PLATFORMS FOR REMOTE SENSING

(Rees Chapter 10)

Platforms

Chapter 10 of Rees is placed at the end of the book since he follows the flow of information from the object to be sensed, through the propagating medium, to the sensor, through the satellite, and finally to the end user. I prefer to discuss platforms first because this discussion provides an overview of the types of remote sensing strategies that are possible.

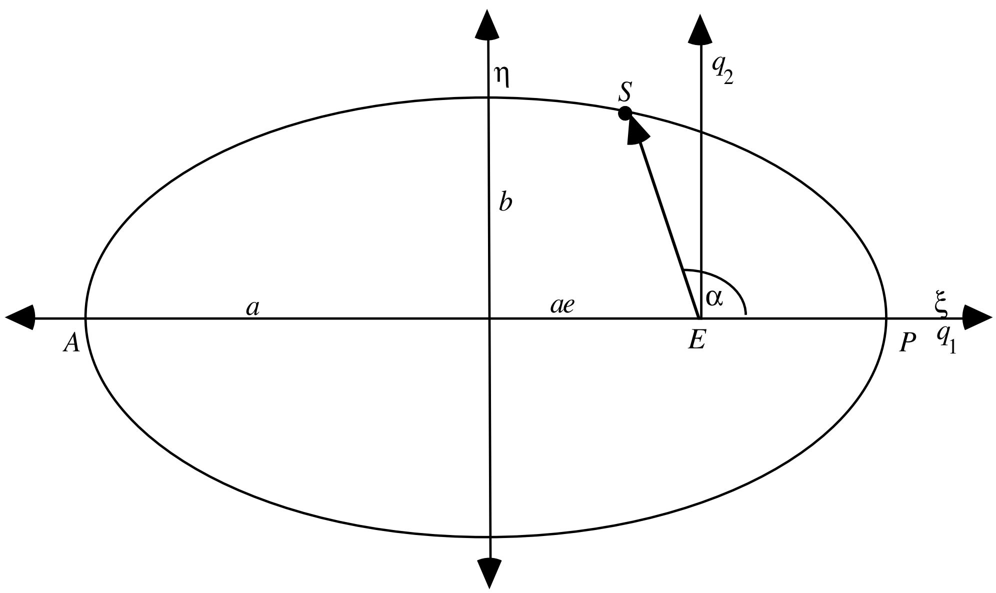
In chapter 10, Rees presents only the final results and equations for simple orbital geometries. Today, I'll include some derivations to highlight some of the physics of orbits. In particular I'll focus on nearly circular orbits that are quite common and also easy to treat mathematically.

Rees notes: "The spatial and temporal variability of the phenomenon to be studied will determine the observing strategy to be employed, and this will have an influence on the choice of operational parameters in the case of an airborne observation or on the orbital parameters in the case of a satellite-borne sensor". Clearly, these considerations will also place limits on the type of sensor to be employed.

The failures of the early designs of Earth Observing System (EOS) were related to a lack of understanding of how each application can require vastly different platform, orbit, and sensor characteristics. The rationale for placing all of the sensors on the same platform was to obtain data from a variety of sensors that would be co-registered in both space and time. While this was a worthy goal, it became obvious to the scientists who would use the data that each sensor has an optimal orbit. For example, a sun-synchronous orbit is best for passive systems relying on uniform illumination direction but this type of orbit is optimally bad for an active sensor like a satellite altimeter because the solar tide is perfectly aliased into the measurements. Moreover, during the 10 years since the original EOS design, it has become possible to track low-orbiting satellites using the constellation of GPS satellites. This has been demonstrated in the case of Topex where orbits having accuracies of a few centimeters are now possible. Thus precise image alignment can be obtained through a precise knowledge of the spacecraft position and attitude.

Satellite Orbits

The geometry of a satellite in an elliptical orbit around the Earth is shown below. These notes are largely derived from a book by Kaula, W. M., *Theory of Satellite Geodesy*, Blaisdell Publishing Co., Waltham, Mass, 124 pp.,1966.



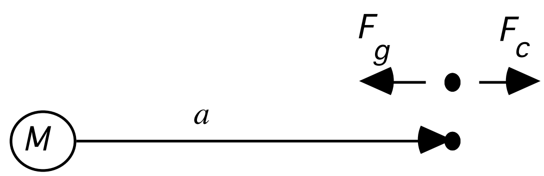
S	-	satellite
E	-	Earth
A	-	apogee
P	-	perigee
a	-	semimajor axis
b	-	semiminor axis
e	-	eccentricity

$$e^2 = \frac{a^2 - b^2}{a^2}$$

Kaula (1966) solves the general force balance for a particle of negligible mass in orbit about a point mass *M*. He shows that the orbit follows and ellipse where the angular momentum $r^2\, d\alpha/dt$ remains constant.

$$\frac{\xi^2}{a^2} + \frac{\eta^2}{b^2} = 1 \quad \Rightarrow \quad r = \frac{a(1-e^2)}{(1 + e\cos\alpha)} \qquad \text{check} \Rightarrow \alpha = 0 \Rightarrow r = a\frac{(1+e)\,(1-e)}{(1+e)}$$

For an elliptical orbit about a point mass, the frequency ω is given by $\omega = \left(\frac{GM}{a^3}\right)^{1/2}$ and the period of the orbit is $P = {}^{2\pi}\!/_{\omega}$ where GM, the gravitational constant times the mass of the Earth. GM (~3.98x10^{14} m^3 s^{-2}) is one of the most accurately determined constants in Earth science. We can check this result for the simple case of a circular orbit. Consider a force balance on a small mass m orbiting the Earth. The inward directed force is the force of gravity F_g while the outward directed force is the centrifugal acceleration F_c.

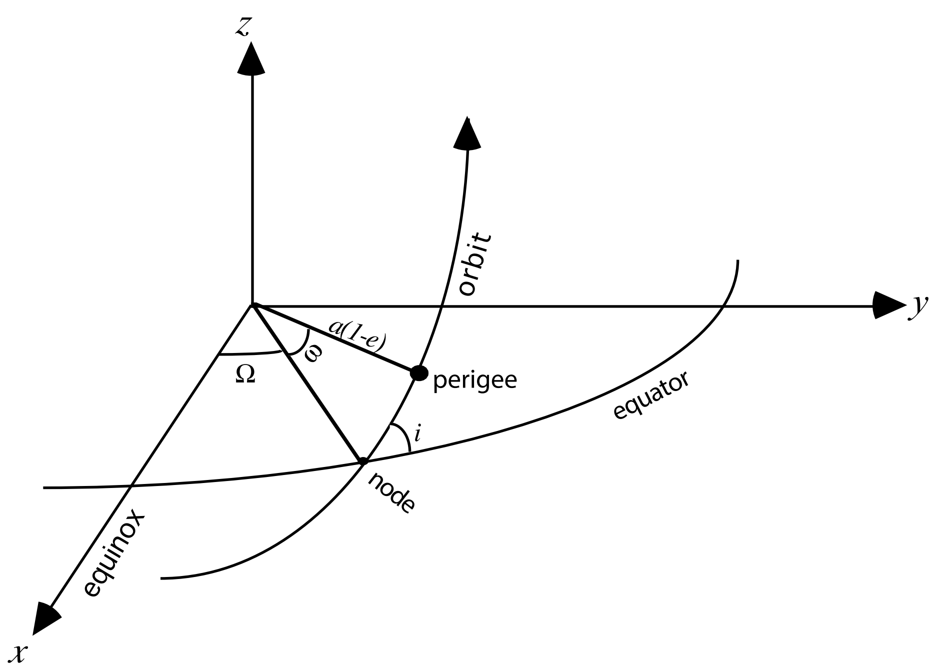


$F_g = m\frac{GM}{a^2}$ and $F_c = m\frac{V^2}{a} = ma\omega^2$ where V is the velocity of the satellite. Equating these forces we find $\omega^2 = \frac{GM}{a^3}$. Note that the orbital frequency increases with decreasing orbit radius a.

Orbital Geometry

The diagram below shows the elliptical satellite orbit places in an Earth-fixed coordinate system.
A complete description of an ideal elliptical orbit is described by 6 Keplerian elements:

α - true anomaly (instaneous angle from satellite to perigee)
ω - argument of perigee
Ω - longitude of node
a - semimajor axis
e - eccentricity
i - inclination



Complications due to Earth Flattening

Because the earth rotates, it is not spherical and is better described by an oblate spheroid. The gravitational attraction of an oblate spheroid has two effects on the orbit. First it decreases the orbit frequency (increases the period). Second it causes the orbit plane to precess with respect to the inertial frame. A more precise description of the Earth's gravity field is

$$V(r,\theta) = \frac{-GM}{r}\left[1 - a_e^2 \frac{J_2}{2r^2}\left(3\sin^2\theta - 1\right)\right]$$

θ - latitude
a_e - equatorial radius
J_2 - dynamic form factor = 1.08×10^{-3}

The nodal period (i.e., the time between successive ascending equator crossings) increases because of the J_2 term in the gravity field.

$$\omega_s = \left(\frac{GM}{a^3}\right)^{1/2}\left[1 + \frac{3J_2}{4}\left(\frac{a_e}{a}\right)^2\left\{\left(1 - 3\cos^2 i\right) + \frac{\left(1 - 5\cos^2 i\right)}{(1-e^2)^2}\right\}\right]^{-1}$$

The second effect is to cause the orbit plane to precess like a top. The precession occurs because the equatorial bulge of the gravity field exerts a torque on the angular momentum vector of the orbit plane. The precession frequency ω_n is given by

$$\frac{\omega_n}{\omega_s} = -\frac{3J_2}{2}\left(\frac{a_e}{a}\right)^2\frac{\cos i}{\left(1-e^2\right)^2}$$

Note the precession is retrograde for $i < 90^\circ$.

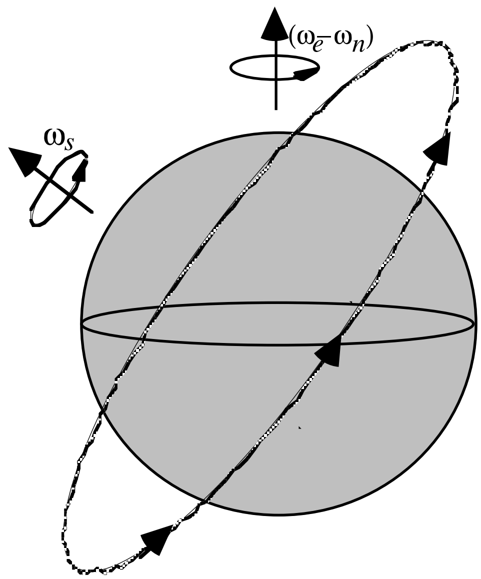
Ground Track of Circular Orbit

In many remote sensing applications we would like to predict the path traced by the subsatellite point along the surface of the Earth. This could be accomplished by running an orbit simulation program. However, in many cases one only needs to know the approximate location or velocity of the satellite.

a) Suppose you know the basic orbit parameters (Keplerian elements) of a Landsat-type satellite and you want to know how long it will take before the satellite is over your target area.

b) You have satellite altimeter profiles and you know the longitudes of their equator crossings. Given an equator crossing you would like to know if the track intersects a particular area; this can be used to design an efficient way to search profiles. A related application is to align altimeter profiles along tracks so they can be differences to look for ocean variability. A crude alignment can be done using Keplerian elements.

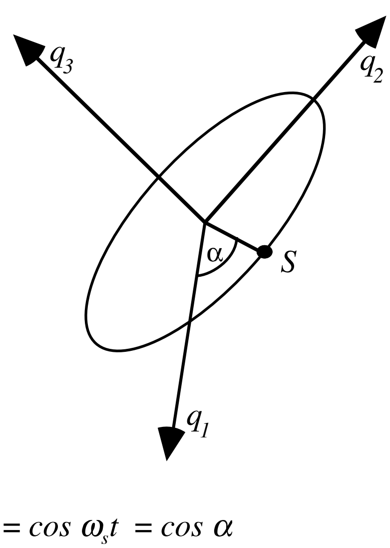
Many remote sensing satellites have nearly circular orbits ($e < 0.01$) so we'll assume $e = 0.0$. Since the orbit is circular the argument of perigee ω is no longer relevant.



ω_s	-	satellite orbit frequency
ω_e	-	Earth rotation frequency $= 2\pi/86164$
ω_n	-	precession frequency

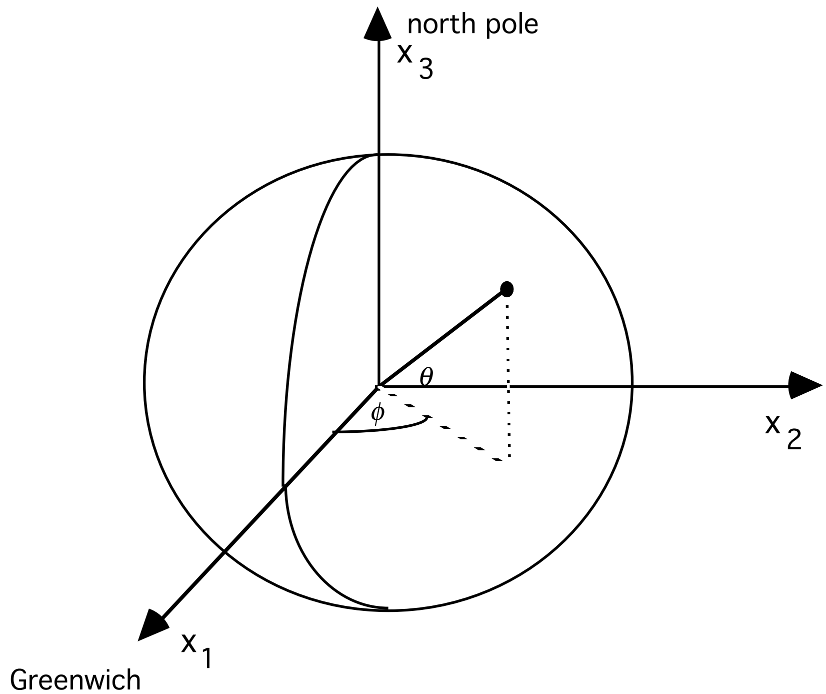
The basic problem is to map the position of a satellite track in a circular orbit (inertial frame) into an Earth-fixed coordinate system. Let $t = 0$ be the time when the satellite crosses the Earth's equator on an ascending orbit at a longitude of ϕ_O.

satellite frame



$$q_1 = cos\ \omega_s t\ = cos\ \alpha$$
$$q_2 = sin \omega_s t\ = sin\ \alpha$$
$$q_3 = 0$$

Earth-fixed frame



$x_1 = cos\ \theta\ cos\ \phi$

$x_2 = cos\ \theta\ sin\ \phi$

$x_3 = sin\ \theta$

Two rotations are needed to align the satellite frame **q** to the Earth-fixed frame **x**. First, the **q**-frame is rotated by an angle $-i$ about the q_1 axis to account for the inclination of the orbit plane with respect to the Earth's equatorial plane. Second, the system is rotated about the #3 axis to account for the Earth's rotation rate minus the precession rate of the orbital plane in inertial space.

Let $\Omega = (\omega_e - \omega_n)\ (t - t_o)$ then the satellite position in the **x**-frame is related to the satellite position on the **q**-frame as follows

$$\mathbf{x} = \mathbf{R}_3[+\Omega]\ \mathbf{R}_1[-i]\ \mathbf{q}$$

By performing the rotations explicitly, one can derive analytical expressions for mapping from the satellite to the Earth-fixed frame and vice versa.

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \cos\Omega & \sin\Omega & 0 \\ -\sin\Omega & \cos\Omega & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos i & \sin -i \\ 0 & \sin i & \cos i \end{bmatrix} \begin{bmatrix} \cos\alpha \\ \sin\alpha \\ 0 \end{bmatrix}$$

Multiplying matrices yields

$$\cos\theta \cos\phi = \cos\Omega \cos\alpha + \sin\Omega \cos i \sin\alpha$$

$$\cos\theta \sin\phi = -\sin\Omega \cos\alpha + \cos\Omega \cos i \sin\alpha$$

$$\sin\theta = \sin i \sin\alpha$$

The last equation relates latitude to time since the equator crossing and vice versa.

$$\theta(t) = \sin^{-1}\left[\sin\omega_s t \ \sin i\right] \qquad t(\theta) = \omega_s^{-1} \sin^{-1}\left[\frac{\sin\theta}{\sin i}\right]$$

The cosine and sine of the longitude (relative to ϕ_o) at some later time are given by

$$\cos\phi = \left[\frac{\cos\omega_e' t \ \cos\omega_s t + \sin\omega_e' t \ \sin\omega_s t \ \cos i}{\cos\theta}\right] \quad \text{and}$$

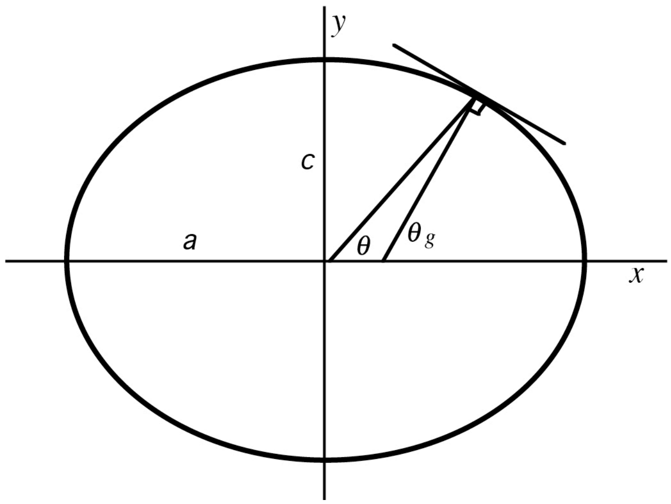
$$\sin\phi = \left[\frac{-\sin\omega_e' t \ \cos\omega_s t + \cos\omega_e' t \ \sin\omega_s t \ \cos i}{\cos\theta}\right]$$

where ω_e' is Earth rotation rate minus the orbit precession. By combining these two expressions, the longitude at a later time is

$$\phi(t) = \tan^{-1}\left[\frac{-\sin\omega_e' t \ \cos\omega_s t + \cos\omega_e' t \ \sin\omega_s t \ \cos i}{\cos\omega_e' t \ \cos\omega_s t + \sin\omega_e' t \ \sin\omega_s t \ \cos i}\right] + \phi_o$$

Another Minor Complication

Throughout this derivation we used *geocentric latitude* θ which is the angle between the equator and a line from the center of the Earth to the point on the surface of the Earth. However, because the way latitude was measured prior to having artificial satellites, *geographic latitude* must be used instead. The conversion between geocentric and geographic latitude depends on the flattening of the Earth *f* as shown in the following diagram.



The spheroid or radius of the Earth versus latitude is $R(\theta) = \left(\frac{\cos^2\theta}{a^2} + \frac{\sin^2\theta}{c^2}\right)^{-1/2}$ where

a	-	equatorial radius	= 6378135 m
c	-	polar radius	= 6356775 m

and the flattening *f* of the Earth is $f = \frac{a - c}{a} \cong \frac{1}{298}$.

Before satellites were available for geodetic work, one would establish latitude by measuring the angle between a local plumb line and an external reference point such as Polaris. Since the local plumb line is perpendicular to the local flattened surface of the Earth, it points to one of the foci of the best fitting elliptical model for the shape of the Earth.

The conversion between geocentric and geographic latitude is straightforward and is left as an exercise. The formulas are

$$\tan\theta = \frac{c^2}{a^2}\tan\theta_g \quad or \quad \tan\theta = (1 - f)^2 \tan\theta_g.$$

To derive this equation one starts with the equation for an ellipse

$$\frac{x^2}{a^2}+\frac{y^2}{c^2}=1\ .$$

Then take the derivative of this equation with respect to *x* and rearrange terms.

(Example: What is the geocentric latitude at a geographic latitude of 45˚. The answer is $\theta =$ 44.8˚ which amounts to a 22 km difference in location!)

Special Orbits

This simple circular orbit calculation provided the mathematical tools to describe several commonly used orbits.

$$\theta(t) = sin^{-1}\left[sin\ \omega_s t\ \ sin\ i\right] \qquad t(\theta) = \omega_s^{-1}\ sin^{-1}\left[\frac{sin\ \theta}{sin\ i}\right]$$

$$\phi(t) = tan^{-1}\left[\frac{-sin\ \omega_e' t\ \ cos\ \omega_s t\ + cos\ \omega_e' t\ \ sin\ \omega_s t\ \ cos\ i}{\cos\ \omega_e' t\ \ cos\ \omega_s t\ + sin \omega_e' t\ \ sin\ \omega_s t\ \ cos\ i}\right] + \phi_o$$

1) Zero Inclination Orbit $i = 0$, $\theta = 0$ and the orbit precession frequency is zero.
Using formulas for the sin and cosine of sums of angles (e.g., sin (a-b) = sin a cos b - cos a sin b) one can simplify the longitude function to the obvious result

$$\phi(t) = tan^{-1}\left[\frac{sin\ (\omega_s - \omega_e)t}{cos\ (\omega_s - \omega_e)t}\right] = (\omega_s - \omega_e)t$$

2) Geostationary Orbit $i = 0$, $e = 0$, a_e = 6378135m
Again the satellite orbit frequency matches the Earth rotation rate $\omega_s = \omega_e$. From the equation above we have an expression for the orbit frequency for a flattened Earth.

$$\omega_s = \left(\frac{GM}{a^3}\right)^{1/2}\left[1 - \frac{9J_2}{2}\left(\frac{a_e}{a}\right)^2\right]^{-1} \qquad \omega_e = 2\pi/86146$$

The radius of the requires orbit is a = 42170 km or about 6.6 times the Earth radius. Usually this type of orbit is used for communications or for monitoring the weather patterns from a global perspective. Coverage of this type of orbit is a small circle of radius 55° centered on the subsatellite point. About 6 satellites are needed to provide a complete equatorial view of the Earth and these orbits are not used for high-latitude coverage.

3) Geosynchronous Orbit $i \neq 0, e = 0, \omega_s = \omega_e$

With a non-zero inclination, this orbit can cover higher latitudes but is spends only 1/2 of its time in the correct hemisphere. Inserting these parameters into the above ground track equations and neglecting the precession frequency of the orbit plane one obtains

$$\theta(t) = \sin^{-1}\left[\sin \omega_e t \ \sin i\right]$$

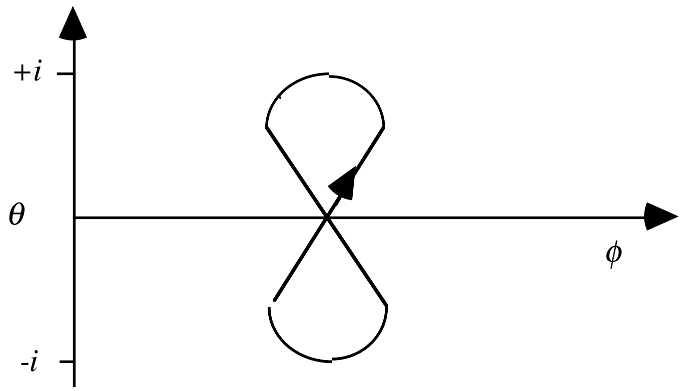
$$\phi(t) = \tan^{-1}\left[\frac{\cos \omega_e t \ \sin \omega_e t \ (\cos i - 1)}{\cos^2 \omega_e t + \cos i \sin^2 \omega_e t}\right]$$

Now consider the case of small inclination so $\cos i \cong 1 - i^2/2$. The denominator is about 1 and the numerator can be simplified so the approximate results for longitude and latitude versus time are

$$\phi(t) = \tan^{-1}\left[\frac{i^2}{2}\frac{1}{2}\sin 2\omega_e t\right] \cong \frac{i^2}{4}\sin 2\omega_e t$$

$$\theta(t) = \sin^{-1}\left[\sin \omega_e t \ \sin i\right] \cong i \sin \omega_e t$$

The latitude varies as a sine wave with a frequency of ω_e while the longitude varies like a sine wave with a frequency of $2\omega_e$. At t = 0 both the latitude and longitude are zero. The ground track of the orbit follows a figure 8.



This orbit spends less than 1/2 of its time at a high latitude in the correct hemisphere. To optimize the time spent in the northern hemisphere, the Soviets developed a special type of orbit called a Molniya orbit. It is highly elliptical with $\omega_s = 2\omega_e$. Apogee placed spent over the former Soviet Union where the satellite spends 92% of its time.

4) Sun-synchronous Orbit

For many remote sensing applications it is important to have the ascending node pass over the equator at the same local time. To create a sun-synchronous orbit, the plane of the orbit must precess in a prograde direction with a period of exactly 1 year.

$\omega_n = 2\pi/(365.25 \text{ x } 86400) = 1.991 \text{ x } 10^{-7} \text{ s}^{-1}$

Remember $\frac{\omega_n}{\omega_s} = -\frac{3J_2}{2}\left(\frac{a_e}{a}\right)^2 \frac{\cos i}{\left(1-e^2\right)^2}$ so prograde precession requires $i > 90°$. Thus the orbital inclination is dictated by the orbital altitude. For example, a sun-synchronous orbit with an orbital radius of a = 7878 km (altitude 1500 km) must have an inclination of 102°.

5) Orbits Tuned for Ocean Altimetry

Criteria:

a) Ascending and descending tracks should cross at a high angle to resolve both components of sea surface slope.

b) One should be aware of the aliasing of Lunar and Solar tides into the altimeter profile.

c) One should choose a repeat cycle that will optimize spatial and temporal coverage.

d) High latitude coverage may be desired for ice altimetry.

Example: Geosat Altimeter

The Geosat radar altimeter was launched in 1985 to measure the topography of the ocean surface in support of the Trident submarine program. The main objective of the first 1.5 years of the mission was to obtain complete marine coverage with tracks that cross at high angles for optimum gravity field recovery. An inclination of 108˚ provides the best compromise (see Fig 9.9 from Rees). However, this retrograde orbit is nearly sun-synchronous which leads to aliasing of daily solar tide into a much longer period of time.

After Geosat completed its 1.5 year mapping mission, it began an extended repeat mission to observe changes in ocean topography associated with mesoscale currents. To measure changes it was placed in an orbit having a ground track that repeats every 17.05 days. Of course this was not optimal in terms of aliasing Lunar and Solar tides.

tide		period - hours	period - days	phase shift after 17.05 days
M2	principal lunar	12.421	.5175	32.95 = 20˚
K^1	lini-solar	23.934	.997	34˚
S2	principal solar	12.00	.500	36˚
O1	diurnal lunar	25.819	1.076	54˚

The Topex orbit was designed to minimize tidal aliasing. It has a repeat cycle of 10 days.

6) Exact Repeat Orbits

For oceanographic applications, where one wants to observe changes in ocean topography, the orbit must retrace its ground track so the large permanent signals associated with the Earth's gravity field can be subtracted out. To accomplish this there must be an integer relationship between the orbit frequency and the rotation rate of the Earth relative to the precessing orbit plane.

$2\pi/(\omega_e - \omega_n)\ n_1$ = time for n_1 rotations of the Earth relative to the orbit plane

$2\pi/\omega_e\ \ n_2$ = time it takes the satellite to complete n_2 orbits

$$\frac{2\pi}{\omega_e - \omega_n} n_1 = \frac{2\pi}{\omega_s} n_2 \Rightarrow \frac{\omega_e - \omega_n}{\omega_s} = \frac{n_1}{n_2}$$

Geosat Example,

$\omega_e = 7.292 \times 10^{-5}\ s^{-1}$
$\omega_n = 4.144 \times 10^{-7}\ s^{-1}$
$\omega_s = 1.041 \times 10^{-3}\ s^{-1}$

$$\frac{\omega_e - \omega_n}{\omega_s} = \frac{17}{244}$$