,d\alpha\,d\theta = 2\pi B\,[\alpha_1{}^{-1} - \alpha_2{}^{-1}] \approx 2\pi \times 10^4\,m^2 = (250\,m)^2$; the mean-square slope is $\displaystyle\int_0^{2\pi}\int_{\alpha_1}^{\alpha_2} \alpha^2\,S\,(\alpha)\,\alpha\,d\alpha\,d\theta = 2\pi B\,(\alpha_2 - \alpha_1) \approx (0\cdot075\,\text{radians})^2$. Under the Pacific the situation is somewhat comparable (W. Farrell, personal communication). From equation (17) we have $\alpha_n = \omega n\pi\gamma H^{-1}/[2\,(k-1)\,N\,(0)]$ for large n. Thus $\epsilon_{0n} \sim n^2\,S\,(\alpha_n)$, and $S\,(\alpha_n) \sim \alpha^{-3} \sim n^{-3}$, so that $\epsilon_{0n} \sim n^{-1}$. We are of course limited to the α-range 10^{-4} to $10^{-3}\,m^{-1}$ to which the spectral bottom analysis applies. This corresponds to a modal range from $n = 2$ to $n = 20$, which is below that required for instability. It would appear that the high internal modes required for instability may be restricted to regions of sharp bottom relief. The final result is

$$\epsilon_{0n} = \tfrac{1}{2}\pi\mathscr{C}'\,n^{-1}\,g\,N\,(0)\,H^{-2}\,A_0{}^2\,B \tag{21}$$

where $\mathscr{C}' = (1 + f^2/\omega^2)\,(1 - f^2/\omega^2)^{-\frac{1}{2}}$. For $H = 4$ km, $A_0 = \frac{1}{2}$m, $B = 1$ m, one obtains $\epsilon_{0n} = 1\cdot2 \times 10^{-6}\,n^{-1}$ ergs g^{-1} sec^{-1}. Over the range $n = 2$ to $n = 20$ the total flux of energy may be 4×10^{-6} ergs g^{-1} sec^{-1}, or 5×10^{18} ergs sec^{-1} for all oceans. This amounts to $1/6$ of the 3×10^{19} ergs sec^{-1} known from astronomical considerations to be dissipated by the tides in the Earth–Moon system (MUNK and MACDONALD, 1960, p. 203; MUNK, 1966, p. 66). The remaining $5/6$ presumably takes place in shallow seas, and in the solid earth.

Energetic considerations. The step from considerations of incipient instability to turbulent diffusion encompasses most of the difficulties in modern hydrodynamics. Energy considerations place some restrictions on what is or is not possible.

In the absence of upwelling, the effect of diffusion would be to raise the center of mass of the stratified ocean. The time variations $(\,\cdot\,)$ are given by

$$\dot{T} = -\kappa\,\bar{T}'', \quad \dot{S} = -\kappa\,S''$$

$$\dot{\rho} = -\rho\,(\alpha\dot{T} - \beta\,\dot{S}) = \kappa\,\rho\,(\alpha\,\bar{T}'' - \beta\,S'') = \kappa\,\rho\,g^{-1}\,(N^2)'$$

with $N^2(z) = N^2(0)\,e^{\tilde{\gamma}z}$. The potential energy is $E_p = \displaystyle\int_0^H g\hat{\rho}z\,dz$; the energy required for diffusion, per unit mass of water, has the average value

$$\epsilon_d = \dot{E}_p/H = \kappa\,N^2\,(0)\,[\tfrac{1}{2}\,(e^\gamma + 1) - \gamma^{-1}\,e^\gamma] \tag{22}$$

subject to the restraint that mass be conserved between $z = 0$ and H. Setting

$\gamma = \bar{\gamma} \cdot H = 3.3$, $\kappa = 1.3 \, \mathrm{cm^2 \, sec^{-1}}$ and $N(0) = 0.49 \times 10^{-3}$ radians $\mathrm{sec^{-1}}$, gives $\epsilon_d = 1.8 \times 10^{-6} \, \mathrm{ergs \, g^{-1} \, sec^{-1}}$, compared to the flux $\epsilon_{0n} = 4 \times 10^{-6} \, \mathrm{ergs \, g^{-1} \, sec^{-1}}$ from surface to internal tides. Almost half of this flux is then required by diffusion, a fraction that is uncomfortably large, but does not immediately rule out diffusion by internal tides.

4. Biological mixing

From the observed interior gradients (vertical and horizontal) of dissolved oxygen, a consumption of $0.7 \times 10^{-10} \, \mathrm{ml/l \, sec^{-1}}$ has been inferred. It is not known to what extent the consumption is by bacteria, or by plankton and nekton. We assume the latter. Heat produced from physiological oxidation of food stuffs varies from $4.69 \, \mathrm{cal/ml} \, O_2$ for pure fat to 5.05 for pure carbohydrate; a good working figure is $4.9 \, \mathrm{cal/ml} \, O_2$. The heat produced would be $4.9 \times 0.7 \times 10^{-10} \, \mathrm{cal \, sec^{-1}}$ per l. of water, or $3.3 \times 10^{-13} \, \mathrm{cal \, sec^{-1}} = 1.4 \times 10^{-5} \, \mathrm{ergs \, sec^{-1}}$ per gram of sea water. For the entire interior oceans this is $3 \times 10^{19} \, \mathrm{ergs \, sec^{-1}}$. It is a remarkable coincidence that the production of chemical energy by marine organisms should be of the same order as that produced by tidal dissipation.

Perhaps 20% of the chemical energy goes into swimming, and we may set $\epsilon = 3 \times 10^{-6} \, \mathrm{ergs \, g^{-1} \, sec^{-1}}$ for the energy dissipated by the perturbed flow. But there may be far more efficient ways for biological mixing than by stirring the water. The most abundant organisms in the abyssal sea, copepods, mysids, euphausiids, squid and the nektonic bristlemouths are all engaged in diurnal migration over a vertical distance of the order 1 km. This migration is associated with some transport of ocean water, whether by respiration of ambient fluid with a time scale of a fraction of a second, or by a continuous renewal of the free body fluid with a long (but unknown) replacement time. Differential feeding has been suggested by KETCHUM (1957) as a possible factor for diffusing radioactive wastes:

" The zooplankton are certainly in the area of the most dense concentration of their food, the phytoplankton, when they are at the surface at night. During the hours of darkness they may therefore be expected to consume the living material in the water, and some of this, at least, would be excreted or passed as faecal pellets at depth in the day time ... There is also evidence that the zooplankton can assimilate dissolved elements from sea water. If elements were assimilated at depth they might be excreted or exchanged near the surface and thus directly modify the vertical distribution in the sea."

A mixing length argument attributes to " Ketchum's yoyo " (as it has been called) a diffusivity of the order $\kappa = L^2 T^{-1} r_1 r_2$, where L is the range of migration and T its period, to be taken at 1 km and 1 day, respectively. This gives $\kappa = 10^5 r_1 r_2 \, \mathrm{cm^2}$ $\mathrm{sec^{-1}}$. r_1 is the migrating biomass per unit mass of water. In the upper 300 m the fractional zooplankton biomass is usually less than 10^{-7} (HELA and LAEVASTU, 1961, p. 64; KETCHUM, 1957) and this is consistent with the estimate by RILEY, STOMMEL and BUMPUS (1949) of 10^{-8} for the fractional mass of carbon. The biomass rapidly diminishes downward, and can hardly exceed $10^{-8} \, \mathrm{g/g}$ at 1 km; hence $\kappa = 10^{-3} r_2$ $\mathrm{cm^2 \, sec^{-1}}$, where r_2 is the fractional body mass involved in the diurnal feeding. Diffusion by defecation appears to be negligible even for $r_2 = 1$, an unlike upper limit.

CONCLUSIONS

The observed distributions of T, S, ^{14}C and O_2 in the interior Pacific give a rather consistent picture, and one that can be reconciled with vertical upwelling and turbulent diffusion as the governing processes. The resulting formulae give convenient receipes for computing typical vertical fluxes and stabilities to within (say) a factor of two.

The required vertical velocity is consistent with some crude notions concerning the formation of antarctic bottom water. I have not succeeded in interpreting the inferred diffusion in terms of known processes. The discussion of boundary and thermodynamic processes is purely qualitative; biological processes appear to be negligible. The effect of tides is considered more fully (because it is possible), and is found to be a marginal source of diffusion, both with respect to the required shear for incipient instability, and the required energy flux for turbulent diffusion.

Horizontal transport processes have been ignored (they may be parametrically disguised under vertical eddy diffusion); these processes play a prominent role in REDFIELD's (1942) and RILEY's (1951) discussion of the nonconservative properties of the deep Atlantic. Until the processes giving rise to diffusion and advection are understood, the resulting differential equations governing the interior distribution, and their solutions, must remain what they have been for so long: a set of recipes.

Appendix—Let $q_i \rho_i$ grams $cm^{-2} sec^{-1}$ designate the grams of sea ice of density ρ_i formed per unit surface area in unit time; this contains $q_i \rho_i S_i$ grams of salt. The same mass of surface water contains $q_i \rho_i S_0$ grams of salt, so that salt is precipitated at the rate $q_i \rho_i (S_0 - S_i)$ grams $cm^{-2} sec^{-1}$. Just beneath the upper boundary there is a layer of thickness δ, and of salinity differential ΔS, within which the salt is transported by molecular diffusion,

$$q_i \rho_i (S_0 - S_i) = \kappa_s \rho \, \Delta S / \delta,$$

The dimension of this layer is determined by the critical Rayleigh Number

$$Ra = \frac{\beta g \Delta S \, \delta^3}{\kappa_s \nu} = \text{order } 10^3;$$

Here $\beta = \rho^{-1} \partial\rho/\partial S = 0{\cdot}8 \times 10^{-3} (\%_0)^{-1}$, $\rho_i = 0{\cdot}9 \, \rho$, $\kappa_s = 2 \times 10^{-5} cm^2 sec^{-1}$, $\nu = 10^{-2} cm^2 sec^{-1}$ is the viscosity, $q_i = 1$ m in 100 days, $S_0 - S_i = 29\%_0$. The equations yield $\delta = 0{\cdot}066$ cm, $\Delta S = 0{\cdot}86\%_0$. Thus the boundary layer is negligibly thin as compared to the roughness of the ice sheet, and the salinity within the remaining column varies by an amount small compared to ΔS.

REFERENCES

BIEN G. S., N. W. RAKESTRAW and G. E. SUESS (1965) Radiocarbon in the Pacific and Indian oceans and its relation to deep water movements. *J. Limnol. Oceanogr.* **10**, (Supp.), R25–R37.

BOLIN B. and H. STOMMEL (1961) On the abyssal circulation of the world ocean—VI. *Deep-Sea Res.* **8**, 95–110.

BUINITSKII V. K. (1956) Features of ice conditions in the Antarctic Ocean. *Vestnik, Leningradskogo- Universiteta* **11**, (6), 80–92, Library of Congress AS 262. L463 Vol. 11.

COX C. (1959) Coupling of surface and internal tides. *Int. Ocean. Cong. Preprints, Am. Ass. Adv. Sci.* p. 398.

COX C. and H. SANDSTROM (1962) Coupling of internal and surface waves in water of variable depth. *J. ocean. Soc. Japan* **20**, 499–513.

CRAIG H. and L. I. GORDON (1966) Isotopic oceanography. *Proceedings Third Spoleto Nuclear Geology Conference.* pp. 9–130, C.N.D.R., Pisa.

ECKART C. (1960) *Hydrodynamics of oceans and atmospheres.* Pergamon, London, 290 pp.

ELLISON T. H. (1957) Turbulent transport of heat and momentum from an infinite rough plane. *J. fluid Mech.* 2, 456–466.

ELLISON T. H. and TURNER J. S. (1959) Turbulent entrainment in stratified flows. *J. fluid Mech.* 6, 423–448.

FOFONOFF N. P (1956) Some properties of sea water influencing the formation of Antarctic bottom water. *Deep-Sea Res.* 4, 32–35.

GOLDBERG E. and M. KOIDE (1963) Rates of sediment accumulation in the Indian ocean. *Earth Science and Meteoritics.* North Holland Publishing. 90–102.

HELA I. and T. LAEVASTU (1961) *Fisheries Hydrography.* Fishing News (Books) Ltd., London. 137 pp.

HOWARD L. N. (1961) Note on a paper by John W. Miles. *J. fluid Mech.* 10, 509–512.

ISELIN C. O'D. (1939) The influence of vertical and lateral turbulence on the characteristics of the waters at mid-depths. *Trans. Am. geophys. Un.* 7, 414–417.

KETCHUM B. (1957) The effects of atomic radiation on oceanography and fisheries. Chapter 5 of *Nat. Acad. Sci., Washington, D.C. Publ.* 551, 52-59.

KNAUSS J. A. (1962) On some aspects of the deep circulation of the Pacific. *J. geophys Res.* 67, 3943-3954.

LAL D. (1962) Cosmic ray produced radionucleides in the sea. *J. oceanogr. Soc. Japan*, 20, 600–614.

LEDENEV V. G. (1963) The influence of evaporation on the processes of formation of the cold Antarctic Waters. *Bull. Soviet. Antarctic Exped.* 44, 35–38.

MILES J. W. (1961) On the stability of heterogeneous shear flows. *J. fluid Mech.* 10, 496–508.

MILES J. W. (1963) On the stability of heterogeneous shear flows. Part 2. *J. fluid Mech.* 16, 209–227.

MIYAKE Y. and Y. SUGIMURA (1964) Uranium and radium in the western North Pacific waters. In: *Studies in Oceanography.* Tokyo, 274–278.

MORTON B. R., G. I. TAYLOR and J. S. TURNER (1956) Turbulent gravitational convection from maintained and instantaneous sources. *Proc. R. Soc.* A 234, 1–23.

MUNK W. H. (1966) Variation of the Earth's rotation in historical time. *Proc. Conf. Earth-Moon System* (in press).

MUNK W. and G. MACDONALD (1960) *The rotation of the Earth.* Cambridge University Press, 323 pp.

PETTERSSON H. (1955) Manganese nodules and oceanic radium. *Deep-Sea Res.* 3, (Suppl.) 335–345.

REDFIELD A. C. (1958) The biological control of chemical factors in the environment. *Am. Scient.* 46, 205–221.

REDFIELD A. C. (1942) The processes determining the concentration of oxygen, phosphate and other organic derivatives within the depths of the Atlantic Ocean. *Pap. phys. oceanogr. Meteorol.* 9, 1–22.

RILEY G. A., H. STOMMEL and D. BUMPUS (1949) Quantitative ecology of the plankton of the western North Atlantic. *Bull. Bingham Oceanogr. Coll.* 12, 43.

RILEY G. A. (1961) Oxygen, phosphate, and nitrate in the Atlantic Ocean. *Bull. Bingham Oceanographic Collection* 13, 1–126.

ROBINSON A. and H. STOMMEL (1959) The oceanic thermocline and the associated thermohaline circulation. *Tellus* 11, 295–308.

SCHIFF L. (1966) Effect of boundary mixing on the distribution in the interior. *Deep-Sea Res.* 13, 621–626.

STERN M. (1960) The " salt-fountain " and thermohaline convection. *Tellus* 12, 172–175.

STEWART R. W. (1958) The problem of diffusion in a stratified fluid. In: *Atmospheric Diffusion and Air Pollution.* Academic Press, New York.

STEWART R. W. (1962) Some aspects of turbulence in the Arctic. *Proc. of the Arctic Basin Symposium*, Arctic Inst. of North America, 122–127.

STOMMEL H. (1958) The abyssal circulation. *Deep-Sea Res.* 5 80–82.

STOMMEL H. and A. B. ARONS (1960) On the abyssal circulation of the world ocean—I and II. *Deep-Sea Res.* 6 140–154 and 217–233.

STOMMEL H., A. B. ARONS and D. C. BLANCHARD (1956) An oceanographical curiosity : the perpetual salt fountain. *Deep-Sea Res.* 3, 152–153.

TOLSTOY I. (1963) The theory of waves in stratified fluids including the effects of gravity and rotation. *Revs. Mod. Phys.* 35, 207–230.

TURNER J. S. (1965) The coupled turbulent transports of salt and heat across a sharp density interface. *Int. J. Heat Transfer* 8, 759–767.

TURNER J. S. and H. STOMMEL (1964) A new case of convection in the presence of combined vertical salinity and temperature gradients. *Proc. Natl. Acad. Sci.* **52,** 1, 45–53.

UNTERSTEINER N. (1964) Calculations of temperature regime and heat budget of sea ice in the central Arctic. *J. geophys. Res.* **69,** 4755–4766.

UNTERSTEINER N. (1966) Sea ice. *Reinhold Encyclopaedia of Earth Sciences* (in press).

WOOSTER W. S. and G. H. VOLKMANN (1960) Indications of deep Pacific circulation from the distribution of properties at five kilometers. *J. geophys. Res.* **65,** (4), 1239–1249.

WYRTKI K. (1962) The oxygen minima in relation to ocean circulation. *Deep-Sea Res.* **9,** 11–28.