

## Passive Microwave Systems

(Rees Chapter 7)

At wavelengths greater than about 2 cm and less than 10 m, the atmosphere and ionosphere are very transparent to E/M waves. Stewart [1985] says "At the longest wavelengths the atmosphere is clearer than the clearest air for visible light and thermal radiation from the Earth dominates". Microwaves penetrate clouds and since the signal is from thermal emissions, passive microwave measurements can be made in all weather and in daytime or nighttime. This is an important consideration since microwave radiometers don't rely on reflected sunlight, they don't need to be placed in sun-synchronous orbits so they can be placed on almost any platform.

- (show introductory slide with transmission versus  $\lambda$  and the various sensors)
- (show infrared BB radiation curves for Earth emissions and reflected sunlight)

### Review of Thermal Radiation Again

Assuming we have a source of radiation that behaves like a blackbody, Planck's Law provides spectral radiance as a function of both wavelength,  $\lambda$  and frequency,  $\nu$ . The frequency form is more commonly used in microwave discussions and calculations although later we'll switch back to the wavelength form.

$$L_{\nu}(\nu) = \frac{2h\nu^3}{c^2} \left[ \exp \frac{h\nu}{kT} - 1 \right]^{-1}$$

|           |   |                      |                                                        |
|-----------|---|----------------------|--------------------------------------------------------|
| $T$       | - | temperature          |                                                        |
| $c$       | - | speed of light       | $2.99 \times 10^8 \text{ m s}^{-1}$                    |
| $h$       | - | Planck's constant    | $6.63 \times 10^{-34} \text{ J s}$                     |
| $k$       | - | Boltzmann's constant | $1.38 \times 10^{-23} \text{ J }^{\circ}\text{K}^{-1}$ |
| $L_{\nu}$ | - | spectral radiance    | $\text{W m}^{-2} \text{ Hz}^{-1} \text{ sr}^{-1}$      |

At long  $\lambda$  (i.e., low  $\nu$ )  $kT \gg h\nu$  so we can expand the exponential in the denominator and keep only the largest terms.

$$\left[ \exp \frac{h\nu}{kT} - 1 \right] = \left[ 1 + \frac{h\nu}{kT} + \frac{1}{2!} \left( \frac{h\nu}{kT} \right)^2 + \dots - 1 \right] \cong \frac{h\nu}{kT}$$

$$L_{\nu} \cong \frac{2h\nu^3}{c^2} \frac{kT}{h\nu} = \frac{2kT}{c^2} \nu^2 = \frac{2kT}{\lambda^2}$$

This is the Rayleigh-Jeans approximation that we developed earlier in the course. It is also the form of the blackbody radiation curve that was developed using purely classical physics.

- (show tungsten radiation curves and Rayleigh-Jeans approximation)

This approximate theory has an accuracy of better than 1% for an object at 300°K viewed at a frequency less than 125 GHz. Most radiometric applications operate in the 10 GHz region where the Rayleigh-Jeans approximation is VERY accurate.

A radiometer can be used to measure spectral radiance. For gray bodies, spectral radiance is reduced because of the emissivity is less than 1.

$$L_{\nu} = \frac{2k}{\lambda^2} T_b = \epsilon \frac{2k}{\lambda^2} T_p$$

|                      |   |                           |   |            |                         |
|----------------------|---|---------------------------|---|------------|-------------------------|
| observed<br>radiance | = | brightness<br>temperature | = | emissivity | physical<br>temperature |
|----------------------|---|---------------------------|---|------------|-------------------------|

This can be rewritten as

$$T_b = \epsilon T_p.$$

Suppose we were to measure the brightness temperature of a surface. If the physical temperature of the object was known, we could infer the emissivity. (This is the case of the radiometer measurements from the Magellan spacecraft where the temperature of the surface of Venus is very well known; one infers the emissivity of the rocks which depends on both the rock type and the roughness of the surface.)

If the emissivity of the surface is known then one can measure the brightness temperature and infer the physical temperature. This is the case of the ocean surface where the emissivity is largely known although it depends on sea state.

- (show figure 7.9 from Rees)

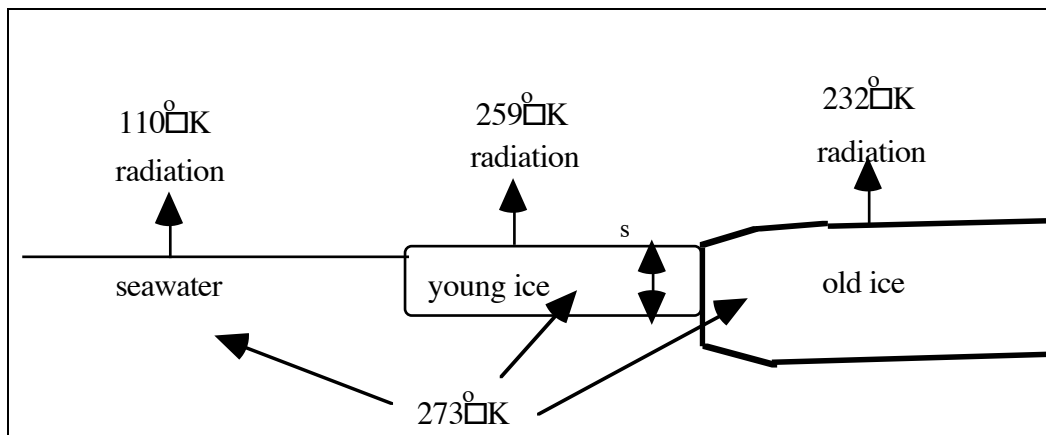
### Example: Polar Ocean

Suppose we measure the thermal emissions at 10 GHz in a polar ocean where there is a mixture of open seawater, young sea ice, and old sea ice. It is a warm day so both the ice and water are at the melting point. At 10 GHz ( $\sim 3$  cm), the E/M waves penetrate about 1 mm into the seawater and about 1 m into the ice.

- (show figure 7.9 from Rees)

|                   |             |      |
|-------------------|-------------|------|
| The emissivity of | seawater is | 0.4  |
|                   | young ice   | 0.95 |
|                   | old ice     | 0.85 |

The brightness temperature observed by the radiometer aboard the spacecraft will reflect the variations in the emissivity of the surface. This is an excellent way to monitor the ice cover of the polar oceans and discriminate first-year ice from multi-year ice.



### Resolving Power of Antennas

As stated above, microwave have three advantages over optical and infrared systems:

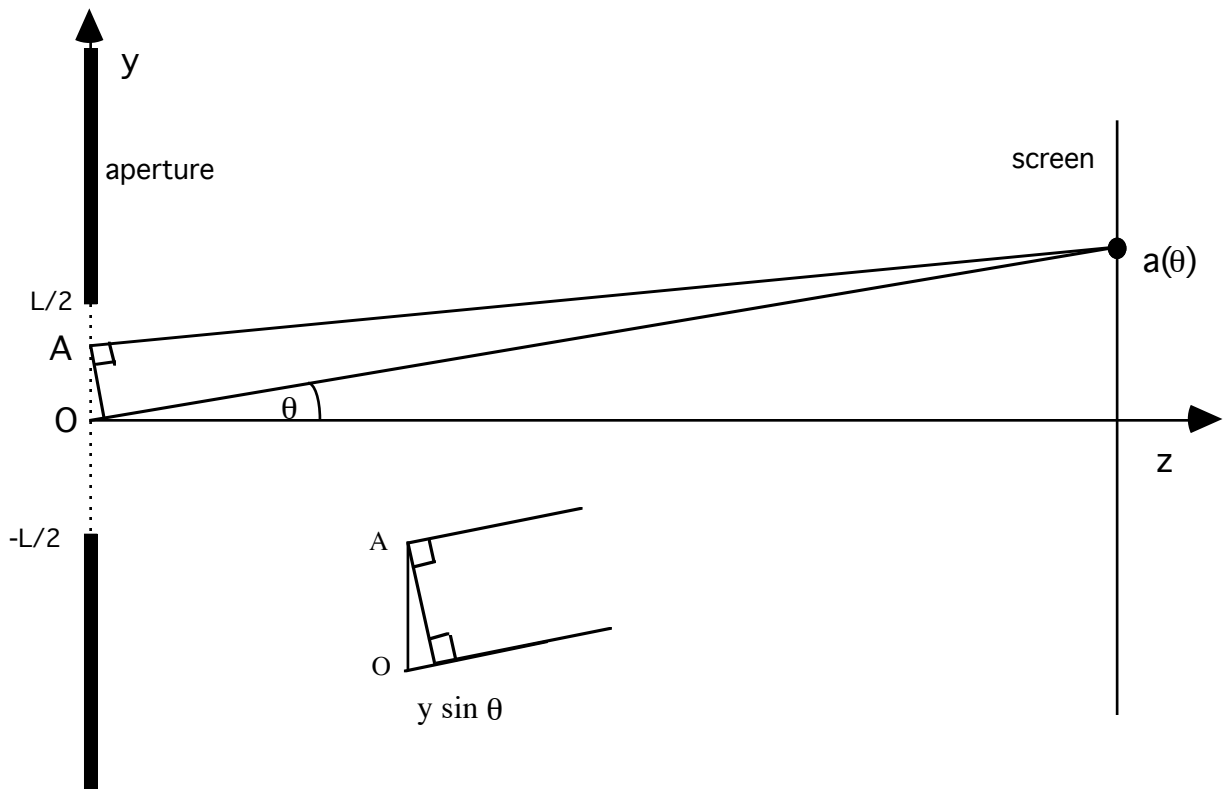
- 1) reflected sunlight is less of a problem;
- 2) for wavelengths greater than a few centimeters, the atmosphere is very transparent;
- 3) the Rayleigh-Jeans formula provides a simple linear relationship between the spectral radiance observed at the sensor and the brightness temperature of the surface so the results are quantitative.

The main disadvantage of operating at such long wavelength  $\lambda$  is that the spatial resolution is poor for reasonable sized antennas. A second disadvantage is that since we are far out on the tail of the Planck radiation curve, the signal from thermal emissions from the Earth are very low. (Note that at Venus the surface temperature ( $750^\circ\text{K}$ ) is nearly three times greater than the Earth's surface temperature so the spectral radiance will be nearly three times larger.) Both of these limitations require large antennas. However in addition to size, care must be taken to reduce the sidelobes of the antenna pattern to minimize unwanted signals.

Stewart [1985] identifies some of the additional major issues with measuring the sea surface temperature (SST) to an accuracy of  $1^\circ\text{K}$ .

- 1) ocean areas must be more than 300 km from any land or the hot land surface will be viewed through one of the sidelobes of the antenna beam pattern.
- 2) radiometer must be out of range of terrestrial transmitters operating in the same frequency range. This is becoming more of a problem with the blossoming of the global cellular phone networks.

Earlier in the course, we developed the amplitude pattern for an antenna or aperture of width  $L$ . Next we'll briefly re-do that calculation to highlight the problems with the sidelobes of the antenna. Then we'll discuss the methods for reducing the sidelobes and finally we discuss electronically steered antennas and phased arrays. As mentioned earlier in the course, the transmit and receive patterns of antennas are the same. For passive radiometers, we want to calculate the receive power pattern but it is easier to think of the transmit problem.



The amplitude of the E/M field at a point on the screen is the complex sum of the contributions from the point sources.

$$a(\theta) = \int_{-\infty}^{\infty} f(y) e^{-iky \sin \theta} dy$$

let  $s = k \sin \theta$

$$a(s) = \int_{-\infty}^{\infty} f(y) e^{-isy} dy$$

**Case 1. Uniform amplitude transmittance function**

$$f(y) = \begin{cases} 0 & y > \frac{L}{2} \\ 1 & -\frac{L}{2} < y < \frac{L}{2} \\ 0 & y < -\frac{L}{2} \end{cases}$$

Then we can do the integral.

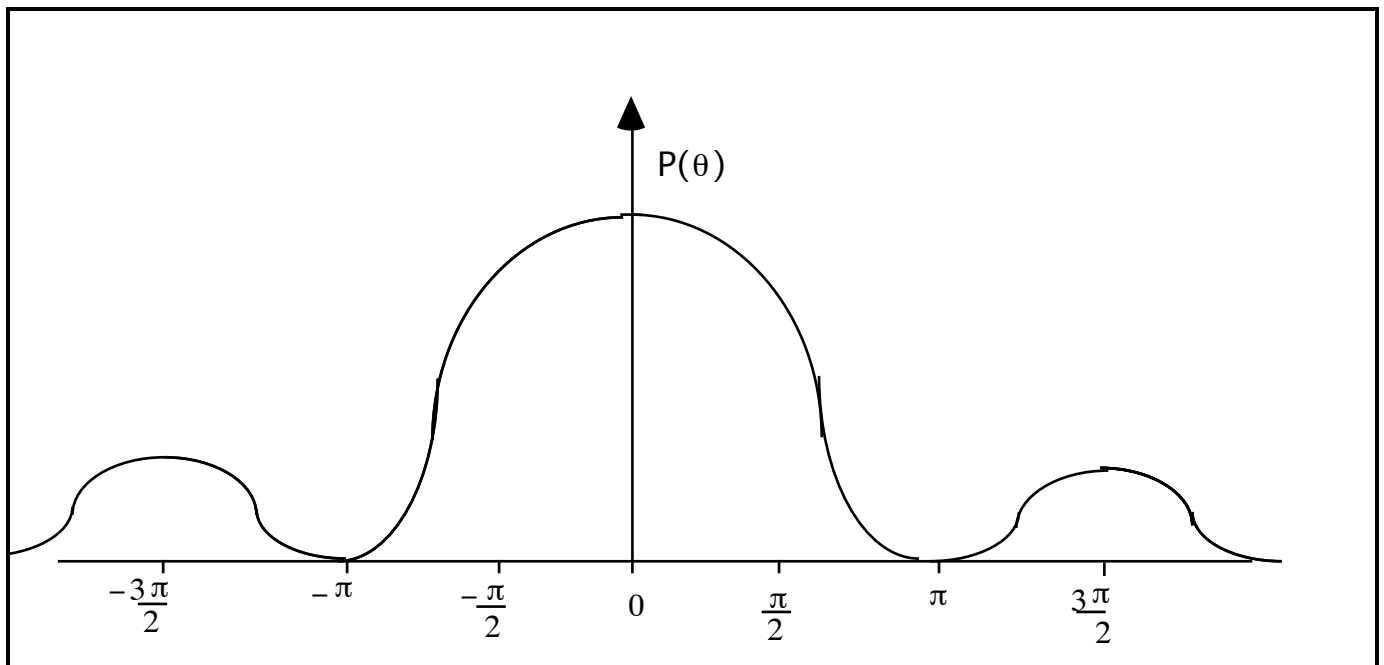
$$a(s) = \int_{-\frac{L}{2}}^{\frac{L}{2}} f(y) e^{-isy} dy = L \frac{\sin\left(\frac{sL}{2}\right)}{\left(\frac{sL}{2}\right)}$$

So the amplitude and power  $P(\theta)$  are given by

$$a(\theta) = L \frac{\sin\left(\frac{kL}{2} \sin \theta\right)}{\left(\frac{kL}{2} \sin \theta\right)} = L \operatorname{sinc}\left(\frac{L}{2\lambda} \sin \theta\right)$$

$$P(\theta) = L^2 \operatorname{sinc}^2\left(\frac{L}{2\lambda} \sin \theta\right)$$

A plot of the power radiated on the screen looks like the following



The first zero crossing occurs when  $\frac{L}{2\lambda} \sin \theta = \pi$  or  $\sin \theta = \frac{2\pi\lambda}{L}$ . The half power point is found by  $\frac{1}{2} = \text{sinc}^2 \left( \frac{L}{2\lambda} \sin \theta_{1/2} \right)$  which corresponds to  $\theta_{1/2} = 83. \frac{\lambda}{L}$  (degrees). The power of the first sidelobe is  $\text{sinc}^2 \left( \frac{3\pi}{2} \right) = \frac{1}{22.2} = -13.4$  dB.

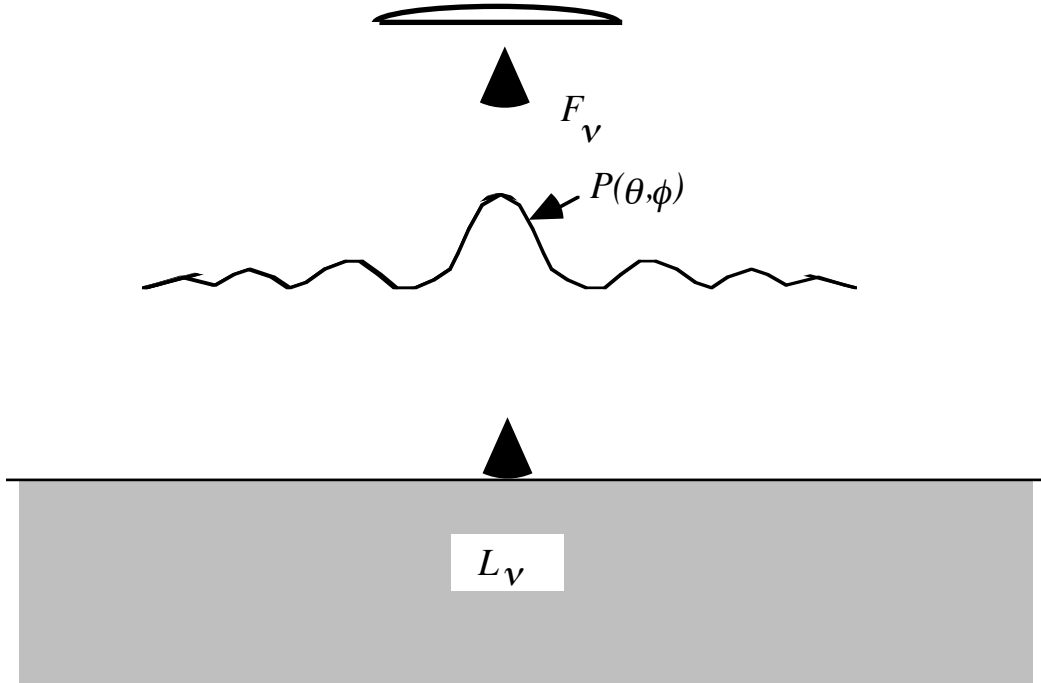
## Case 2. Cosine amplitude transmittance function

$$a(s) = \int_{-\frac{L}{2}}^{\frac{L}{2}} \cos\left(\frac{\pi y}{L}\right) e^{-isy} dy$$

- (show Figure 9.2 from Stewart, [1985])

A well designed antenna has sidelobes of -20dB or lower. A box transmittance function produces sidelobes of -13dB while a cosine transmittance function has lower sidelobes of -23 dB. However, the side lobes are reduced at the expense of reducing the angular resolution of the main lobe.

Now consider this antenna pointing toward the Earth. The antenna is designed to observe radiation at a given frequency over a narrow frequency range (bandwidth)  $\nu \pm \frac{\Delta\nu}{2}$ .



Under the Rayleigh-Jeans approximation, the spectral radiance is related to the physical temperature of the surface  $T_p$  and the emissivity  $\epsilon$ .

$$L_{\nu} = \frac{2k}{\lambda^2} T_b = \epsilon \frac{2k}{\lambda^2} T_p$$

The brightness temperature  $T_b$  is the product of the physical temperature and the emissivity. Notice the spectral flux density received by the antenna  $F_{\nu}$  has units of  $\text{Wm}^{-2}\text{Hz}^{-1}$  while the spectral radiance  $L_{\nu}$ , emitted by the Earth, has units of  $\text{Wm}^{-2}\text{Hz}^{-1}\text{sr}^{-1}$ . Thus we need to integrate over the power pattern of the antenna to equate the two energy measures.

$$F_{\nu} = \frac{2k}{\lambda^2} \int_{4\pi} T_b(\theta, \phi) P(\theta, \phi) \sin \theta d\theta d\phi$$

We can define something called the antenna temperature as



$$T_A = \frac{\int_{4\pi} T_b(\theta, \phi) P(\theta, \phi) d\Omega}{\int_{4\pi} P(\theta, \phi) d\Omega} .$$

Note this is the integral of the weights times the brightness temperature divided by the integral of the weights.

### Sensitivity

It is customary to express the power per bandwidth reaching the antenna in terms of the antenna temperature  $T_A$  although this is not really the physical temperature of the antenna. If the antenna is not at absolute zero, then it will emit thermal radiation into its own microwave detector. This can be considered as thermal noise. Ideally we would like to have an antenna with a reflectivity of 1 and an emissivity of 0 so that no thermal noise is emitted into the antenna feed.. The physical temperature of the antenna times the emissivity of the antenna is called the system temperature  $T_{sys}$ .

To improve the signal to noise ratio of a passive microwave sensor, the measurements are averaged over a period of time  $\Delta t$ . The number of independent measurements for this time interval is  $N = \Delta\nu \Delta t$  . For a typical bandwidth of 200 MHz and an integration time of 0.1 sec, there are 20 million independent measurements. This will reduce the noise by  $\sqrt{N}$  or 5000 times for this example. The sensitivity of the radiometer to temperature changes  $\Delta T$  is

$$\Delta T = C T_{sys} (\Delta\nu \Delta t)^{-1/2}$$

where  $C$  is a dimensionless constant having a value of 5-10 depending on the antenna and electronics. The absolute calibration of the radiometer is achieved by looking at a known source or out into cold space at the microwave background radiation.