Tidal conversion or Three frequencies and two slopes

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Outline

1. Introduction
2. The Weak Topography Approximation (WTA)
3. Beyond the WTA
4. Questions and some answers

Introduction

2.4 TW energy loss in Earth-Moon system = tidal dissipation.
Link to climate: maintenance of oceanic stratification?

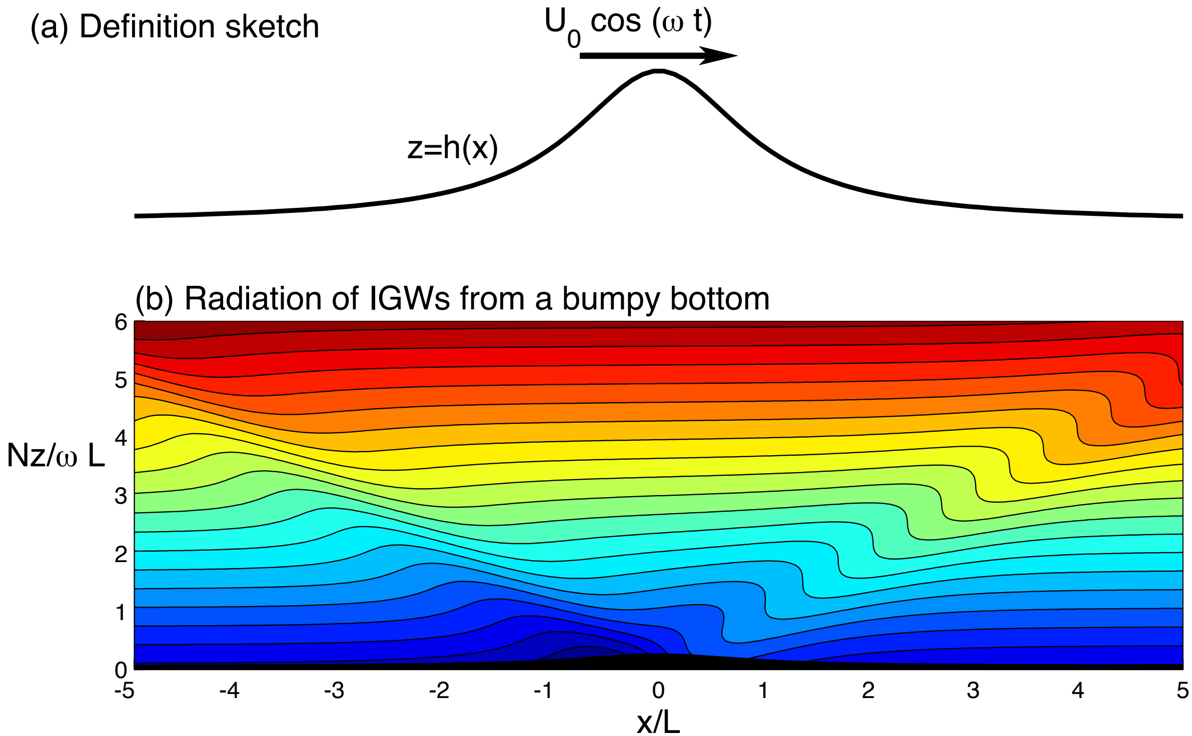
Two mechanisms:
(1) Bottom drag in shallow regions with $\mathcal{E} = 0.0025 \times \rho_0 |u|^3$ (W m^{-2}).
(2) Flow of stratified fluid over topography with $\mathcal{E} = c\rho_0 u^2 N \ell$ (W m^{-2}).

N: buoyancy frequency.
ℓ: a length related to the topography. Link to geophysics.

This talk is about mechanism (2).

Topography; ω, N, f $\Rightarrow$ This Talk $\Rightarrow$ $\mathcal{E}$ (W m^{-2}) $\Rightarrow$ $\varepsilon/5 = \kappa_v N^2$

Two slopes



Ray slope $\alpha = \sqrt{(\omega^2 - f^2)/(N^2 - \omega^2)} \sim$ 0.1–0.4.

Topographic slope $s \sim \mathrm{d}h/\mathrm{d}x$ (very small above).

$s \ll \alpha$ is the Weak Topography Approximation.

Weak Topography Approximation $s \ll \alpha$

Bell (1975):

$$\mathcal{E} = \frac{\rho_0}{8\pi^2} N_\mathrm{B} \sqrt{1 - \frac{f^2}{\omega^2}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{U_+^2 k'^2 + U_-^2 l'^2}{\sqrt{k'^2 + l'^2}} \phi(k', l') \, \mathrm{d}k' \mathrm{d}l' \text{ (W m}^{-2}\text{)}.$$

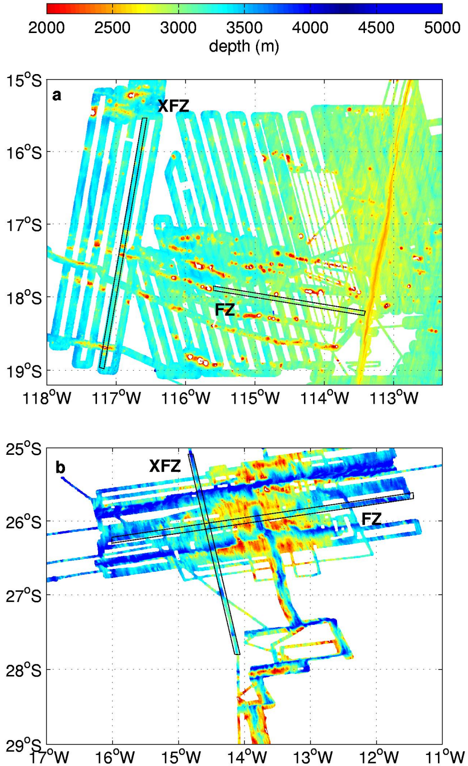
$U_\pm$: tidal velocity along semi-major and -minor axes of tidal ellipse.
N_B: buoyancy frequency at the bottom.

$$h_{rms}^2 = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \phi(k, l) \, \mathrm{d}k \, \mathrm{d}l = \frac{1}{A} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} |\tilde{h}(k, l)|^2 \, \mathrm{d}k \, \mathrm{d}l.$$

Is this formula ever useful, i.e. is $s \ll \alpha$?

For that matter, how do we define s?

St. Laurent and Garrett's proposed definition of s



Multibeam bathymetry data for the (a) EPR and (b) MAR. FZ: along fracture zone; XFZ: across fracture zone.

St. Laurent and Garrett's proposed definition of s (contd.)

Define

$$s(k) \equiv \sqrt{\int_{k_0}^{k} k'^2 \phi(k')\, \mathrm{d}k'}$$
$$= \text{“slope in topographic wavelengths longer than } 2\pi/k\text{”.}$$

The critical wavenumber k_c is defined by

$$s(k_c) = \alpha.$$

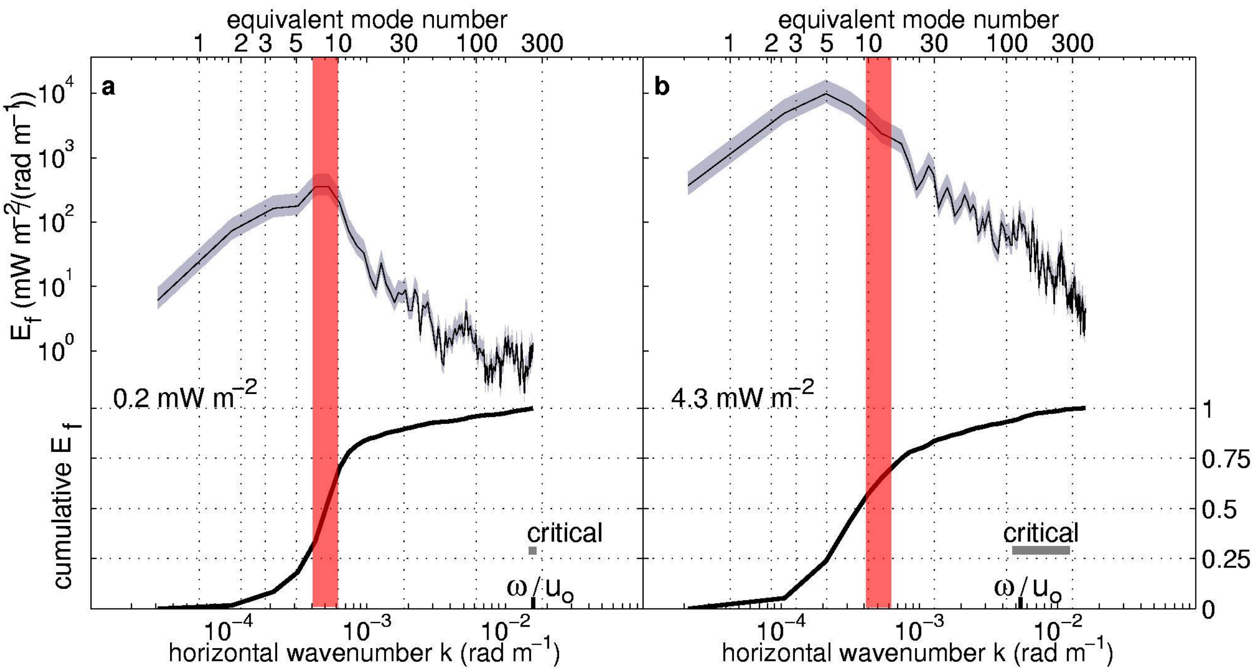
For the EPR and the MAR, k_c is roughly 1 km.

$\Rightarrow$ "Most" of the EPR and MAR is subcritical.

$\Rightarrow$ The WTA is valid for the EPR and MAR.

$\Rightarrow$ Perhaps all ridge systems are subcritical.

Energy flux spectra (St. Laurent and Garrett)



Energy flux spectra computed using the WTA for the (a) EPR and (b) MAR.

⇒ The WTA formula plateaus before $k = k_c \sim 2\pi/(1\,\text{km})$.

⇒ Spectral extrapolation might work...

⇒ ABYSS resolution at $k \sim 2/\pi/(12\,\text{km})$ is marginal.

Another WTA application: Seamounts (LSY 2002)

According to Jordan, Menard and Smith (1983) seamounts cover about 6% of the seafloor. There are about 1.4×10^6 seamounts in total.

For a Gaussian seamount with $h = h_{max} \exp\left(-r^2/2a^2\right)$, the radiated power is

$$\mathcal{C} = \frac{\pi^{3/2}}{8} \rho_0 N_B a (U_+^2 + U_-^2) \sqrt{1 - \frac{f^2}{\omega^2}} h_{max}^2 \text{ (Watts)}.$$

With $a = 1.6$ km, $h_{max} = a/5$, $f = 10^{-4}$ s^{-1}, $\omega = 2f$, $N_B = 5f$ and $U_+ = U_- = 1$ cm s^{-1},

$$s/\alpha \approx 1/3, \quad \mathcal{C} = 10^4 \text{ Watts.}$$

Note geophysical rule of thumb: $h_{max} = a/5 \Rightarrow \mathcal{C} \propto a^3$.

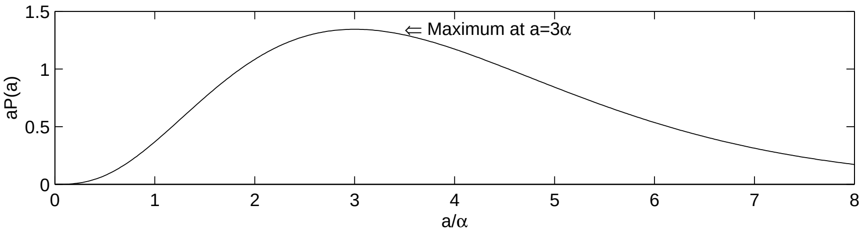
Big seamounts produce more conversion. But there are fewer big seamounts.

Seamounts (LSY 2002 contd.)

According to Jordan et al. (1983), the pdf of the seamount radius is

$$P(a) = \alpha^{-1} \exp(-a/\alpha), \qquad \alpha = 1.6 \text{ km}.$$

Typically $h_{max} = a/5$ and $\mathcal{C} \propto a^3$, so the total conversion is dominated by seamounts with $a = 3\alpha \sim 4.8$ km.



Summing over 1.4×10^6 seamounts using the pdf above gives 90 GW of conversion globally.

This 90 GW is not negligible: Ray and Mitchum (1996) estimated 15 GW of conversion into the first baroclinic mode by the Hawaiian Ridge. Munk (1997) estimated that 50,000 km of submarine ridges produce 200 GW of M_2

$3\alpha = 4.8$ km is below resolution of ABYSS.

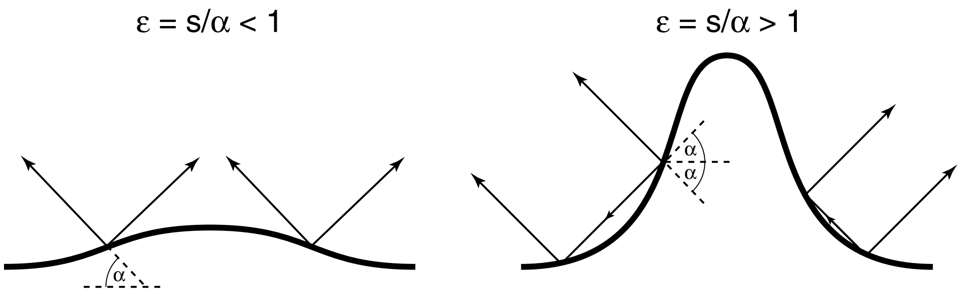
Beyond the WTA i.e. $\epsilon \equiv s/\alpha \geq 1.$

The WTA allows us to superpose topography and suggests that spectral characterizations of topography are enough.

How accurate is WTA if $\epsilon \geq O(1)$?

How can one calculate $\mathcal{C}$ for strong topography?

Is the dominant effect $\mathcal{C} \sim h^2_{max}$ as suggested by WTA?

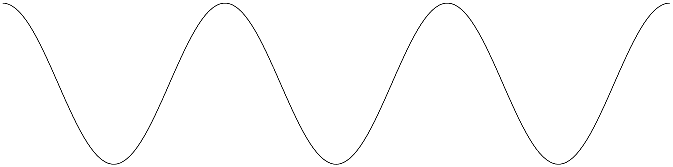


Subcritical and supercritical topography.

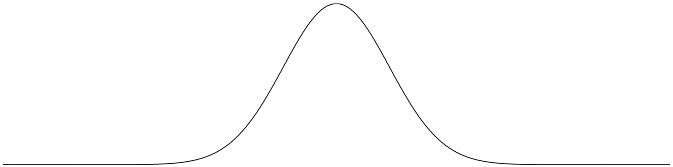
No subterranean rays even if $\epsilon > 1$!

Balmforth, Ierley and Young (2002)

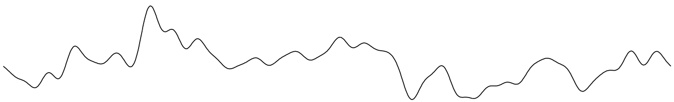
BIY considered sinusoidal topography, a Gaussian ridge, and a random bumpy bottom using the spectral model of Goff and Jordan (1988).



$$\mathcal{C}(\epsilon = 1) = 1.56 \times \mathcal{C}(\epsilon = 0)$$



$$\mathcal{C}(\epsilon = 1) = 1.14 \times \mathcal{C}(\epsilon = 0)$$



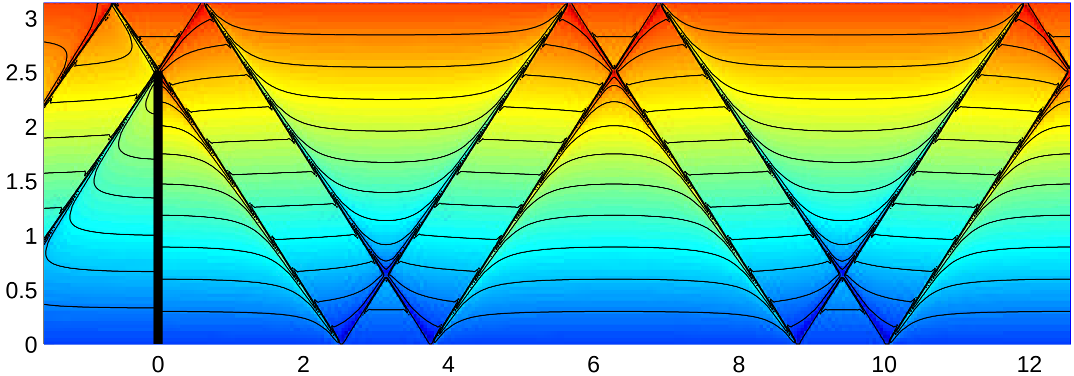
$$\mathcal{C}(\epsilon = 1) = 1.06 \times \mathcal{C}(\epsilon = 0)$$

$\Rightarrow$ WTA is accurate up to $\epsilon = 1$.

$\Rightarrow$ $\mathcal{C} \propto h_{max}^2$ is OK for moderate topography.

Llewellyn Smith and Young (2002) – $\epsilon = \infty$ and $\phi(k) = 0$

"Maximally supercritical" case: knife-edge ridge.



Isopycnal surfaces for conversion by an idealized knife-edge ridge.

$$\mathcal{C} = \rho_0 U_0^2 N \sqrt{1 - \frac{f^2}{\omega^2}} \frac{h_{max}^2}{\pi^2} K(B) \text{ (Watts)},$$

where $B = h_{max}/$depth of ocean.

As $B \to 1$, $\mathcal{C} \to \infty$.

Even though $\phi(k) = 0$, $\mathcal{C}$ is a steeply increasing function of h_{max}.

This is a very simple model of an island arc.

Questions and some answers

How relevant is the WTA to the "real ocean" and the "real bottom"? Depends

Good for ridge zones and seamounts ($s/\alpha < 1$). Not so good for island arcs ($s/\alpha > 1$).

How much resolution is enough? To resolve down to the k_c of St. L+G, possibly 1 km.

However, the conversion rate plateaus before $k = k_c \Rightarrow$ theory + extrapolation might work.

Is a spectral characterization of ocean topography all that is needed? Yes and no

1) The knife-edge topography has zero spectral content. Yet it yields a non-zero conversion rate.
2) Different realizations of topography, constructed using the same spectral recipe, give different conversion rates (Monte Carlo simulations of BIY).

Does all of the important at localized steep features? No

Localized steep features, like the Hawaiian islands, require dedicated modelling. But global budgets require consideration of gentle and widespread features such as seamounts and ridge zones.

Is spectral extrapolation a good strategy? Even if the answer is no, extrapolation is necessary

How do we extrapolate? Subject for another seminar

Will ABYSS access the power-law regime of the topography?